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Modeling Epileptic Seizure: An Alternative Perspective Through Integral Equation Reformulation

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نمذجة النوبة الصرعية: منظور بديل من خلال إعادة صياغة المعادلة التكاملية

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ABSTRACT

Epileptic seizures, characterized by abnormal electrical activity in the brain, pose a significant challenge for diagnosis and treatment. Electroencephalography (EEG), measuring the brain's electrical activity, plays a crucial role in understanding these events. However, analysing epileptic EEG signals, often resembling a complex storm of electrical activity, can be challenging. Usually, these signals have been modelled using integral equations, but the limitations remain in capturing the full dynamics of seizures. This study proposes a novel approach to analysing EEG data during epileptic seizures by reformulating the integral equation. Instead of viewing the chaotic patterns as mere noise, the study proposes an alternative perspective that delves deeper into the underlying structure. By reformulating the integral equation, it is aimed at to capturing not just the individual components of the electrical activity but also the intricate relationships between them, potentially revealing hidden patterns and dynamics associated with epileptic seizures. This new approach holds the potential for a deeper understanding of seizure mechanisms. By uncovering the hidden order amidst the seemingly chaotic EEG signals, we might gain valuable insights into the brain's behaviour during seizures. This understanding could pave the way for enhanced diagnostics, more effective treatment strategies, and ultimately, better outcomes for patients suffering from epilepsy.

Keywords: Epileptic seizures, EEG signals, Integral Equation, convolution equation.

الملخص

تشكل النوبات الصرعية والتي تتميز بنشاط كهربائي غير طبيعي في الدماغ تحديًا كبيرًا للتشخيص والعلاج. يلعب جهاز تخطيط كهربية الدماغ (EEG) الذي يقيس النشاط الكهربائي للدماغ دورًا مهمًا في فهم هذه الأحداث. ومع ذلك، يمكن أن تكون تحليل إشارات تخطيط كهربية الدماغ الصرعية والتي غالبًا ما تشبه عاصفة معقدة من النشاط الكهربائي أمرًا صعبًا وعادةً ما يتم نمذجة هذه الإشارات باستخدام المعادلات التكاملية، ولكن لا تزال هناك قيود في التقاط الديناميكيات الكاملة للصرع. تقترح هذه الدراسة نهجًا جديدًا لتحليل بيانات تخطيط كهربية الدماغ أثناء النوبات الصرعية عن طريق إعادة صياغة المعادلة التكاملية بدلاً من النظر إلى الأنماط الفوضوية على أنها مجرد ضوضاء، إننا نقترح منظورًا بديلاً يتعمق في البنية الأساسية من خلال إعادة صياغة المعادلة التكاملية، كما نهدف إلى التقاط ليس فقط المكونات الفردية للنشاط الكهربائي ولكن أيضًا العلاقات المعقدة بينها، مما قد يكشف عن أنماط وديناميكيات مخفية مرتبطة بالنوبات الصرعية. يوفر هذا النهج الجديد إمكانية فهم أعمق لآليات النوبات. من خلال الكشف عن النظام الخفي وسط إشارات تخطيط كهربية الدماغ التي تبدو فوضوية، فقد نكتسب رؤى قيمة حول سلوك الدماغ أثناء النوبة الصرعية. يمكن أن يمهد هذا الفهم الطريق لتحسين التشخيص واستراتيجيات علاج أكثر فعالية، وفي النهاية نتائج أفضل للمرضى الذين يعانون من الصرع.

كلمات مفتاحية: النوبة الصرعية، جهاز تخطيط كهربية الدماغ (EEG)، معادلة تكاملية، معادلة الالتفاف.

1. INTRODUCTION

Epileptic seizures, a diverse symphony of abnormal brain activity, present a spectrum of challenges in diagnosis and management, from dramatic convulsions to subtle lapses in awareness, their manifestations demand precise identification [1]. a non-invasive technique capturing the brain's electrical whispers, serves as a crucial window for diagnosis [2]. During seizures, the rhythmic EEG patterns morph into distinctive signatures, offering valuable markers [3].

EEG data, a treasure trove of information, offer glimpses into the timing and location of underlying neural activity [4]. Analysing these signals through tools like chaos theory helps unveil hidden pathological patterns within the epileptic brain [5]. However, traditional methods often rely on subjective visual inspection, limiting objectivity and standardization [6]. While techniques like Fourier Transform (FT) provide frequency domain insights, alternative approaches are needed to unlock a full potential of seizure-related EEG data [7].

This paper proposes a novel conductor: mathematical modelling. By translating the complex dynamics of seizures into a mathematical framework, we aim to unlock deeper insights into their hidden mechanisms. This shift in perspective transcends the limitations of traditional analysis, paving the way for advanced seizure detection and classification.

Mathematical modelling offers a powerful symphony of advantages. First, it replaces subjective interpretation with precise mathematical tools, ensuring objective and quantifiable analysis [8]. This allows for robust statistical analysis, leading to more reliable conclusions. Second, these models go beyond observed activity, capturing the hidden dynamics driving seizures. Simulating the seizure process mathematically sheds light on the intricate interactions between brain regions involved, revealing the true "fingerprint" of seizures within EEG signals. Furthermore, these models act as a bridge between data and clinical applications. By simulating diverse seizure scenarios, we can virtually test and refine potential diagnostic and therapeutic interventions before applying them to patients. This "in silico" testing reduces risks and allows for personalized treatment strategies tailored to individual patients' unique brain activity [9].

Ultimately, unlocking the secrets hidden within EEG signals holds immense potential to transform lives. By embracing the power of mathematical modelling, we can move beyond the limitations of traditional methods, gain a deeper understanding of seizure dynamics, and develop more effective tools for diagnosis, prediction, and treatment. This symphony of knowledge promises a brighter future for those living with epilepsy.

2. LITERATURE REVIEW AND FOUNDATIONAL CONCEPTS

The past few decades have seen a continuous quest to understand and model the complex dynamics of epileptic seizures using mathematical tools. While this journey has yielded significant progress, it has also unveiled the inherent challenges and nuances of the problem [10]. Early attempts at modelling seizures often focused on the concept of "chaos" suggesting a complete loss of predictable behaviour within the epileptic brain. Nevertheless, subsequent study cast doubt on this simplistic explanation, attributing the observed chaotic patterns to noise within the data itself [11]. This realization underscored the need for more refined and robust modelling approaches.

Driven by the crucial goal of seizure prediction, scientists continued to explore various modelling methods. Initially, the focus shifted towards nonlinear models, as they were believed to better capture the intricate non-linear relationships within the brain during epileptic seizures. However, surprisingly, linear models also demonstrated promising outcomes, highlighting the potential for simpler methods in achieving accurate predictions [12]. The recent surge in sophisticated mathematical tools has further fuelled the development of even more intricate seizure models. Some of these cutting-edge approaches involve utilizing techniques like Burg autoregressive coefficients and fast independent component analysis, allowing for a deeper dive into the hidden dynamics of EEG signals. While these advanced models have demonstrated exceptional accuracy in seizure detection, their computational complexity remains a significant hurdle, demanding significant computing resources and expertise to implement [13].

As research progresses, the field of mathematical modelling in epilepsy continues to evolve, tackling the complex interplay between model accuracy, computational efficiency, and interpretability. The ultimate aim is to create models that not only excel at seizure prediction but also provide valuable insights into the underlying mechanisms of these debilitating events. This deeper understanding can pave the way for the development of more effective preventive and therapeutic strategies, ultimately improving the lives of individuals living with epilepsy [14].

In 2024, Barja [15] proposed a novel mathematical model that describes EEG signals during epileptic seizures as an integral equation of the form

$$\phi(t) = \int_{t}^{\infty} K(\tau)\, \upsilon(t-\tau)\mu(\tau)d\tau \qquad (2.1)$$

The equation (2.1), with its kernel function $K(\tau)$, pre-seizure signal $\upsilon(t-\tau)$, and seizure function $\mu(\tau)$, served as a key tool for unlocking the connection between the observed EEG signal $\phi(t)$ during a seizure and the underlying brain activity $\mu(\tau)$. By solving (2.1), we could estimate the seizure activity using the measured EEG signal. This breakthrough had profound implications, allowing us to pinpoint and localize the source of seizures within the brain, as demonstrated in Figure 1.

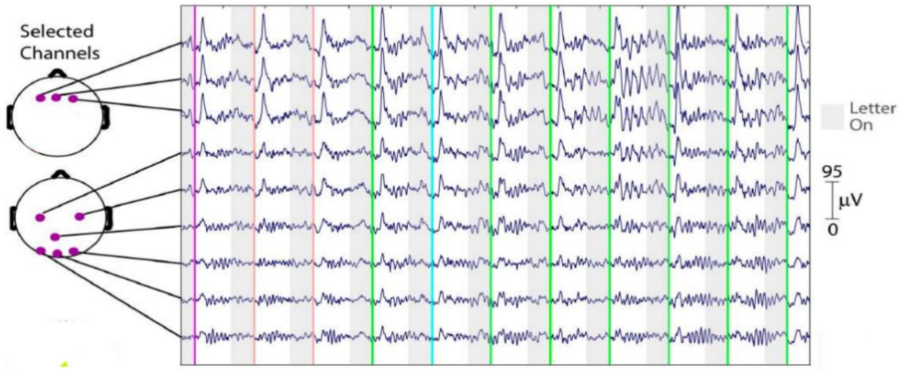
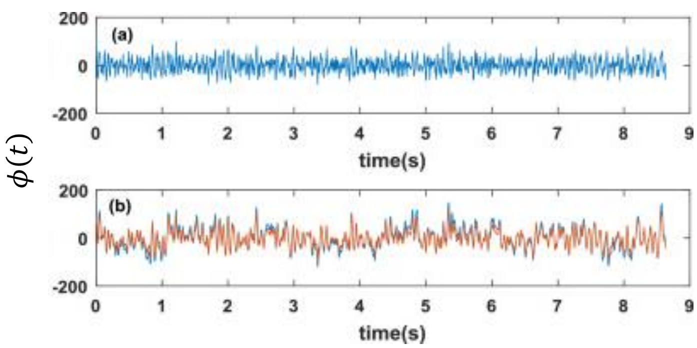


Figure (1): Brain activity captured by EEG during an epileptic episode.**[16]**

Furthermore, the prior integral equation (2.1) was implemented in MATLAB, yielding unambiguous results in the referenced study as shown in figure 2.



Figure(2): An integral equation mapping for EEG signals during a seizure

2.1 Definition [17]: The kernel function (kerf) is a function that appears within the integral equation itself. It represents the relationship between the unknown function and its integral. Typically, it's a continuous, bounded, and symmetric real function.

2.2 Definition [18]: A convolution equation is an integral equation that expresses the output of a system as a convolution of the input with a kernel function. It has the general form: $y(t) =$

$x(t) \odot k(t)$ where: $y(t)$ is the output of the system, $x(t)$ is the input to the system, $k(t)$ is the kernel function, and $\odot$ denotes convolution.

2.3 Leibniz integral theorem [19]: Let $f(x,t)$ be a function such that both $f(x,t)$ and its partial derivative $f_x(x,t)$ are continuous in t and x in some region of the xt-plane, including $\alpha(x) \leq t \leq \beta(x), x_0 \leq x \leq x$. Also suppose that the functions $\alpha(x)$ and $\beta(x)$ are both continuous and both have continuous derivatives for $x_0 \leq x \leq x$. Then

$$\frac{d}{dx}\left(\int_{\alpha(x)}^{\beta(x)} f(x,t)dt\right) = f\big(x,\beta(x)\big).\frac{d}{dx}\beta(x) - f\big(x,\alpha(x)\big).\frac{d}{dx}\alpha(x) + \int_{\alpha(x)}^{\beta(x)} \frac{\partial}{\partial x} f(x,t)dt$$

3. FINDINGS AND ANALYSIS

Building upon the groundwork laid in the previous section, this section presents the principal outcomes of the investigation. These results demonstrate the existence of alternative forms of the integral equation (2.1) that dictate the behavior of EEG signals during epileptic seizures. To derive the alternative forms of (2.1), we can use the following theorems:

3.1 Theorem: The integral equation (2.1) for EEG signals during an epileptic seizure can be expressed in the following form:

$$\phi(t) = \int_0^t K(t-\tau)\mu(\tau)d\tau + \, \upsilon(t)$$

Proof.

Since $\phi(t) = \int_t^\infty K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$, with the seizure onset starting at $t = 0$

When we change the limits of integration, we obtain

$$\phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau + \int_t^\infty K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$$

Since $\upsilon(t-\tau) = 0$ for $\tau > t$, the second integral becomes zero. Therefore, we are left with the following:

$$\phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$$

Now, pick $\omega = t - \tau$. Then $d\omega = -d\tau$ and $\tau = t - \omega$. Substituting into the above equation, we get:

$$\phi(t) = -\int_0^t K(t-\omega)\upsilon(\omega)\mu(t-\omega)d\omega = \int_t^0 K(t-\omega)\upsilon(\omega)\mu(t-\omega)d\omega$$

Changing the limits of integration back to 0 and t, we obtain:

$$\phi(t) = \int_0^t K(t-\tau)\upsilon(\tau)\mu(t-\tau)d\tau = \int_0^t K(t-\tau)\,[\phi(\tau) - \upsilon(\tau) + \upsilon(\tau)]\mu(t-\tau)d\tau$$

Adding and subtracting $\upsilon(t)$ from the integrand, we obtain:

$$\phi(t) = \int_0^t K(t-\tau)\,[\phi(\tau) - \upsilon(\tau)]\mu(t-\tau)d\tau + \int_0^t K(t-\tau)\,\upsilon(t)\mu(t-\tau)d\tau$$

Since the first integral on the right-hand side is the same as the integral in the original equation (2.1), and the second integral on the right-hand side is equal to $\upsilon(t)$, because the integral of $K(t-\tau)\ \mu(t-\tau)$ over the interval $[0, t]$ is equal to 1. Therefore, we have:

$$\phi(t) = \upsilon(t) + \int_0^t K(t-\tau)\,[\phi(\tau) - \upsilon(\tau)]d\tau\,,$$ as required.

Subsequent theorems will address a distinct form of the integral equation (2.1) for EEG signals associated with epileptic seizures.

3.2 Theorem: The integral equation (2.1) for EEG signals during an epileptic seizure takes the following form:

$$\mu(t) = \frac{1}{K(t)}\left[\phi(t) - \int_0^t K(\tau)\,\upsilon(\tau)d\tau\right]$$

Proof.

Suppose that $\phi(t) = \int_t^\infty K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$;

where $\phi(t)$ represents the EEG signals during the seizure

Multiply both sides by $\mu^{-1}(t)$, we obtain $\mu^{-1}(t)\phi(t) = \int_t^\infty K(\tau)\upsilon(t-\tau)d\tau$

$$\Longrightarrow \mu^{-1}(t)\phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)d\tau + \mu^{-1}(t)\phi(t) = \int_t^\infty K(\tau)\upsilon(t-\tau)d\tau$$

since the second integral on the right hand side is zero because $\upsilon(t-\tau) = 0 \; for \; \tau > t$.

therefore, we left with:

$$\mu^{-1}(t)\phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)d\tau \Longrightarrow K(t)\mu^{-1}(t)\phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)d\tau$$

$\Longrightarrow \int K(t)\mu^{-1}(t)\phi(t)dt = \int \int_0^t K(\tau)\upsilon(t-\tau)d\tau\, dt$; by integrate both sides with respect to t

$$\Longrightarrow \int K(t)\mu^{-1}(t)\phi(t)dt = \int_0^t \int K(\tau)\upsilon(t-\tau)dtd\tau = \int_0^t K(\tau)\left[\int \upsilon(t-\tau)dt\right]d\tau$$

Since the inner integral is equal to $\upsilon(t-\tau)$, hence we are left with:

$$\int K(t)\mu^{-1}(t)\phi(t)dt = \int_0^t K(\tau)\upsilon(t-\tau)d\tau$$

$$\Rightarrow \mu(t)\int K(t)\mu^{-1}(t)\phi(t)dt = \mu(t)\int_0^t K(\tau)\upsilon(t-\tau)d\tau$$

$\Rightarrow \phi(t) = \int_0^t K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$, by simplifies both sides using 2.2 difinntion

$$\Rightarrow \phi(t) - \int_0^t K(\tau)\upsilon(\tau)d\tau = \int_0^t K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau - \int_0^t K(\tau)\upsilon(\tau)d\tau$$

$$\Rightarrow \phi(t) - \int_0^t K(\tau)\upsilon(\tau)d\tau = \int_0^t K(\tau)[\upsilon(t-\tau) - \upsilon(\tau)]\mu(\tau)d\tau$$

$$\Rightarrow \phi(t) - \int_0^t K(\tau)\upsilon(\tau)d\tau$$

$= \upsilon(t)\int_0^t K(\tau)\left[1 - \frac{\upsilon(\tau)}{\upsilon(t)}\right]\mu(\tau)d\tau$, by factoring out $\upsilon(t)$ from the integration

$\Rightarrow \mu(t) = \frac{1}{K(t)}\left[\phi(t) - \int_0^t K(\tau)\,\upsilon(\tau)d\tau\right]$, by rearranging the previous equation, as required.

3.3 Theorem: The integral equation (2.1) for EEG signals during an epileptic seizure can be expressed in the following form:

$$\frac{d\phi}{dt} = \int_0^t K(\tau)\,\upsilon'(t-\tau)\mu(\tau)d\tau$$

Proof.

Differentiate both sides of equation (2.1), we obtain:

$$\frac{d\phi}{dt} = \frac{d}{dt}\int_{t}^{\infty} K(\tau)\upsilon(t-\tau)\mu(\tau)d\tau$$

$$\Rightarrow \frac{d\phi}{dt} = K(t)\upsilon(0)\mu(t) + \int_{t}^{\infty} \frac{\partial}{\partial t}[K(\tau)\upsilon(t-\tau)\mu(\tau)]d\tau \; \dots\dots (*)$$

now, $\frac{\partial}{\partial t}[K(\tau)\upsilon(t-\tau)\mu(\tau)] = -K(\tau)\frac{\partial}{\partial t}[\upsilon(t-\tau)]\mu(\tau) = K(\tau)\upsilon'(t-\tau)\mu(\tau)$

substituting in $(*)$, we obtain:

$$\frac{d\phi}{dt} = K(t)\upsilon(0)\mu(t) + \int_{t}^{\infty} K(\tau)\upsilon'(t-\tau)\mu(\tau)d\tau$$

$$\Rightarrow \frac{d\phi}{dt} = K(t)\upsilon(0)\mu(t) + \int_{0}^{t} K(\tau)\upsilon'(t-\tau)\mu(\tau)d\tau$$

Since $\upsilon(0) = 0$, this is because $\upsilon(t)$ represents the pre-seizure EEG signals, and the seizure is assumed to start at $t = 0$. Therefore, we are left with:

$\frac{d\phi}{dt} = \int_{0}^{t} K(\tau)\upsilon'(t-\tau)\,\mu(\tau)d\tau$, as required.

4. DISCUSSION

The three formulations presented in this paper provide an alternative to the integral equation (2.1) describing the relationship between the EEG signal during seizures, the EEG signal preceding seizures, seizure activity, and seizure kernel These alternative approaches this can be used for a variety of purposes, such as a study of the mechanisms of allergic rhinitis; methods of identifying and formulating results. 3.1 Theorem provides an alternative form of the integral equation for numerical analysis. This analogical approach can be used to develop real-time tumor detection methods. Theorem 3.2 presents a new form of integral equation that focuses on the role of seizure activity in epileptic seizure dynamics, this equation can be used to study the timing of seizures, and to develop methods for predicting how the tumor is severe. Theorem 3.3 provides an alternative form of integral equation that is more convenient for theoretical analysis. This form of equation can be used to study the relationship between the pre-attack EEG signal and the seizure itself.

5. CONCLUSION

This paper offers three new concepts that provide a powerful framework for understanding and managing epilepsy. These considerations provide a novel approach to the integral equation governing EEG signals during seizures, and provide researchers and clinicians with new tools These tools hold promise for improvements in various fields. First, they pave the way for sophisticated methods of analyzing EEG data, potentially improving diagnosis. Second, it provides deeper insight into tumor dynamics and suggests the development of prognostic tools. Ultimately, the established framework can guide the search for new treatment options, ultimately improving the quality of life for people with epilepsy.

6. FUTURE STUDIES:

- Develop approaches to use the new forms of the integral equation to detect resistance in real time.
- Study the role of the seizure function in the dynamics of epileptic seizures.
- Study the role of the seizure function in the dynamics of epileptic seizures.

7. LIST OF ABBREVIATIONS

EEG: Electroencephalography

APD: Average Potential Differences

Kerf: Kernel Function

FT: Fourier Transform

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