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Optimization Of The Procedure of Iterative Data Transformations

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Optimization Of The Procedure of Iterative Data Transformations

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Abstract .In this article done the review for results of research on algorithms for iterative transformations of noise-immune codes. where the similar decoding procedures provide a significant increase in the efficiency of soft decoders classification of such algorithms is given. The optimality of the algorithm is proved by the criterion of the rate of achievement of the final result when using the step-by-step correction of soft solutions.

Keywords : Iterative Data , soft decoders , noise-immune codes

1- Introduction

Iterative decoding algorithms for channels with unknown phase have attracted an increasing interest in the recent literature, wide application in modern control systems of radio communication means poses a number of new tasks, the solution of which is aimed at the effective use of soft decoders of noise-immune codes. The main advantage of such decoders is the possibility of implementing iterative data transformations in them to obtain additional energy gain in the communication system [1, 2, 3]. However, the implementation of these transformations may unnecessarily increase the processing time of information, and therefore the procedure of iterative data transformations needs to be optimized in terms of the speed parameter for obtaining the final result. The goal of the work is optimization and unification of the procedure of iterative transformations of soft decoder solutions by the criterion of minimum iterations when calculating the most probable solution.

2- Formulation of the problem

Let there be given a closed set of sequences of finite length, which are words of a systematic corrective code. The symbols of these words are chosen from the finite alphabet $X = \{x\}$ and fixed by the receiver in the form of hard decisions. Consider a binary code for which x takes the values 0 or 1. We denote by $x_0 = \left(x_0^{(1)},\ldots,x_0^{(n)}\right)$ the channel transmitted along the channel, and by $x_s = \left(x_s^{(1)},\ldots,x_s^{(n)}\right)$, $s = \overline{1,S}$ other sequences of the set under consideration. The problem is to estimate the conditions under which, after passing through the channel of the sequence x_0 and performing the procedure of iterative transformations over the corresponding soft solutions, the likelihood functions C_S of all alternative sequences x_s will be less than the likelihood function C_0 of the sequence x_0.

With soft decoding, each *i*-th bit, $i = \overline{1,n}$ is represented as a hard decision, accompanied by a Soft Decision in the form of some real α_i, $\alpha = \overline{\alpha_{\min}, \alpha_{\max}}$ [4, 5]. Denoting the hard solution 0 by the sign «-», and the solution 1 by «+», for the tuple of binary data …1 0 0 1 1… we obtain a sequence of the form$\ldots + \alpha_i - \alpha_{i+1} - \alpha_{i+2} + \alpha_{i+3} + \alpha_{i+4} \ldots$ which is processed by a soft decoder according to the rule:

$$L(ª_{ki}) + L(ª_{pc}) \approx (-1)^{1-m} \times sign[L(ª_{ki})] \times sign[L(ª_{pc})] \times \min(|L(ª_{ki})|, |L(ª_{pc})|)\ , \tag{1}$$

where the function $sign(\bullet)$ returns the sign of its argument;

$L(ª_{ki})$ – index of the soft-decision symbols involved in the formation of a check bit;

$L(ª_{pc})$ – index of the soft-decision symbol verification;

m – the number of positive soft-solutions excluded from the analysis included in the corrected vector [5].

The procedure (1) helps to increase the value of C_0, but its outcome is not unambiguous. We denote by $(+pc)$ the execution of the parity condition on the receiving side for the received code vector. Otherwise, the receiver fixes the value $(-pc)$. The work of a decoder with a system of iterative transformations and parity checks is expedient to be described by an objective function of the form.

$$Q\{S; M(ª); \pm(ª)\} \Rightarrow \underset{S\to(-pc)}{sign(S)}; \underset{\max}{|M(ª)|}; \underset{\min}{\pm(ª)},$$

where S – The parity value for all information bits of the received vector;

parameter $|M(ª)|$ – the absolute value of the mean value of the tuple of soft symbol solutions;

parameter $\pm(ª)$ is an indicator of the spread of soft solutions, calculated by the rule:

$$\pm(ª) = (1/n-1)\sum_{i=1}^{n}(|M(ª)| - |ª_i|)^2.$$

Studies have shown that individually presents parameters are not informative and do not allow to evaluate the order of processing of several code vectors in a system with the product of codes [6]. In accordance with $Q\{\bullet\}$, the decoder performs the parity check in the first decoding step, estimates the average value of the received soft-decision indices of the symbols in the second data processing step and, lastly, determines the scatter of the receiver fixed indices. The maximum value of $|M(ª)|$ corresponds to the high confidence of the received symbols, but a lot of identical values of $|M(ª)|$ can be obtained for a different set of estimates, therefore it is necessary to additionally estimate the parameter $\pm(ª)$. If there is a situation of uncertainty, when $|M_i(ª)| = |M_j(ª)|$ when $i \neq j$, then the priority for subsequent data processing is a combination of which $\pm_i(ª) < \pm_j(ª)$. This fully corresponds to the principle of spreading confidence during the decoding of a group of code combinations [6].

3- The properties of an iterative process that converges to significant estimates

The procedure for correcting two information bits from a sequence of lengths n with values $L(ª_{k1})$ и $L(ª_{k2})$, executed according to (1), for the step of iteration with the number j has the form:

$$\Omega_j = \begin{cases} \min(sign[L(ª_{k1}) + L(ª_{cork2})_j] \times signL(ª_{pc})) \approx sign(ª_{cork2})_{j+1} \times (-1)^{1-m}; \\ \min(sign[L(ª_{k2}) + L(ª_{cork1})_j] \times signL(ª_{pc})) \approx sign(ª_{cork1})_{j+1} \times (-1)^{1-m}. \end{cases} \tag{2}$$

In accordance with the Bayes principle when $j = 1$ in the first step of the iteration, a posteriori estimates $ª_{cork1_1} = ª_{cork2_1} = 0$ [3]. From (2) it follows that when $L(ª_{k1}) = L(ª_{k2})$, any $L(ª_{pc})$ and realization of the condition $(-pc)$, the correction procedure loses its meaning due to the equality of a posteriori estimates $a_{cork1j} = a_{cork2j}$, from here $C_S = C_0$. Consequently, for $(-pc)$ the condition $L(a_{k1}) \neq L(a_{k2})$ is necessary for changing the values of the likelihood function. When $(+pc)$ is realized and the condition $a_{cork1} = a_{cork2}$ is satisfied, the inequality $C_S < C_0$ is always guaranteed Let $L(\breve{a}_{ki})$ or $L(\widehat{a}_{ki})$ correspond to a decrease or increase in the index of a soft solution. The iterative process is defined as converging to significant estimates, if, when some $j = j_{cs}$ is reached, the values of $L(\breve{a}_{ki})$ and $L(\widehat{a}_{ki})$ do not change their sign for all $j > j_{cs}$. For example, on the receiving side a vector with a soft-solution of the form is obtained: $V = +7 + 6 + \hat{3}\langle -7 \rangle$, where in the angle brackets the test digit is shown, and the symbol of the $\hat{d}$ form represents an erroneous discharge of the code combination. In the example for the reduced vector, the condition $(-pc)$ is fulfilled, therefore in the preliminary step of data processing the decoder excludes from the combination the symbol $+7$, which is the most reliable. Since this symbol is followed by a «+» sign, the value of m is assumed to be 1, and the subsequent steps of the iterative transformations are shown below.

$$\Omega_1 = \begin{cases} [+3+0]\langle -7 \rangle = -3; \\ [+6+0]\langle -7 \rangle = -6. \end{cases} \quad \Omega_2 = \begin{cases} [+3-6]\langle -7 \rangle = +3; \\ [+6-3]\langle -7 \rangle = -3. \end{cases} \quad \Omega_3 = \begin{cases} [+3-3]\langle -7 \rangle = 0; \\ [+6+3]\langle -7 \rangle = -6. \end{cases}$$

$$\Omega_4 = \begin{cases} [+3-7]\langle -7 \rangle = +4; \\ [+6+0]\langle -7 \rangle = -6. \end{cases} \quad \Omega_5 = \begin{cases} [+3-6]\langle -7 \rangle = +3; \\ [+6+4]\langle -7 \rangle = -7. \end{cases} \quad \Omega_6 = \begin{cases} [+3-7]\langle -7 \rangle = +4; \\ [+6+3]\langle -7 \rangle = -7. \end{cases}$$

$$\Omega_7 = \begin{cases} [+3-7]\langle -7 \rangle = +4; \\ [+6+4]\langle -7 \rangle = -7. \end{cases} \dots$$

It is noteworthy that, starting from Ω_4, the values of a posteriori estimates do not change their sign, and the results of calculations for Ω_6, Ω_7 and subsequent do not change. Based on this Ω_4 is taken as a significant estimate. The result of the full cycle of transformations is the sequence:

$$V = +7 \quad (+6+4) \quad (+\hat{3}-7) \quad \langle -7 \rangle = +7 \quad +10 \quad -4 \quad \langle -7 \rangle .$$

There was a reconstruction of the wrongly received vector, and the condition $(+pc)$ is fulfilled for it. The researcher estimate the variations between the values of $L(a_{k1})$, $L(a_{k2})$, and $L(a_{pc})$, at which the inequality is reached $C_S < C_0$.

When $(+pc)$ and $L(a_{pc}) = a_{\max}$, the $L(a_{ki})$ adjustment is performed in just one iteration step. If $L(a_{k1}) + L(a_{cork2}) \geq a_{\max}$, then $L(\widehat{a}_{ki}) > a_{\max}$ is possible and $C_S < C_0$ is satisfied. To conserve the receiver processor's bitmap grid at $L(\widehat{a}_{ki}) > a_{\max}$, it is advisable to perform the equality $L(\widehat{a}_{ki}) = a_{\max}$.

If $L(a_{k1}) + L(a_{cork2})_j < a_{\max}$ (corrected symbols have low soft decision indices), but $(+pc)$, correction to level $L(\widehat{a}_{ki}) = a_{\max}$ is carried out in several steps, even if one of the corrected soft-decision indexes $L(a_{ki}) = a_{\min} = 0$.

in case of $(-pc)$ and $L(a_{pc}) = a_{\max}$ the result of correction depends on the ratio of the value modules $L(a_{k1})$ and $L(a_{k2})$. The correction procedure is always successful if the estimates that fall under the control are sufficiently distinct, for example, $L(a_{k1}) = a_{\min}$ and $L(a_{k2}) = a_{\max}$,when the symbol with the value of $a_{\min}$ is erroneous. At the first step of the iterative transformations $a_{cork1_1} = a_{cork2_1} = 0$. Hence the new values for the corrected symbols are equal $L(a_{k1}) = L(a_{k1}) + a_{\max}$ and $L(a_{k2}) = L(a_{k2}) + L(a_{\min})$, that is, the sign is inversed for $L(a_{k1})$ and preservation of the mark for $L(a_{k2})$. If $L(a_{k1}) \neq a_{\min}$, but close to this value, then the uncertainty situation is solved in several iterations and the condition $C_S < C_0$ is satisfied.

Let $a_{cork1} < a_{cork2}$ and $L(a_{k2})$ represent an erroneous discharge, and the value $L(a_{pc}) = a_{\max}$. Due to the proximity of the $L(a_{k1})$ and $L(a_{k2})$ indices, the correction procedure will be erroneous, error propagation occurs and $C_S > C_0$. Therefore, it is necessary to minimize the correlation of high indicators of soft-decisions with errors [6]. The multiplier $\min(\bullet)$ in (1) indicates the advisability of forming integer soft-solutions, since the increment of the values of the estimates is determined by the relation $\Delta = |a_{k1} - a_{k2}|$, and for rational indices of soft-solutions a larger volume of iterations is needed.

4- Iterative process with step-by-step correction of soft-solutions

An iterative process in which a posteriori estimates of the j-th step are used to correct the indices of soft-solutions obtained at the $(j-1)$-th step is defined as an algorithm for stepwise correction of the values of soft-solutions. Suppose given $L(a_{k1})$, $L(a_{k2})$, $L(a_{pc})$, let $L(a_{k1})$ and $L(a_{k2})$ They are not equal in absolute values and have different signs. Since the $a_{cork1_1} = a_{cork2_1} = 0$, then the first step of the iteration of the classical algorithm is skipped and replaced by the calculation of the algebraic sum $L(a_{k1})$ and $L(a_{k2})$. Thus, for the above example, the iteration for Ω_1 is formal and can be excluded from the data transformation algorithm. On the basis of the commutativity property, the operation for Ω_2 can be performed only once to obtain $a_{cork1_2} = a_{cork2_2}$. Therefore, instead of four cycles of iterations in the new algorithm, only one cycle of computations with results $L(a_{k1_2}) = L(a_{k1_1}) + a_{cork1_2}$ and $L(a_{k2_2}) = L(a_{k2_1}) + a_{cork2_2}$. The repetition of this procedure provides a sharp increase in the estimates, in contrast to the smooth growth of similar indicators in the performance of the classical algorithm. The resulting gain in the number of iterations is shown in the figure. 1..

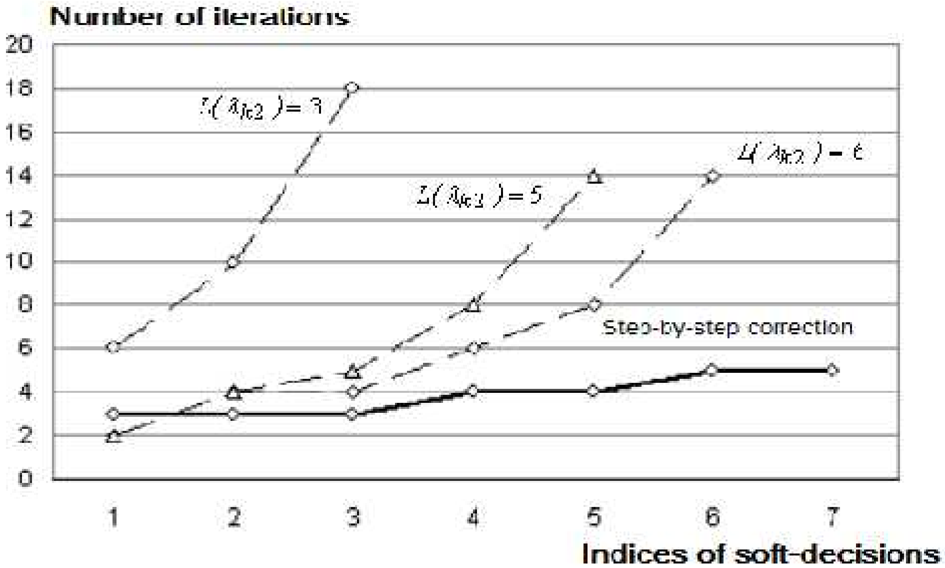

Figure 1. Comparative characteristics of the method of significant values and step-by-step correction by the number of iterations

The proposed algorithm is not effective in minimizing the number of iterative cycles if $L(\text{a}_{k1})$ and $L(\text{a}_{k2})$ have the same signs. In this case, it is suggested to perform cyclic shifts of the symbols, $L(\text{a}_{k1})$, $L(\text{a}_{k2})$ and $L(\text{a}_{pc})$ to the left (right) so that in place of the verification symbol, there are soft solutions with the largest index from the alternative $L(\text{a}_{k1})$ or $L(\text{a}_{k2})$. Naturally, after performing the necessary number of iterations in the algorithm, an inverse cyclic shift of the symbols to the right (left) is provided for the subsequent processing of the symbols.

Under the condition of $L(\text{a}_{k1}) = L(\text{a}_{k2})$, the execution of the algorithm leads to a situation of uncertainty, which can only be resolved through cross checks on other verification relations. We will show this with an example. Suppose that in a product of second-order codes on the receiving side a matrix of the form is formed:

$$\begin{array}{cccccc}
+5 & -\hat{1} & +7 & \langle +7 \rangle & -5 & 8 \\
-7 & +6 & +7 & \langle -7 \rangle & +6{,}75 & 0{,}25 \\
+6 & +\hat{3} & +4 & \langle -3 \rangle & -4{,}00 & 2{,}00 \\
\langle -7 \rangle & \langle -7 \rangle & \langle +6 \rangle & \langle +7 \rangle & +6{,}75 & 0{,}25 \\
+6{,}25 & +4{,}25 & +6 & 6{,}00 & 0 & 0 \\
0{,}92 & 7{,}58 & 2{,}0 & 4{,}00 & 0 & 0.
\end{array}$$

In this matrix, the right column and the bottom row represent the values $\pm(\text{a})$, and the previous column and row represent values $|M(\text{a})|$, The sign of this parameter indicates that condition $(+pc)$ or $(-pc)$. Since in the first line of this matrix the value of $M(\text{a})$ is negative and it is the largest in absolute value among the detected negative values of $M(\text{a})$, the decoder selects this line to correct the data. It temporarily removes the symbol $(+7)$, so $m = 1$. The string takes the form:

$$+5 \quad -\hat{1} \quad (+7) \quad \langle +7 \rangle \quad \Rightarrow \quad +5 \quad -\hat{1} \quad \langle +7 \rangle .$$

Further, in accordance with the algorithm of step-by-step correction:

$$[-1+5] \quad \langle +7 \rangle = +4 - \text{ correction score for the symbol (+5);}$$

$$[+5-1] \quad \langle +7 \rangle = +4 - \text{ correction score for the symbol } (-\hat{1}).$$

The result of the correction for the selected line: $(+5+4)(-1+4)+7\langle +7 \rangle = +7 \quad +3 \quad +7 \quad \langle +7 \rangle$.

To store the bit grid of the MP indexes, their values greater than 7 in absolute value change to a value of 7.

The matrix takes the form:

$$\begin{matrix} +5 & +3 & +7 & \langle +7 \rangle & 5{,}50 & 3{,}67 \\ -7 & +6 & +7 & \langle -7 \rangle & +6{,}75 & 0{,}25 \\ +6 & +\hat{3} & +4 & \langle -3 \rangle & -4{,}00 & 2{,}00 \\ \langle -7 \rangle & \langle -7 \rangle & \langle +6 \rangle & \langle +7 \rangle & +6{,}75 & 0{,}25 \\ +6{,}25 & -4{,}75 & +6{,}00 & 6{,}00 & 0 & 0 \\ 0{,}92 & 4{,}25 & 2{,}00 & 4{,}00 & 0 & 0. \end{matrix}$$

In accordance with the objective function $Q\{\bullet\}$ For subsequent correction, a sequence $(+3) \quad +6 \quad +\hat{3} \quad \langle -7 \rangle \quad \Rightarrow \quad +6 \quad +\hat{3} \quad \langle -7 \rangle$, wherein $m=1$. The symbol (+3) is deleted, because it was corrected at the previous step and its reliability is high enough (the essence of cross checking). This index in absolute value in this sequence is equal to the index of another corrected symbol. With the same signs of the corrected symbols, the iterative transformation process becomes undefined. To eliminate this negative effect, the sequence $+6 \quad +\hat{3} \quad \langle -7 \rangle$ is cyclically shifted one step to the left. The new sequence $+\hat{3} \quad \langle -7 \rangle \quad +6$ thus obtained is processed in the usual way:

$$[-7+3]+6 = -4 - \text{ new a posteriori score for a character } (+\hat{3});$$
$$[+3-7]+6 = -4 - \text{ new a posteriori score for a character } (-7).$$

After correction, the sequence $+\hat{3} \quad \langle -7 \rangle \quad +6$ is reduced to the form $-1\langle -14 \rangle + 6 = -1\langle -7 \rangle + 6$ and in order to increase the reliability of the result obtained, the second step of the iterative transformations is performed, which ensures the index of soft decisions for the symbol is hanged -1. This iteration step is implemented by algorithm similar to the computational procedure of the first step:

$$[-7-1]+6 = -6 - \text{ new a posteriori score for a character } (-1);$$
$$[-1-7]+6 = -6 - \text{ new a posteriori score for a character } (-7).$$

The obtained sequence $-7\langle -14\rangle +6$ is reduced to the form $-7\langle -7\rangle +6$ and after the cyclic shift to the right and the addition of the crossed out symbol, the final data series is formed: $+3 \quad +6 \quad -7 \quad -7.$

The matrix is restored and takes the form:

$$\begin{array}{cccccc} +5 & +3 & +7 & +7 & +5{,}50 & 3{,}67 \\ -7 & +6 & +7 & -7 & +6{,}75 & 0{,}25 \\ +6 & -7 & +4 & -3 & +5{,}00 & 3{,}33 \\ -7 & -7 & +6 & +7 & +6{,}75 & 0{,}25 \\ +6{,}25 & +5{,}75 & +6{,}00 & +6{,}00 & 0 & 0 \\ 0{,}92 & 3{,}58 & 2{,}00 & 4{,}00 & 0 & 0. \end{array}$$

5- Conclusion

Comparison of the presented methods of iterative transformations of soft solutions of symbols of code combinations shows a significant advantage of the stepwise correction method with respect to the method of significant estimates.

When implementing decoders of noise-immune codes using iterative transformations, it is advisable to exclude situations where the values of the corrected symbols are equal and, in the event of such situations, resort to cross-checks, for example, when using code products of dimension greater than two. When using such codes, it is necessary to apply the proposed objective function with the determination of the fulfillment of the parity rules, the modulus of the average value of soft solutions, and the spread of estimates.

It is inadmissible to use soft solutions with low reliability indicators as index tests. In the case of such situations, it is permissible to use cyclic shifts of symbols subjected to iterative processing with subsequent inverse transformation by cyclic permutations.

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