Topological folding of multiple chaotic graphs with density variation

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Abstract

Chaotic graph is a graph which carries physical characters with density variation; the density of chaotic graphs can be fixed and unique or different, according to this the representation of the chaotic graphs by matrices is different to normal chaotic graphs and this research is a following discussion to previous research done, which is "Folding simple chaotic graphs with density variation, Journal of Humanities and Applied Science (JHAS)".[1]

Firstly, we will discuss the idea of topological folding of multiple chaotic graphs with density variation and we start by introducing the definition of multiple graphs, and we will define the incidence matrix representing the topological folding of this type of multiple chaotic graphs, also the limit of this folding will be deduced. In each case we will discuss the decrease or increase the degree of density.

Keywords: Geometric graph, chaotic graphs, density, incidence matrix, topological folding.

I. Introduction

There are many physical systems whose performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. If we change a resistor to a capacitor, generally some of the properties (such as an input impedance of the network) also change. This indicates that the performance of a system depends on the characteristics of the components. If, on the other hand, we change the location of one resistor, the input impedance again may change, which shows that the topology of the system is influencing the system's performance. There are systems constructed of only one kind of component so that the system's performance depends only on its topology. An example of such a system is a single-contact switching circuit. Similar situations can be seen in nonphysical systems such as structures of administration. Hence it is important to represent a system so that its topology can be visualized clearly.

One simple way of displaying a structure of a system is to draw a diagram consisting of points called "vertices" and line segments called "edges" which connect these vertices so that such vertices and edges indicate components and relationships between these components. Such a diagram is called a "Linear graph" whose name depends on the kind of physical system we deal with. This means that it may be called a network, a net, a circuit, a graph, a diagram, a structure, and so on.

Instead of indicating the physical structure of a system, we frequently indicate its mathematical model or its abstract model by a "Linear graph". Under such a circumstance, a linear graph is referred to as a flow graph, a signal flow graph, a flow chart, a state diagram, an organization diagram, and so forth.

The generalization of this graph is the "fuzzy graph" and the most generalization of them is the "chaotic graph", which applied in many uncertain circuits, resonance, perturbation theory and many other applications. More advanced applications using the more complicated graphs are the chaotic graphs [1,2,3].

Generally, **a chaotic graph** is a geometric graph that carries many other graphs or physical characters, these geometric graphs might have similar properties or different [2].

Chaotic graph with density variation: is a geometric graph that carries many other graphs or physical characteristics, these geometric graphs might have similar properties or different, that density have two cases equal densities or different densities.

Simple graph:

A "simple " graph is a graph with no loops or multiple edges [3,4].

Multiple edges:

Two or more edges joining the same pair of vertices are called " multiple edges" [4].

Multiple graphs:

A multiple graph has multiple edges. Sometimes the multiple graphs are called a general graph or simply a graph[5].

The incidence matrix: Let G be a graph without loops, with n-vertices labeled $1,2,3,...,n$ and m- edges labeled $1,2,3,...,m$.The" incidence matrix" $I(G)$ is the $n \times m$ matrix in which the entry in row i and column j is 1 if vertex i is incident with edge j and 0 otherwise [4, 5, 6].

Null graph: Is a graph consists of a set of vertices and no edges [7,8,9].

Loop: A loop is an edge which starts and ends on the same vertex [6, 7, 8].

Example: Consider the multiple graph G in Figure (1):

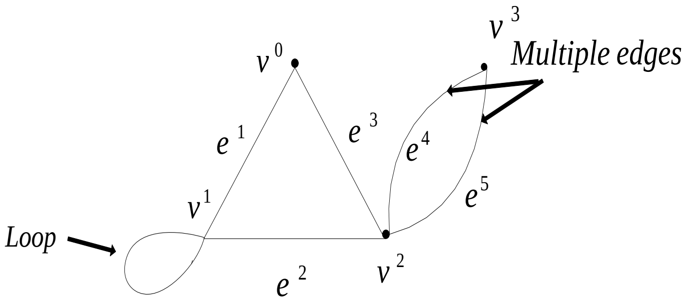


Figure (1). Multiple graph

The incidence matrix $I(G)$ is: $I(G)=\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1^1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

Noting that the symbol (1^1) in the second row and first column means that we have one loop at the vertex v^1 with the edge e^1. Also, if we have two loops at the same above vertex, we symbolize it by 1^{11}. Moreover, if we have an infinite number of loops at any vertex, say v^i, we use the symbol $1^{1111...}$ to represent them [1].

Density (d): Is a physical property of matter, as each element and compound have a unique density associated with it [1].

Folding:

The Field of folding began with S. A. Robertson’s work, in 1977 [10], on isometric folding of Riemannian manifold M into another N, which send any piecewise geodesic path in M to a piecewise geodesic path with the same length in N. More studies on the folding of manifolds are studied by M. El-Ghoul.

This paper the physical character is presented by density, the density might be constant everywhere or vary from place to another place, for example the color of a plant leaves is a perfect green, or magnetic field waves have the same velocity. We will denote the degree of each area on the chaotic graph by d_{pq}, where p denotes levels of chaotic graph, while q denotes different areas on each level of chaotic graph.

1- Folding multiple chaotic graphs with density variation

There are two fundamental types of folding of any graph, especially chaotic graphs:

1- One contracts the distances (the edges) between the vertices in the multiple graphs under consideration. (Topological Folding).

2- The other type of folding has multiple choices, it may be folding a vertex to a vertex, folding an edge to another one, folding of a loop to another loop, and folding of an edge to loop.

1.1. Topological folding

Definition of topological Folding:

Generally topological folding can be defined as:

Let $F: G \to \overline{G}$ be a map between any two graphs G and $\overline{G}$ (not necessarily to be simple) such that if $(u,v) \in G, (f(u), f(v)) \in \overline{G}$; then f is called a "topological folding" of G and $\overline{G}$ provided that $d(f(u), f(v)) \le d(u,v)$.

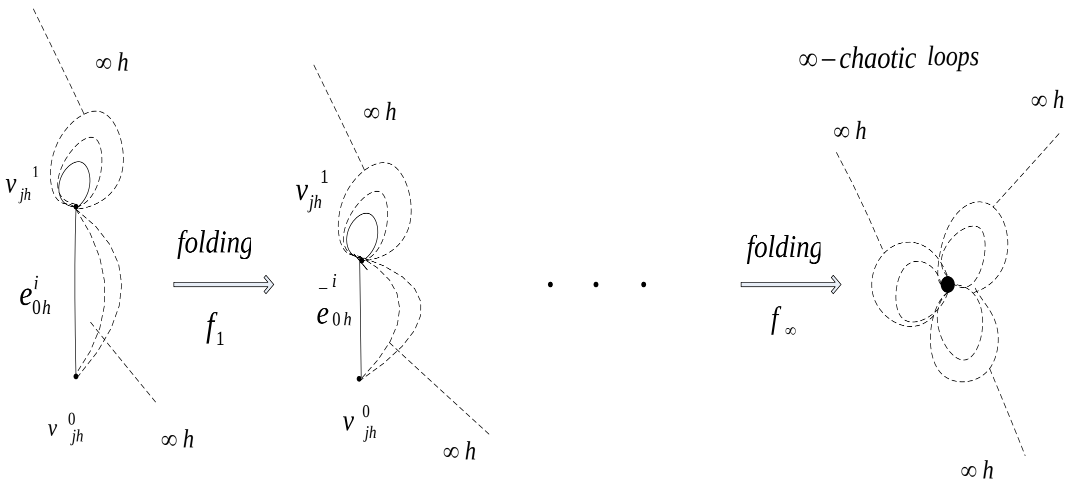
So we can generalise this to chaotic graph as:

If $F: G \to \overline{G}$ be a map between any two chaotic graphs G_h and $\overline{G}_h$ (not necessarily to be simple) such that if $(v_{ih}^{i}, v_{ih}^{i+1}) \in G_h, i, j = 0,1,2,.., (f(v_{ih}^{i}), f(v_{ih}^{i+1})) \in \overline{G}_h$; then f is called a "topological folding" of G_h and $\overline{G}_h$ provided that $d(f(v_{ih}^{i}), f(v_{ih}^{i+1})) \le d(v_{ih}^{i}, v_{ih}^{i+1})$. [11]

Her we will divide the topological folding into subtypes as follows:

(a)-The folding (contraction) is restricted on the geometric graph only, but not on the chaotic edges or chaotic loops.

Consider a chaotic multiple chaotic graph $G_h^{(m)d}$, where h shows that the graph is chaotic graph and m shows that a graph has multiple edges; and it has a geometric loop α_{0h} at v_{jh}^1 ,overlapped on an infinite number of different chaotic loops $\alpha_{1h}, \alpha_{2h}, \alpha_{3h}, ...\alpha_{\infty h}$ .The successive folding's of this chaotic graph with density are $G_h^{(m1)d}, G_h^{(m2)d^{\bullet}}, ...G_h^{(m\infty)d^{\bullet\bullet\infty}}$.(See figure (1.1.1))



$$G_h^{(m)d}\ (multiple) \Longrightarrow G_h^{(m1)\dot{d}}\ (multiple) \Longrightarrow \cdot\ \cdot\ \cdot \Longrightarrow G_h^{(m\infty)\overset{\bullet\bullet\bullet\infty}{d}}\ (semi-multiple)$$

$$(d < \dot{d} < \ddot{d} .... < \overset{\bullet\bullet\bullet\bullet\infty}{d})$$

Figure (1.1)

The end limit of successive folding sequence is a geometric vertex overlapped on by infinitely different chaotic loops with density without any geometric loops resulting a semi multiple chaotic graphs and each chaotic edge keeps its own density as before folding process, while the geometric edge and geometric loop changed by folding into one vertex with higher density whatever if the density is constant or varies on the geometric edge and geometric loop, in all cases the density will increase.

The chaotic incidence matrix with density representing the original chaotic multiple graph $G_h^{(m)d}$ is as follows I_1:

$$I_1 = \begin{bmatrix} 1_{(012\ldots\infty)d_{pq}h} & 0 & . & . & . & . & . \\ 1_{(012\ldots\infty)d_{pq}h} & 1_{(012\ldots\infty)d_{pq}h} & 0 & . & . & . & . \\ 0 & 1_{(012\ldots\infty)d_{pq}h} & 1_{(012\ldots\infty)d_{pq}h} & 0 & . & . & . \\ 0 & 0 & 1_{(012\ldots\infty)d_{pq}h} & 1_{(012\ldots\infty)d_{pq}h} & 0 & . & . \\ . & 0 & 0 & 1^{1_{(012..\infty)d_{pq}h}}_{(012\ldots\infty)d_{pq}h} & 1_{(012\ldots\infty)d_{pq}h} & 0 & . \\ . & . & . & 0 & 1_{(012\ldots\infty)d_{pq}h} & 1_{(012\ldots\infty)d_{pq}h} & . \\ . & . & . & . & . & . & . \end{bmatrix}$$

Also, the incidence matrix representing folding chaotic multiple graphs with density variation $G_h^{(m1)d}$ (density has increased) of the given chaotic multiple graphs with density G_h^{md} is I_2 where,

$$I_2 = \begin{bmatrix} 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 0 & . & . & . & . & . \\ 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 0 & . & . & . & . \\ 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 0 & . & . & . \\ 0 & 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 0 & . & . \\ . & 0 & 0 & 1^{1_{(012..\infty)d^{\bullet}{}_{pq}h}}_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 0 & . \\ . & . & . & 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & 1_{(012\ldots\infty)d^{\bullet}{}_{pq}h} & . \\ . & . & . & . & . & . & . \end{bmatrix}$$

And then the incidence matrix representing the chaotic multiple graph $G_h^{(m\infty)d}$ induced from the limit of the successive folding sequence of the given chaotic multiple graphs with density $G_h^{(m)d}$ is I_∞ where ,

$$I_{\infty} = \begin{bmatrix} . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & . \\ . & 0 & . & 0 & 0 & . & . \\ . & . & 0 & 0^{1_{(12..\infty)d^{\bullet\bullet\bullet\infty}}{}_{pqh}1_{(12..\infty)d^{\bullet\bullet\bullet\infty}}{}_{pqh....}} & 0 & 0 & . \\ . & . & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . \end{bmatrix}$$

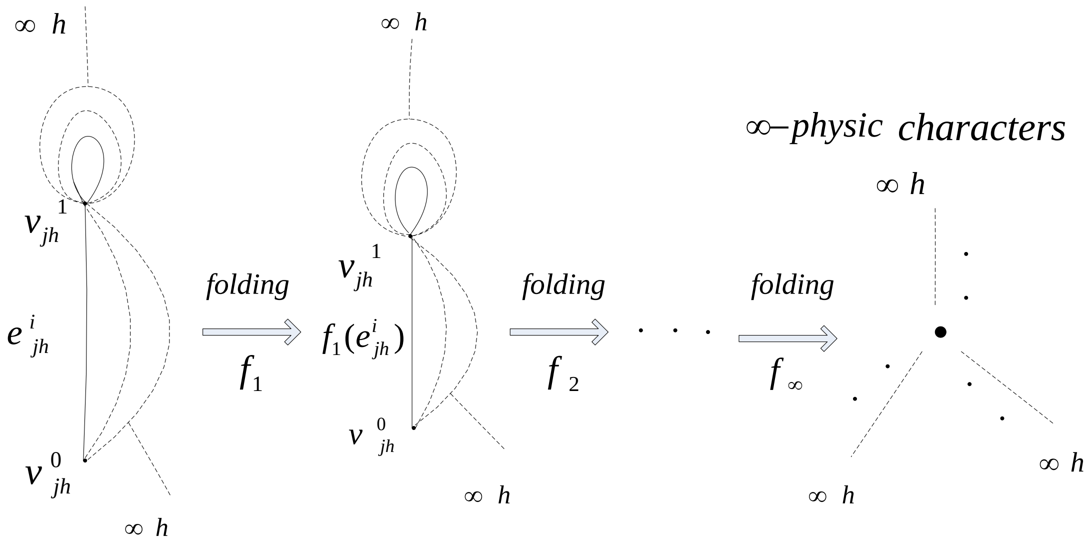
From the all above, we can formulate the following theorem;

Theorem (1.1.1):

The chaotic incidence matrices representing each of the given and the induced chaotic multiple graphs are similar to each other but differ from the incidence matrix representing the chaotic (semi- multiple) graph induced from the limit of successive folding and the density increases each time we fold a chaotic edge.

(b)- The second type of topological folding (the contraction) is to fold both the geometric and the chaotic edges, in this case the end limit of successive folding is one vertex has greater density with no chaotic edges (i.e. null graph), so the density has increased more than in the previous case, so if we want to increase rate of density, it is preferred to choose this kind of folding rather than the previous folding, because the rate of density increases each time we fold a chaotic edge, not only when we fold the geometric edge (i.e. more density, less distance for the graph).

Consider a multiple chaotic graph $G_h^{(m)d}$ with a geometric loop α_{0h} at V_{jh}^{1} ,overlapped on an infinite number of different chaotic loops $\alpha_{1h}, \alpha_{2h}, \alpha_{3h}, ...\alpha_{\infty h}$.The successive folding of this chaotic multiple graph with density $G_h^{(m)d}$ are the chaotic multiple graphs with density $G_h^{(m1)d}, G_h^{(m2)d^{\bullet}}, ...G_h^{(m\infty)d^{\bullet\bullet\bullet\infty}}$. (See figure (1.1.2))



$$G_h^{(m)d}\ \text{(multiple)} \rightarrow G_h^{(m1)\dot{d}}\ \text{(multiple)} \rightarrow \ldots\ldots\ G_h^{(m\infty)\overset{\bullet\bullet\bullet\infty}{d}}\ \text{(Null-multiple)}$$

$$d < \dot{d} < \ddot{d} < \dddot{d} < \ldots.. < \overset{\bullet\bullet\bullet\infty}{d}$$

Figure (1.1.2)

In similar way, the chaotic incidence matrix representing the given chaotic multiple graphs with density $G_h^{(m)d}$ is I_1 where,

$$I_1 = \begin{bmatrix} 1_{(012...\infty)d_{pq}h} & 0 & . & . & . & . & . \\ 1_{(012...\infty)d_{pq}h} & 1_{(012...\infty)d_{pq}h} & 0 & . & . & . & . \\ 0 & 1_{(012...\infty)d_{pq}h} & 1_{(012...\infty)d_{pq}h} & 0 & . & . & . \\ 0 & 0 & 1_{(012...\infty)d_{pq}h} & 1_{(012...\infty)d_{pq}h} & 0 & . & . \\ . & 0 & 0 & 1^{(012..\infty)d_{pq}h}_{(012...\infty)d_{pq}h} & 1_{(012...\infty)d_{pq}h} & 0 & . \\ . & . & . & 0 & 1_{(012...\infty)d_{pq}h} & 1_{(012...\infty)d_{pq}h} & . \\ . & . & . & . & . & . & . \end{bmatrix}$$

The incidence matrix representing the folding chaotic multiple graph $G_h^{(m1)d}$ (density has increased) of the given chaotic multiple graphs with density G_h^{md} is I_2 where,

$$I_2 = \begin{bmatrix} 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 0 & . & . & . & . & . \\ 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 0 & . & . & . & . \\ 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 0 & . & . & . \\ 0 & 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 0 & . & . \\ . & 0 & 0 & 1^{1_{(012..\infty)d^{\bullet}{}_{pqh}}}_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 0 & . \\ . & . & . & 0 & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & 1_{(012\ldots\infty)d^{\bullet}{}_{pqh}} & . \\ . & . & . & . & . & . & . \end{bmatrix}$$

And then the incidence matrix representing the final chaotic graph $G_h^{(m\infty)d}$ induced from the end limit of successive folding sequence of the given chaotic multiple graph $G_h^{(m)d}$ is I_∞ where,

$$I_\infty = \begin{bmatrix} . & . & . & & . & & . & . & . \\ 0 & . & . & & . & & . & . & . \\ 0 & 0 & . & & . & & . & . & . \\ . & 0 & . & & 0 & & 0 & . & . \\ . & . & 0 & 0 & 0_{(12..\infty)d^{\bullet\bullet\bullet\infty}{}_{pqh}} \, 0_{(12..\infty)d^{\bullet\bullet\bullet\infty}{}_{pqh\ldots}} & & 0 & 0 & . \\ . & . & 0 & & 0 & & . & 0 & 0 \\ . & . & . & & . & & . & . & . \end{bmatrix}$$

Since the final graph resulted from folding process is the null graph, according to this the matrix is incidence matrix representing null graph is the zero matrix.

The end limit of topological folding to the geometric graph and chaotic edges is the null graph with great density matrix.

$$f_1 : G_h^{(m)d} \longrightarrow G_h^{(m1)d} \ , f_2 : G_h^{(m1)d} \longrightarrow G_h^{(m2)d} , f_3 : G_h^{(m2)d} \longrightarrow G_h^{(m3)d} ,\ldots\ldots, \lim_{n\to\infty} f_n(G_h)^{(m(n-1))d} = G_h^{(mn)d} = G^D$$

D means greates density

Each folding reduces the length of the graph and increase its density, and each time we repeat the process, the graph is reduced more and the density increases more than before, until we reach the end limit of folding the geometric edge and all chaotic edges and both vertices folded on each other, so we end up with one vertex has greater density than before and this exactly the null graph, see figure (1.1.3).

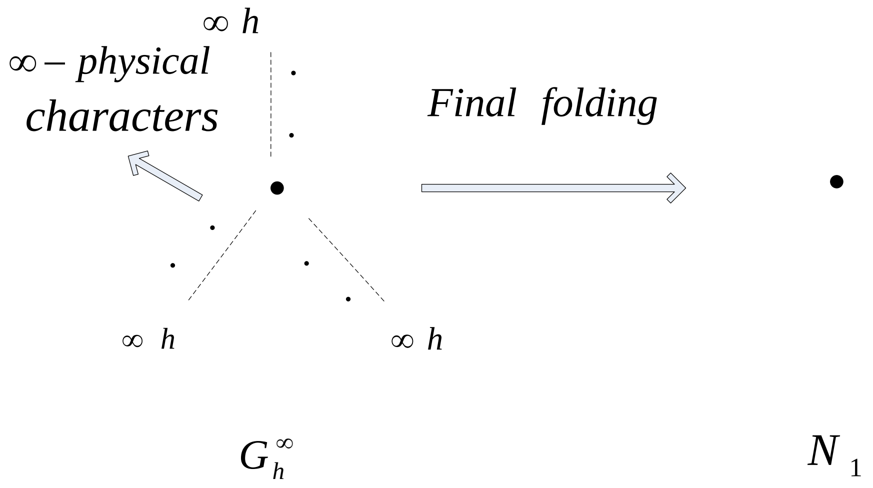


Figure (1.1.3)

And the incidence matrix will take the form (i.e., zero matrix)

$$I_{final} = \begin{bmatrix} . & . & . & . & . & . \\ 0 & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . \\ . & 0 & 0 & 0 & 0 & . \\ . & . & . & 0 & 0 & . \\ . & . & . & . & . & . \end{bmatrix}$$

Theorem (1.1.3):

The chaotic incidence matrices representing each of the given induced folding of chaotic multiple graphs of type (b) are similar to each other, but differs from what representing the null chaotic graph induced from the limit of successive folding, and this is exactly the incidence matrix representing the null chaotic graph.

Corollary (1.1.4): The end limit of successive folding (i.e., the final folding) of any chaotic multiple graphs under folding of type **(b)** is the geometric null graph and the incidence matrix representing this induced graph is the zero matrix. See figure (1.1.3)

Conclusion

This paper discussed the idea of folding of chaotic multiple graphs with density variation; the incidence and adjacency matrix were obtained; two types of topological folding were studied. Generally, the topological folding increases the density and reduces the length of the graph. The limit of successive folding a vertex into another vertex of a geometric graph only is a geometric vertex overlapped on by different chaotic loops and each loop has its own density characters, while the limit of successive folding of a vertex into another vertex of a geometric graph with folding chaotic edges too is a geometric vertex without any loops and it has greater density than before and this is exactly the null chaotic graph; indeed the end limit folding of this type of folding induces a null graph (i.e. non-multiple graph) and it has the zero matrix.

As a future study, we can extend the idea of folding into chaotic folding of multiple chaotic graphs, which is a more complicated than the topological folding.

Applications:

- Folding a plant leaves, most of plant leaves have variation of green color, according to these chaotic graphs can present the variation of green color of the leaves according to the density character.
- Folding a balloon, the density of the balloon color increase, while the length of the balloon decrease.
- An effective example of chaotic graph with density variations is the nerve system human body such that the nerve system in the body carries many different signals a very such a different signal represents a 1-chaotic graph, where the signals are different and depends on the mission it carries.

- The perturbation of magnetic field waves and the resonance of the waves are the chaotic graphs, since every single wave of magnetic field has different wavelength and speeds, and the wave length varies on the periodic time.

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