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Reliability extrema of a complex circuit on bi-variate slice classes

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Abstract

Within the frame of reliability models, the geometry of constant level sets of the reliability function of a complex circuit — regarded as hypersurfaces, reveals properties which provide useful information on the relation between the reliability of the circuit and its components. A special role plays the study of intersections of these hypersurfaces with 2-dimensional plane slices, which provide a foliation by pencils of algebraic curves. The present study classifies these pencils and consequently, it allows: (i) to evaluate the possible bounds of the bivariate slice-reliability in terms of the circuit components; (ii) to compensate the impact of a slice-component malfunction on the slice-reliability, by tuning the appropriate pairing slice-component; (iii) to flag out the cases when the slice-reliability is linear (mono- or bi-variate) in the slice-components, or constant along the whole slice; (iv) in the quadratic case, to make use the convex/concave mutual dependence of slice-components along the curves of constant-slice reliability, in order to maintain or increase the circuit reliability.

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Keywords: Reliability; Network model; Algebraic curves; Optimal design

1. Introduction

Numerous studies describe multiple methods to derive the reliability of a complex circuit: the minimal cut method, the sum of disjoint product, the Boolean truth table, the inclusion-exclusion method, etc (see, e.g., [1—23]). Recent research proficiently use geometry tools in studying the reliability function, and prove that the properties of the specific algebraic varieties provide useful information for the study of the modeled complex system ([4—6,12,16]).

In this study, after presenting a brief introduction in reliability of complex systems, we investigate the reliability of a classic bridge complex system, by classifying the pencils of curves of constant reliability lying inside 2-dimensional slices of the foliation given

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by the constant level reliability hypersurfaces. We use this classification to enhance the design of the system in order to maintain or increase its reliability.

In Section 1 we obtain, by using the path tracing method, the polynomial reliability function of the system, which is further used to derive the constrained reliability function (*slice-reliability*) which depends on two arguments, while the remaining three ones are considered as fixed parameters.

For each choice of selecting the two slice variables, we determine the extremal values of the system slice-reliability in terms of the fixed three parameters, and provide the conditions for which the slice-reliability is quadratic/linear/constant.

As well, for each choice, we show that the slice-reliability may exhibit a *quadratic* (hyperbolic convex or concave) dependence, an *affine* (bi- or mono-variate) dependence, or *constancy* within the slice.

In the *quadratic* case, the pencils of constant slice-reliability curves prove to be mostly sets of hyperbolic-type pencils of conics, parametrized by the arbitrarily admissible reliability level. It is shown that, for constant slice-reliability, the hyperbolic dependence between the free variables provides a center of symmetry with a specific location (0101 and 1010 Cohen-Southerland codes relative to the main clipping domain $[0,1]^2$ of the two active variables), which ensures that a gradual failure of one component may be compensated by improving the pairing component by a technique which uses the convexity/concavity property of the pencil class.

In the *linear* case, the slice-reliability is shown to generally depend on one single variable (the complementary one having an obsolete role), with one singular case exception, when *both the active variables* effectively influence the slice-reliability of the system.

We further illustrate this classification both by pencil representatives of algebraic curves and by the Monge surfaces of the slice-reliability mappings. The classification is then used to describe the dependence of the slice-reliability on the components of the complex system, aiming — by tuning the selected slice-components — to maintain or improve its reliability.

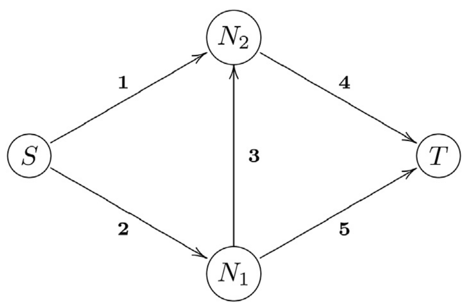

Fig. 1. A bridge-type network.

2. Basic reliability facts

We will briefly describe the basic concepts of network topology and of graph theory [12,13,16,22], commonly used in the network reliability models.

We represent our directed networks as graphs $G = (V,E)$, where V is the set of *vertices* (or *nodes*) and E the set of *edges*, or *arcs*.

In such a configuration (see Fig. 1), one node is considered as the *source* (node S), and a second one is regarded as a *sink*, or *target* (node T). The nodes of the network are joined by numbered arcs.

A *failure of an arc* is equivalent by the failing of the communication along the respective arc (removing, or cutting the arc). The system is *successful* if there exists a valid path which joins the source to the sink. The system is said to *fail* if no such path exists. The *reliability of the system* is the probability that there exist one or more successful paths from the source to the sink.

2.1. Definitions

a) A *path* of the network is a set of arcs, whose success imply the system being successful. For example, the system from Fig. 1 admits as paths the sets $\{1,4\}$, $\{2,5\}$ and $\{2,3,4\}$.
b) A *minimal path* (briefly, *min-path*) of the system is a set of arcs which contains a path, such that the removal of any of its arcs produces a subset which no longer contains paths. In other words, if any of the arcs of the minimal path fails, then the system will fail along the remaining arcs of this path. In terms of the network model, the minimal path corresponds to a simple path from the source to the sink in the network.

Considering the set of components of a minimal path $A = \{i_1,\ldots,i_r\}$, and denoting by X_i the indicator for the success of the component a_i, for each $i \in \{i_1,\ldots,i_r\}$, the event of A being successful is the binary function $\Pi_{i=1}^{r} X_i$, and the event of failed path A is $(1 - \Pi_{i=1}^{r} X_i)$, In our case, we have the min-paths

$$A_1 = \{1,4\},\quad A_2 = \{2,5\},\quad \text{and}\quad A_3 = \{2,3,4\}.$$

Table 1
Slice-reliability and its limiting extremal values.

#	i,j	Slice-reliability $\varphi(x,y)$	range $\varphi = [r_{min},r_{max}]$
1.	1,2	$\alpha xy+xd+y(e+cd-cde)$ where $\alpha=-d[c(1-e)+e]$	$[0,d+e(1-d)]$
2.	1,3	$\alpha xy+xd(1-be)+ybd(1-e)+be$ where $\alpha=-bd(1-e)$	$[be,d+be(1-d)]$
3.	1,4	$\alpha xy+ybc(1-e)+be$ where $\alpha=1-b[c+e(1-c)]$	$[be,1]$
4.	1,5	$\alpha xy+xd(1-bc)+yb(1-cd)+bcd$ where $\alpha=-bd(1-c)$	$[bcd,b+d(1-b)]$
5.	2,3	$\alpha xy+xe(1-ad)+ad$ where $\alpha=d(1-a)(1-e)$	$[ad,d+e(1-d)]$
6.	2,4	$\alpha xy+xe+ya$ where $\alpha=c(1-a)(1-e)-ae$	$[0,1-(1-a)(1-b)(1-c)]$
7.	2,5	$\alpha xy+xcd(1-a)+ad$ where $\alpha=1-d[a+c(1-a)]$	$[ad,1]$
8.	3,4	$\alpha xy+ya(1-be)+be$ where $\alpha=b(1-a)(1-e)$	$[be,a+b(1-a)]$
9.	3,5	$\alpha xy+xbd(1-a)+yb(1-ad)+ad$ where $\alpha=-bd(1-a)$	$[ad,b+ad(1-b)]$
10.	4,5	$\alpha xy+x(a+bc-abc)+yb$ where $\alpha=-b[a+c(1-a)]$	$[0,a+b(1-a)]$

Then the system is successful if there exists a minimal path which does not fail, and the structure function of the complex system is $\Phi(X) = 1-\Pi_{k=1}^{3}(1-\Pi_{i\in A_k}X_i)$, and hence we get *the expectation* [10,22]

$$\Phi(X) = 1-(1-X_1X_4)(1-X_2X_5)(1-X_2X_3X_4)$$
$$= X_1X_4+X_2X_5+X_2X_3X_4-X_1X_2X_3X_4^2-X_1X_2X_4X_5-X_2^2X_3X_4X_5+X_1X_2^2X_3X_4^2X_5,$$

where $X_k = R_k(t)$, $k=\overline{1,5}$, stands for the time-dependent expectation for each component of the system. The states of the time-dependent structure function of the system $\Phi(X)$ are Poisson variables [8,10,15], this leading to considering the Boolean degree reduction $R_k^p(t) = R_k(t)$, $(X_k^p = X_k)$, for all $p \geq 2$, $k\in\overline{1,5}$, in order to obtain *the survival function*

$$\Phi_s(X) = X_1X_4+X_2X_5+X_2X_3X_4-X_1X_2X_3X_4 -X_1X_2X_4X_5-X_2X_3X_4X_5+X_1X_2X_3X_4X_5.$$

Then, by considering the variables as time-dependent probabilities of functioning of the system components, $p_i = R_i(t) = X_i$, and denoting $p = (p_1,\ldots,p_5)$, we obtain the exact expression of the system reliability polynomial function,

$$R(p) = p_1p_4+p_2p_5+p_2p_3p_4+p_1p_2p_3p_4p_5-p_1p_2p_3p_4 -p_1p_2p_4p_5-p_2p_3p_4p_5, \tag{2.1}$$

which depends on the five components variable $p = (p_1,p_2,\ldots,p_5) \in [0,1]^5$.

3. Classification of slice-pencils and range extremal values of slice-reliability within classes

We shall further examine the reliability mapping $R(p)$ from (2.1). To this aim, we firstly select two components (further called *slice-components*) out of the five components of p:

$$(x,y) = (p_i,p_j)\in[0,1]^2,\ (1\leq i<j\leq 5).$$

We note that the number of such choices is 10. While we fix one such choice (i,j), the remaining three components, $p_k \in [0,1]$, for $k\in\overline{1,5}\setminus\{i,j\}$ (further called *complementary components*), will be regarded as parameters, which we arbitrarily fix; then the 5-couple will vary in $\mathbb{R}^5$ within the 2-plane slice π_{ij}, defined by

$$\pi_{ij} = \{p|p_k = \rho_k\in[0,1], k\in\overline{1,5}\setminus\{\mathrm{i,j}\}\},$$

where $\rho_k\in[0,1]\subset\mathbb{R}$ are constant and arbitrarily fixed, and $(x,y) = (p_i,p_j)$ are the inner variables of the slice.

On the other hand we readily remark that the 5-th degree polynomial (2.1) provides by means of the Monge chart $p_6 = R(p)$, $p \in [0,1]^5$, an imbedded 5-dimensional ruled hypersurface in $\mathbb{R}^6$, well-known in literature (e.g., see Ref. [24]).

The relation $r = R(p)$ for $r \in [0,1]$ provides in $\mathbb{R}^5$ an $r-$ dependent family of constant level hypersurfaces $\Sigma_r = \{p\in[0,1]^5 \big| R(p) = r\}$, $r \in [0,1]$, which intersect each slice π_{ij} along *pencils of algebraic curves*, described generically by equations of the form

$$r = \varphi(x,y),\ (x,y)\in[0,1]^2, \tag{3.1}$$

where

$$\varphi(x,y) = R(p)|_{(p_i,p_j)=(x,y);\ p_k=\rho_k,\ \forall k\in\overline{1,5}\setminus\{i,j\}} \tag{3.2}$$

Table 2
Quadratic case. Generic locations of centers of symmetry.

$\mathbb{D} = O \cup S_1 \cup S_2 \cup D$	$\mathbb{D}' = O' \cup S'_1 \cup S'_2 \cup D'$
$D = \{(x, y) \mid x < 0, y < 0\}$	$D' = \{(x, y) \mid x > 1, y > 1\}$
$S_1 = \{(x, 0) \mid x < 0\}$	$S'_1 = \{(x, 1) \mid x > 1\}$
$S_2 = \{(0, y) \mid y < 0\}$	$S'_2 = \{(1, y) \mid y > 1\}$
$O = \{(0,0)\}$	$O' = \{(1, 1)\}$

is the induced on the slice π_{ij} reliability function, further called *slice-reliability*, where we shall examine hereafter all the ten choices of selecting the slice π_{ij}.

The slice-reliability (3.2) naturally provides pencils of constant level curves $r = \varphi(x,y)$ for $r \in [0,1]$, which implicitly depend on the complementary components, re-denoted for convenience as:

$$(\rho_1, \rho_2, \rho_3, \rho_4, \rho_5) = (a, b, c, d, e) \in [0, 1]^5.$$

We note that equation (3.1) generically describes an algebraic curve of order at most two, as shown in Table 1. Here, for all the ten choices, the extremal admissible values of the circuit reliability $r = \varphi(x,y) \in [r_{min}, r_{max}]$ are described for $(x,y) \in [0,1]^2$, with the complementary components arbitrarily fixed.[2]

The ordering of the slice-reliability range limits from Table 1 can be justified as follows: first we note that the pencil equation has the form

$$r = \varphi(x, y) \Leftrightarrow r = \alpha(x - x_C)(y - y_C) + k,$$

with α and k depending on the three complementary components. Then we consider the center-dependent function $\tau(C) = \text{sign}\,(\alpha \cdot (\chi_{\mathbb{D}}(C) - \chi_{\mathbb{D}'}(C)))$, where χ is the characteristic function of its index set; τ has for each of the ten choices constant a sign, $\varphi(x,y)$ is lexicographically increasing/decreasing in the couple of its arguments for $\tau = +1/\tau = -1$, and the range $[r_{min}, r_{max}]$ of the slice-reliability φ will be, accordingly, $[\varphi(0,0), \varphi(1,1)]$ vs. $[\varphi(1,1), \varphi(0,0)]$.

We note that the algebraic curve given by (3.1) is of order two (namely, a conic of hyperbolic type), only if the indicator α from Table 1 is non-zero. In this case the slice-reliability φ is quadratic in the variables x,y.

In the quadratic case, the displacement of *centers of symmetry* of a pencil

$$\Gamma_r = \left\{(x, y) \in [0, 1]^2 \middle| r = \varphi(x, y)\right\}, \text{ for } r \in [r_{min}, r_{max}] \subset [0, 1], \tag{3.3}$$

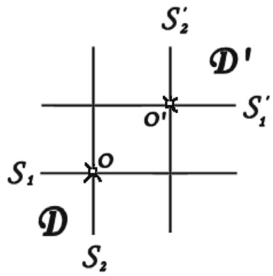

Fig. 2. Admissible locations for the centers of nondegenerate constant reliability paths.

plays an essential role for describing the geometric features of the pencil of curves and is able to provide a two-fold classification of the pencils for each of the ten choices.

The possible regions of the xOy plane which contain the centers of symmetry $C = (x_c, y_c) \in \mathbb{R}^2$ of the hyperbolic paths (3.3) characterized by the condition $\delta \equiv -\alpha^2/4 < 0$ are generally specified in Table 2, while Fig. 2 illustrates these locations.

More specifically, in the quadratic ($\alpha \neq 0$) case, Table 3 indicates the coordinates of the centers of the pencils $\{\Gamma_r\}$ for each of the 10 cases, where we denote

$$\pi = \frac{ae}{(1-a)(1-e)}, \quad n = (1-a)(1-e)(c-\pi),$$

$$(x_1, y_1) = \left(1 + \frac{e(1-d)}{d(c+e-ce)}, 1 + \frac{(1-c)(1-e)}{c+e-ce}\right),$$

$$(x_{10}, y_{10}) = \left(1 + \frac{(1-a)(1-c)}{a+c-ac}, 1 + \frac{a(1-b)}{b(a+c-ac)}\right).$$

We easily note that the location $\mathbb{D}$ corresponds to pencils of *convex* hyperbolic clipped branches, while $\mathbb{D}'$, to concave pencils.

The complementary classes to the one of quadratic dependence, are the ones where either the slice-reliability φ varies in affine manner in terms of (x,y), or it is constant. The corresponding ten choices for linear and constant slice-reliability function are detailed[3] in Table 4, where we point out, like above, the equations of constant reliability paths, and the slice-reliability admissible range $[r_{min}, r_{max}]$.

[2] Here $x = R_i$, $y = R_j$ and the complementary components are appropriately selected from the parameters 5-uple $(\rho_1, \rho_2, \rho_3, \rho_4, \rho_5) = (a,b,c,d,e)$.

[3] In Table 4 we denoted $\psi(x,y) = xd(1-b) + yb(1-d) + bd$, $\mu_{uv} = u + v - uv$ and $\sigma(a,e,c) = [a \neq 1 \wedge e \neq 1 \wedge a + e \leq 1 \wedge c = ae/(1-e)(1-a)]$.

Table 3
Location of centers of symmetry in the quadratic slice-reliability case.

#	Hyperbolicity conditions	Center (x_C, y_C)	Center location
1.	$d\neq 0\wedge(c\neq 0\vee e\neq 0)$	(x_1, y_1)	$\mathbb{D}'$
2.	$b\neq 0\wedge d\neq 0\vee e\neq 1$	$(1, 1+1-b/b(1-e))$	$\{O'\}\cup S'_2\subset\mathbb{D}'$
3.	$b\neq 1\vee(c\neq 1\wedge e\neq 1)$	$(-bc(1-e)/1$ $-b(c+e-ce), 0)$	$\{O\}\cup S_1\subset\mathbb{D}$
4.	$b\neq 0\wedge d\neq 0\wedge c\neq 1$	$(1+1-d/d(1-c),$ $1+1-b/b(1-c))$	$\mathbb{D}'$
5.	$a\neq 1\wedge d\neq 0\wedge e\neq 1$	$(0, -(1-ad)e$ $/d(1-a)(1-e))$	$\{O\}\cup S_2\subset\mathbb{D}$
6.	$a=1\wedge e\neq 0$	$(1+1-e/e, 1)$	$S'_1\subset\mathbb{D}'$
	$e=1\wedge a\neq 0$	$(1, 1+1-a/a)$	$S'_2\subset\mathbb{D}'$
	$a\neq 1\wedge e\neq 1\wedge c<\pi$	$(-a/n, -e/n)$	$D'\subset\mathbb{D}'$
	$a\neq 1\wedge e\neq 1\wedge c>\pi$		$D\subset\mathbb{D}$
7.	$a\neq 1\vee(c\neq 1\wedge d\neq 1)$	$(0, -cd(1-a)$ $/1-d(a+c-ac))$	$\{O\}\cup S_2\subset\mathbb{D}$
8.	$b\neq 0\wedge a\neq 1\wedge e\neq 1$	$(-1-be$ $/b(1-a)(1-e), 0)$	$\{O\}\cup S_1\subset\mathbb{D}$
9.	$b\neq 0\wedge d\neq 0\wedge a\neq 1$	$(1+1-d/d(1-a), 1)$	$\{O\}'\cup S'_1\subset\mathbb{D}'$
10.	$b\neq 0\wedge(a\neq 0\vee c\neq 0)$	(x_{10}, y_{10})	$\mathbb{D}'$

Table 4
Linear and constant slice reliability classes.

#	Constraints	Subcase	$\varphi(x,y)$	range φ
1.	$d=0\vee(c=e=0)$	$d=0, e\neq 0$	ey	$[0,e]$
		$c=e=0$	dx	$[0,d]$
		$d=e=0$	0	$\{0\}$
2.	$b=0\vee d=0\vee e=1$	$b=0, d\neq 0$	dx	$[0,d]$
		$d=0$	be	$\{be\}$
		$e=1, d\neq 0, b\neq 1$	$b+dx(1-b)$	$[b,\mu_{bd}]$
		$b=d=0$	0	$\{0\}$
3.	$b=1\wedge(c=1\vee e=1)$	$b=e=1$	$r=1$	$\{1\}$
		$b=c=1, e\neq 1$	$e+y(1-e)$	$[e,1]$
4.	$b=0\vee d=0\vee c=1$	$b=0, d\neq 0$	dx	$[0,d]$
		$d=0, b\neq 0$	by	$[bdc,\mu_{bd}]$
		$c=1$	$\psi(x,y)$	$[bd,\mu_{bd}]$
		$b=d=0$	0	$\{0\}$
5.	$d=0\vee a=1\vee e=1$	$a=1, e\neq 0, d\neq 1$	$d+xe(1-d)$	$[d,\mu_{de}]$
		$e=1\wedge(d\neq 1\vee a\neq 1)$	$ad+x(1-ad)$	$[ad,1]$
		$a=1, e=0$	d	$\{d\}$
6.	$c(1-e)(1-a)-ae=0$	$\sigma(a,e,c)$	$xe+ay$	$[0,e+a]$
		$e=1\wedge a=0$	x	$[0,1]$
		$a=1\wedge e=0$	y	$[0,1]$
		$a=e=0$	0	$\{0\}$
7.	$a=1\wedge c=1\wedge d=1$	$a=d=1$	1	$\{1\}$
		$c=d=1, a\neq 1$	$a+x(1-a)$	$[a,1]$
8.	$b=0\vee a=1\vee e=1$	$b=0, a\neq 0$	ay	$[0,a]$
		$e=1, a\neq 0, b\neq 1$	$b+ya(1-b)$	$[b,\mu_{ab}]$
		$a=1\wedge(b\neq 1\vee e\neq 1)$	$be+y(1-be)$	$[be,1]$
		$a=0, b=e=1$	be	$\{be\}$
9.	$b=0\vee d=0\vee a=1$	$b=0$	ad	$\{ad\}$
		$d=0\wedge b\neq 0$	by	$[0,b]$
		$a=1\wedge b\neq 0\wedge d\neq 1$	$d+yb(1-d)$	$[d,\mu_{bd}]$
		$a=d=1$	1	$\{1\}$
10.	$b=0\vee(c=a=0)$	$b=0, a\neq 0$	ax	$[0,a]$
		$a=c=0\wedge b\neq 0$	by	$[0,b]$
		$a=1\wedge b\neq 0\wedge d\neq 1$	$d+yb(1-d)$	$[d,\mu_{bd}]$
		$a=b=0$	$r=0$	$\{0\}$

Table 5
Sample quadratic slice-reliability range and symmetry center data.

#	i,j	$[r_{min}, r_{max}]$	Center	C-S
1.	1,2	[0., 0.9125]	(1.2421, 1.1173)	1010
2.	1,3	[0.5525, 0.8881]	(1., 1.5042)	1010
3.	1,4	[0.5525, 1]	(−0.8704, 0.)	0101
4.	1,5	[0.4462, 0.9625]	(2.1111, 1.5882)	1010
5.	2,3	[0.6000, 0.9125]	(0.,−4.9524)	0101
6.	2,4	[0., 0.9790]	(1.6985, 1.3800)	1010
7.	2,5	[0.6000, 1]	(0., −0.3559)	0101
8.	3,4	[0.5525, 0.9700]	(−6.0168, 0.)	0101
9.	3,5	[0.6000, 0.9400]	(2.6667, 1.)	1010
10.	4,5	[0., 0.9700]	(1.0638, 1.1502)	1010

4. Illustrative examples of slice-reliability classes

We illustrate below the slice-reliability classes (quadratic/linear/constant) of the constant reliability paths (3.3). To this aim, for the quadratic case, we numerically fix inside φ, by means of (3.2) the complementary components from the set of values

$$(\rho_1, \rho_2, \rho_3, \rho_4, \rho_5) = (0.8, 0.85, 0.7, 0.75, 0.65). \qquad (4.1)$$

Then the expression of the slice-reliability φ given by (3.2) will effectively depend only on the variables $(x,y)\in[0,1]^2$. The obtained constant level curves, for the above data (4.1) are arcs of hyperbolas. The locations of their centers of symmetry $C\in\mathbb{R}^2$, referred to the admissible clipping square $[0,1]^2$, can be described by means of the Cohen-Southerland binary code $TBRL_2$ = top/bottom/right/left, in this particular case and for the choices of slice-components, are described[4] in Table 5, which provides, as well, the admissible slice-reliability intervals $[r_{min}, r_{max}]$ for each choice $(p_i, p_j)=(x,y)$.

In Fig. 3 we plot the two hyperbolic families both in plane (as pencils) and in space (as Monge surfaces which project the h-cuts onto the pencil).

The two depicted subclasses[5] contain convex/concave pencils of hyperbolic branches clipped by the square domain $[0,1]^2$.

The remaining classes — when the slice-reliability is either linear in x, in y, in both x and y, or constant within the slice[6] — are illustrated in Fig. 4.

[4] The Cohen-Southerland codes correspond to the classified locations of centers via $[1010]\supset\mathbb{D}$, and $[0101]\supset\mathbb{D}'$.

[5] The plots for the first subclass were obtained for the choice #2 and $p=(x,\rho_2,y,\rho_4,\rho_5)$, while the second ones, for the choice #7, and $p=(\rho_1,x,\rho_3,\rho_4,y)$.

[6] The four subclasses plots were obtained, respectively, for: choice #1, $p=(x,y,\rho_3,0,\rho_5)$; choice #2, $p=(x,0,y,\rho_4,\rho_5)$; choice #2, $p=(x,\rho_2,y,0,\rho_5)$; and choice #6, $p = (1/2, x, \rho_3, y, 1/2)$.

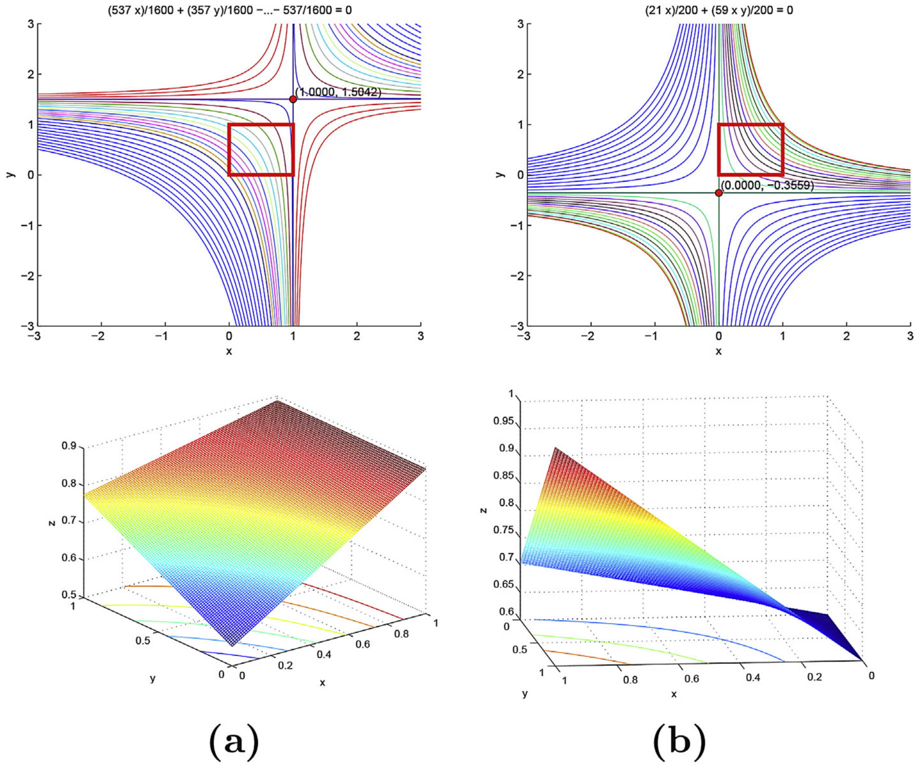

Fig. 3. Quadratic subclasses: (a) concave; (b) convex.

5. Slice-reliability classes and the tuning of the complex system

The classes of slice-reliability constant level (3.3) can be used to tune the components of the complex system, which exhibits a freedom in the properties of its slice-components. Tables 1–4 allow to develop specific techniques for preserving or enhancing the reliability of the system, as follows:

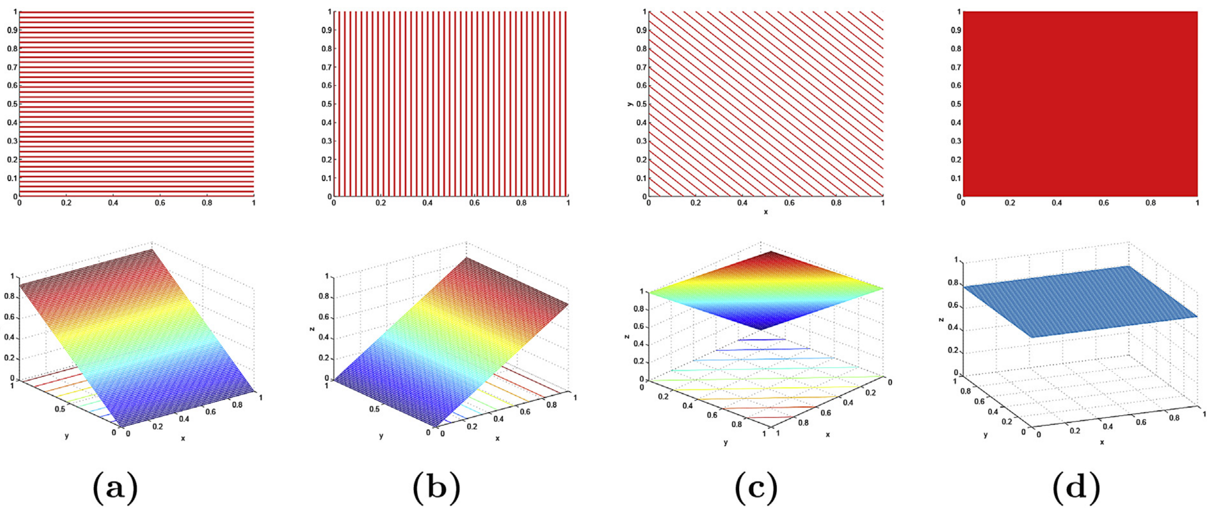

Fig. 4. Secondary subclasses. Linear slice-reliability: (a) in *x*; (b) in *y*; (c) in *x, y*. Constant slice-reliability: (d).

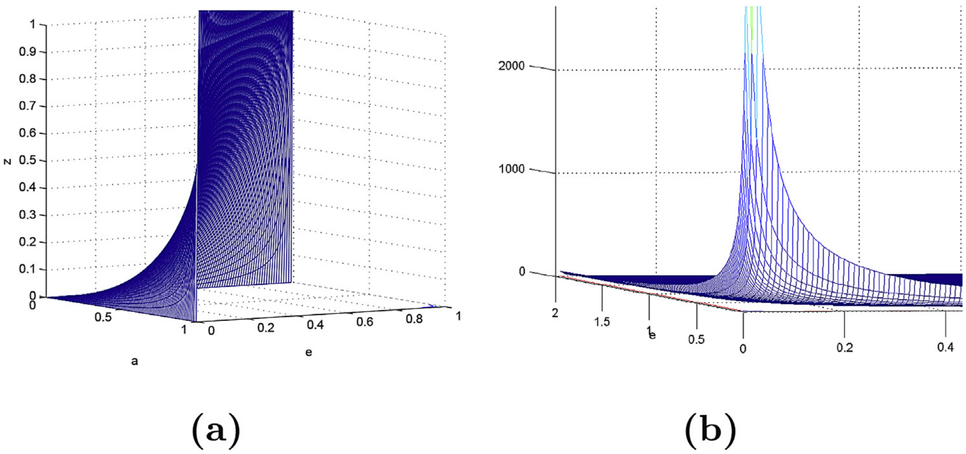

Fig. 5. The degeneracy surface in case 6 (a) and its Tzitzeica sibling (b).

1. In both quadratic and linear cases, the slice-reliability maxima and minima can be easily evaluated in terms of the complementary components, by using Tables 1 and 4, which specify in concrete applications the feasible reliability intervals.
2. In the quadratic slice-reliability case, centers of symmetry C, according to Table 3, reside only in the Cohen-Southerland regions 1010 ($\mathbb{D}$), or 0101 ($\mathbb{D}'$). This has a direct effect on the shape of the slice-reliability constant level curves (see, e.g., Table 5 and Fig. 3). Since the curves are monotonic (decreasing), a gradual decrease of one slice-component can be compensated by a corresponding gradual increase of the pairing one, in order to preserve the system reliability level. Moreover, in the convex subcase (when the symmetry center C lies in $\mathbb{D}$, for the pair of slice-components (x,y) located on the constant level slice-reliability curve $r=\varphi(x,y)$ ($r=const.\in[r_{min},r_{max}]\subset[0,1]$), if[7] $(x_*,y_*)\in(0,1)^2$ and $x<<x_*$, a slight drop in the probability of success of the component $x=R_i$ requires a big increase in the probability of the component $y=R_j$, in order to maintain the system reliability. A similar occurrence is valid in the concave case (where $C\in\mathbb{D}'$), but for $x>>x_*$.
3. In the linear case (except of the choice 6), Table 4 flags out the situations when the slice-reliability depends on just one of the two slice-components; in these cases the pairing active component plays no role in improving or decaying the system reliability.
4. As well, Table 4 flags out the choices and chosen parameters for which the reliability remains constant, in spite of the variation of both active components, which play no role in the functioning of the system.
5. The choice 6 in Table 4 reveals the possibility of linear slice-dependence on both the slice-components, which both *linearly* affect the constrained system reliability. We note in this case that the particular surface $c=ae/(1-e)(1-a)$ − represented in the system of coordinates $Oaec$ in Fig. 5(a)- is diffeomorphically equivalent via the mapping[8] $(a,e,c)\to(a',e',c')=(a^{-1}-1,e^{-1}-1,c)$, with the Tzitzeica surface $a'e'c'=1$ from Fig. 5 (b).

6. Conclusions

In the present study, the reliability function of the bridge complex system specified in Fig. 1 was explicitly determined. The system reliability function (2.1) induces for each choice of slice-components the slice-reliability given by (3.2), which is much more tractable than the overall 5-variable reliability mapping.

From geometric point of view, the reliability constant level hypersurfaces intersected with the canonic coordinate 2-planes provide (via ten distinct choices)

[7] Here $(x_*,y_*)\in\mathbb{R}^2$ satisfies the system $\{\varphi(x,y)=r;\ \varphi_x(x,y)=-\varphi_y(x,y)\}$, whose solutions are the two locations where the slope of the conic is −1. Among the solutions, a single one (if any) *might* belong to the clipping domain $(0,1)^2$; this is further considered.

[8] This is essentially a product of 1D-inversions and translations in each of the first two coordinates.

slices which contain pencils of algebraic curves, which are classified according to: algebraic order, convexity and type of dependence on the slice-variables.

Illustrative examples of class representative pencils are provided for the basic classes of slice-reliability: quadratic, linear and constant; the associated subclasses are exemplified as well (the quadratic convex/concave, and three subtypes of linear dependency).

The developed classification of the associated constant level slice-curves (3.3) is shown to provide the ranges of the attendable slice-reliability (Tables 1 and 4) and hence easily evaluate extremal bounds of constrained system reliability. As well, in Section 5 it is shown that the classification allows to develop specific component-replacement methods to easily redesign the system while having a failing component, by modifying the (success probability of) its pairing slice-component, considering the need to maintain or increase the system reliability.

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