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On Harmonic Univalent Functions Defined by Dziok-Srivastava Operator

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ABSTRACT

The purpose of this work is to present a class of harmonic univalent functions defined by the Dziok-Srivastava operator. Some geometric properties like coefficients conditions, distortion theorem, convolution (Hadamard product), convex combination and extreme points are investigated.

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1. Introduction

Let A denote the class of functions $f(z)$ which are analytic in the open unit disk $U=\{z:|z|<1\}$. Each $f \in$ A can be represented by $f = h + \overline{g}$, with h and g of analytic type in U. We say that h is an analytic part and g the related co-analytic part of f (see[2]). Thus for $f = h + \overline{g} \in$ A , we can write

$h(z) = z + \sum_{n=2}^{\infty} a_n z^n$, $g(z) = \sum_{n=1}^{\infty} b_n z^n$, $(0 \leq |b_1| < 1)$ …… (1)

Let $f = h + \overline{g}$ given by (1) and $F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda}$ is Dziok-Srivastava operator of f and is given by [8]

$$F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z) = z + \sum_{n=2}^{\infty} \frac{\Pi_{j=1}^{p}(\lambda_j+1)_{n-1}}{\Pi_{j=1}^{q}(\mu_j+1)_{n-1}} \left(\frac{a+1}{a+n}\right)^s \frac{\Lambda(a+n,b,s,\lambda)}{\Lambda(a+1,b,s,\lambda)} a_n \frac{z^n}{n!}$$

…. (2)

Where

$(p,q \in N_O;\ \lambda_j \in C,\ \ (j = 1,2,\dots,p);\ \ a,\mu_j \in \frac{C}{Z_O^-},$ $(j = 1,2,\dots,q);\ Z_O^- = \{0,1,2\dots\};$ $s,z \in C),$

but

$$\Phi_n(s,a;b,\lambda) = \frac{\Pi_{j=1}^{p}(\lambda_j+1)_{n-1}}{\Pi_{j=1}^{q}(\mu_j+1)_{n-1} n!} \left(\frac{a+1}{a+n}\right)^s \frac{\Lambda(a+n,b,s,\lambda)}{\Lambda(a+1,b,s,\lambda)}.$$

…… (3)

Let $B_H(\lambda,\alpha,k)$ be the family of functions harmonic type of the form (1) by

$$Re\left\{\frac{F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z)}{z(F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z))'}\right\} > \alpha,\quad 0 \leq \alpha < 1\,,$$

where $F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z)$ is defined by (2). …. (4)

Let $\overline{B_H}(\lambda,\alpha,k)$ denoted that the subclass of $B_H(\lambda,\alpha,k)$such that h and g are the from

$h(z) = z - \sum_{n=2}^{\infty} a_n z^n$, $g(z) = (-1)^n \sum_{n=1}^{\infty} b_n z^n, |b_1| < 1.$ … (5)

2. The Main Results

In this section, the main important results are stated and proved an enough coefficient to functions of harmonic univalent types.

Theorem 2.1: Let $f = h + \overline{g}$ be given by (1), if

$$\sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + \sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\overline{z}|^n \leq (1-\alpha),\ \dots\dots (6)$$

where $a_1 = 1, 0 \leq \alpha < 1$, then, f harmonic of sense-preserving type to U with

$$\Phi_n(s,a;b,\lambda) = \frac{\Pi_{j=1}^{p}(\lambda_j+1)_{n-1}}{\Pi_{j=1}^{q}(\mu_j+1)_{n-1} n!} \left(\frac{a+1}{a+n}\right)^s \frac{\Lambda(a+n,b,s,\lambda)}{\Lambda(a+1,b,s,\lambda)}.$$

Proof: For $z_1 \neq z_2$, we have

$$\left|\frac{f(z_1)-f(z_2)}{h(z_1)-h(z_2)}\right| \geq 1 - \left|\frac{g(z_1)-g(z_2)}{h(z_1)-h(z_2)}\right|$$

$= 1- \left|\frac{\sum_{n=1}^{\infty} b_n(z_1^n - z_2^n)}{(z_1-z_2)\sum_{n=2}^{\infty} a_n(z_1^n - z_2^n)}\right| \geq 1 - \frac{\sum_{n=1}^{\infty} n|b_n|}{1-\sum_{n=2}^{\infty} n|a_n|}$

$\geq 1 - \frac{\frac{(1-\alpha n)|\varphi_n(s,a;b,\lambda)|}{1-\alpha}|b_n|}{\frac{(1-\alpha n)|\varphi_n(s,a;b,\lambda)|}{1-\alpha}|a_n|} \geq 0$.

Hence proved univalent, since

$|h'(z)| \geq 1 - \sum_{n=2}^{\infty} n|a_n||z|^{n-1} > 1 - \sum_{n=2}^{\infty} n|a_n| \geq \sum_{n=2}^{\infty} \frac{(1-\alpha n)|\varphi_n(s,a;b,\lambda)|}{1-\alpha}|a_n|$

$\geq 1 - \sum_{n=1}^{\infty} \frac{(1-\alpha n)|\varphi_n(s,a;b,\lambda)|}{1-\alpha}|b_n| \geq \sum_{n=1}^{\infty} n|b_n|$

$> \sum_{n=1}^{\infty} n|b_n||z|^{n-1} \geq |g'(z)|$.

By using the case of $Re(w) \geq \alpha, \Leftrightarrow |1-\alpha+w| \geq |1+\alpha-w|$ for $(0 \leq \alpha < 1)$, it show that

$|A(z) + (1-\alpha)B(z)| - |A(z) - (1+\alpha)B(z)| \geq 0,$ ….(7)

where

$A(z) = F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z)$, $B(z) = z(F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z))'$.

Consider $|A(z) + (1-\alpha)B(z)|$

$\leq \sum_{n=2}^{\infty} (1+n-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + \sum_{n=1}^{\infty} (1+n-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\bar{z}|^n + 2 - \alpha$. ……..(8)

And

$|A(z) - (1+\alpha)B(z)|$

$=$

$|z + \sum_{n=2}^{\infty} \Phi_n(s,a;b,\lambda)a_n z^n + (-1)^k \sum_{n=1}^{\infty} \Phi_n(s,a;b,\lambda)b_n\bar{z}^n - (1+\alpha)[z + \sum_{n=2}^{\infty} n\Phi_n(s,a;b,\lambda)a_n z^n + (-1)^k \sum_{n=1}^{\infty} n\Phi_n(s,a;b,\lambda)b_n\bar{z}^n|$

$\leq \sum_{n=2}^{\infty} (1-n-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + \sum_{n=1}^{\infty} (1-n-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\bar{z}|^n + \alpha$. ……(9)

Substituting (8) and (9) in (7), we obtain

$2\sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + 2\sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\bar{z}|^n + 2(1-\alpha),$

so

$\sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + \sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\bar{z}|^n \leq (1-\alpha).$

Theorem 2.2: Let $f = h + \bar{g}$ is formulated by (5). Then $f(z) \in \overline{B_H}(\lambda,\alpha,k)$ if and only if

$\sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n||z|^n + \sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n||\bar{z}|^n \leq (1-\alpha),$ ….(10)

where

$\varphi_n(s,a;b,\lambda) = \frac{\Pi_{j=1}^{p}(\lambda_j+1)_{n-1}}{\Pi_{j=1}^{q}(\mu_j+1)_{n-1}n!}\left(\frac{a+1}{a+n}\right)^s \frac{\wedge(a+n,b,s,\lambda)}{\wedge(a+1,b,s,\lambda)},$

and $\lambda_j \in C\ (j=1,\dots,p), \mu_j \in \frac{C}{Z_O}\ (j=1,\dots,q)$

Proof: Since $\overline{B_H}(\lambda,\alpha,k) \subset B_H(\lambda,\alpha,k)$, we just need to prove the only if part of the theorem. We notice that the condition (5) is equation to

$Re\left\{\frac{F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z)}{z(F_{(\lambda\rho)(\mu q),b}^{s,a,\lambda} f(z))'}\right\} > \alpha, \quad 0 \leq \alpha < 1$

$$Re\left[\frac{z - \sum_{n=2}^{\infty} |\Phi_n(s,a;b,\lambda)||a_n|z^n + (-1)^{2s+1}\sum_{n=1}^{\infty} |\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^n}{z - \sum_{n=2}^{\infty} n|\Phi_n(s,a;b,\lambda)||a_n|z^n + (-1)^{2s}\sum_{n=1}^{\infty} n|\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^n}\right] > \alpha.$$

$$Re\left[\frac{z - \sum_{n=2}^{\infty} |\Phi_n(s,a;b,\lambda)||a_n|z^n + (-1)^{2s+1}\sum_{n=1}^{\infty} |\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^n - \alpha\{z - \sum_{n=2}^{\infty} n|\Phi_n(s,a;b,\lambda)||a_n|z^n + (-1)^{2s}\sum_{n=1}^{\infty} n|\Phi_n(s,a;b,\lambda)||b_n|\underline{z}^n\}}{z - \sum_{n=2}^{\infty} n|\Phi_n(s,a;b,\lambda)||a_n|z^n + (-1)^{2s}\sum_{n=1}^{\infty} n|\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^n}\right] \geq 0.$$

$$Re\left[\frac{(1-\alpha) - \sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n|z^{n-1} - \frac{\bar{z}}{z}\sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^{n-1}}{1 - \sum_{n=2}^{\infty} |\Phi_n(s,a;b,\lambda)||a_n|z^{n-1} + \frac{\bar{z}}{z}\sum_{n=1}^{\infty} |\Phi_n(s,a;b,\lambda)||b_n|\bar{z}^{n-1}}\right] \geq 0. \quad \dots(11)$$

The condition (11) must satisfy for all values of z on the positive real axis, where $|z| \in (0, \mu)$, we must have

$$Re\left[\frac{(1-\alpha) - \sum_{n=2}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||a_n|\mu^{n-1} - \frac{\bar{z}}{z}\sum_{n=1}^{\infty} (1-\alpha n)|\Phi_n(s,a;b,\lambda)||b_n|\bar{\mu}^{n-1}}{1 - \sum_{n=2}^{\infty} |\Phi_n(s,a;b,\lambda)||a_n|\mu^{n-1} + \frac{\bar{z}}{z}\sum_{n=1}^{\infty} |\Phi_n(s,a;b,\lambda)||b_n|\bar{\mu}^{n-1}}\right] \geq 0. \quad \dots(12)$$

If the condition (6) does not hold then the numerator in (8), when goes to 1 is negative. This is a contradiction with the situation case where $f \in \overline{B_H}(\lambda,\alpha,k)$ and so the proof is accomplished.

3. The Distortion Theorem

Theorem 3.1: Suppose $f \in \overline{B_H}(\alpha,\beta,\lambda)$. Then $|z| = r < 1$, we can get

$|f(z)| \leq (1+|b_1|)r + \frac{1}{\Phi_2(s,a;b,\lambda)}\left[\frac{1-2\alpha}{1-\alpha} - \frac{1+2\alpha}{1-\alpha}|b_1|\right]r^2$, $|z| = r < 1$

and

$|f(z)| \geq (1+|b_1|)r - \frac{1}{\Phi_2(s,a;b,\lambda)}\left[\frac{1-2n}{1-\alpha} - \frac{1+2n}{1-\alpha}|b_1|\right]r^2,$ $|z| = r < 1.$

Proof: We have

$|f(z)| \leq (1+|b_1|)r + [(|a_n| + |b_n|)]r^n$

$\leq (1+|b_1|)r + [(|a_n| + |b_n|)]r^2$

$= (1+|b_1|)r + \frac{1-\alpha}{\Phi_2(s,a;b,\lambda)(1-2\alpha)}\left[\frac{1-2\alpha}{1-\alpha}|a_n| + \frac{1-2\alpha}{1-\alpha}|b_n|\right]|\Phi_2(s,a;b,\lambda)|r^2 \leq (1+|b_1|)r + \frac{1}{|\Phi_2(s,a;b,\lambda)|(1-2\alpha)}\left(1 - \frac{1+2\alpha}{1-\alpha}|b_1|\right)r^2 \leq (1+|b_1|)r + \frac{1}{|\Phi_2(s,a;b,\lambda)|}\left[\frac{1-\alpha n}{1-\alpha} - \frac{1+2\alpha}{1-\alpha}|b_1|\right]r^2$.

4. The Convolution (Hadamard product)

Let

$f(z) = z - \sum_{n=2}^{\infty} |a_n|z^n + (-1)^k\sum_{n=1}^{\infty} |b_n|\bar{z}^n$

and

$g(z) = z - \sum_{n=2}^{\infty} |c_n| z^n + (-1)^k \sum_{n=1}^{\infty} |d_n| \overline{z}^n$.

Then, form the convolution of $f(z)$ and $g(z)$, we can obtain

$(f * g)(z) = f(z) * g(z) = z - \sum_{n=2}^{\infty} |a_n||c_n| z^n + (-1)^k \sum_{n=1}^{\infty} |b_n||d_n| \overline{z}^n$.

Theorem 4.1: Let $f(z) \in \overline{B_H}(\lambda,\alpha,k)$ and $g(z) \in \overline{B_H}(\lambda,\beta,k)$. Then for $0 \le \beta \le \alpha < 1$, we have $(f * g)(z) \in \overline{B_H}(\lambda,\alpha,k) \subset \overline{B_H}(\lambda,\beta,k)$, then satisfy (6) and since $(|c_n| \le 1, |d_n| \le 1)$,

we write

$$\sum_{n=1}^{\infty} \left(\frac{(1-\alpha n)}{1-\alpha}|a_n c_n| + \frac{(1-\alpha n)}{1-\alpha}|b_n d_n|\right)|\Phi_n(s,a;b,\lambda)|$$
$$\le \sum_{n=1}^{\infty} \left(\frac{(1-\alpha n)}{1-\alpha}|a_n| + \frac{(1-\alpha n)}{1-\alpha}|b_n|\right)|\Phi_n(s,a;b,\lambda)|$$

The last inequality is bounded of the right hand side above by(1), then

$f * g \in \overline{B_H}(\lambda,\alpha,k) \subset \overline{B_H}(\lambda,\beta,k)$.

5. The Convex Combination

This section, is devoted to prove that the space $\overline{B_H}(\lambda,\alpha,k)$ is closed under convex combination. Suppose that the $f_i(z)$ is formulated for $i = 1,2,3,\dots,m$, by the following form

$f_i(z) = z - \sum_{n=2}^{\infty} |a_{n,i}| z^n + (-1)^k \sum_{n=1}^{\infty} |b_{n,i}| \overline{z}^n$. ….(13)

Theorem 5.1: Assume that the functions f_i is given by (13) be in the class $\overline{B_H}(\lambda,\alpha,k)$, for every $i = 1,2,\dots,m$. Then the functions $\tau_i(z)$ well-defined by $\tau_i(z) = \sum_{n=1}^{\infty} C_i f_{i(z)}$, $0 \le c_i \le 1$ are also in the class $\overline{B_H}(\lambda,\alpha,k)$ where $\sum_{n=1}^{\infty} C_i = 1$.

Proof. In view of the $t_i(z)$ definition, we can write reformulate

$\tau_i(z) = z - \sum_{n=2}^{\infty} \left(\sum_{i=1}^{\infty} C_i|a_{n,i}|\right) z^n + (-1)^k \sum_{n=1}^{\infty} \left(\sum_{i=1}^{\infty} C_i|b_{n,i}|\right)\overline{z}^n$ …. (14)

Further, since $f_i(z)$ are in $\overline{B_H}(\lambda,\alpha,k)$, for every $i = 1,2,\dots,m$, then we have

$$\sum_{n=2}^{\infty} \frac{(1-\alpha n)}{1-\alpha}\left(\sum_{i=1}^{\infty} C_i|a_{n,i}|\right)z^n + \sum_{n=1}^{\infty} \frac{(1-\alpha n)}{1-\alpha}\left(\sum_{i=1}^{\infty} C_i|b_{n,i}|\right)\overline{z}^n$$
$$= \sum_{i=1}^{\infty} C_i\left(\sum_{n=2}^{\infty} (1-\alpha n)|\varphi_n(s,a;b,\lambda)||a_{n,i}| + \sum_{n=1}^{\infty} (1-\alpha n)|\varphi_n(s,a;b,\lambda)||b_{n,i}|\right)$$
$$\le \sum_{i=1}^{\infty} C_i(1-\alpha) \le (1-\alpha).$$

6. The Extreme Point

In this part, we get the extreme points for the class $\overline{B_H}(\alpha,\beta,\lambda)$.

Theorem 6.1: Suppose f be given by (5). Then $f \in \overline{B_H}(\alpha,\beta,\lambda)$, $\Leftrightarrow$

$f(z) = \sum_{n=1}^{\infty} (T_n h_n(z) + S_n g_n(z))$, …(15)

where

$h_1(z) = z$,

$h_n(z) = z - \left(\frac{(1-\alpha)}{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}\right) z^n$, $n = 2,3,\dots$

and

$g_n(z) = z + (-1)^{k-1}\left(\frac{(1-\alpha)}{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}\right)\overline{z}^n$, $n = 1,2,\dots$

$\sum_{i=1}^{\infty} (T_n + S_n) = 1$, $T_n \ge 0$ *and* $S_n \ge 0$.

In particular, the extreme points of $f \in \overline{B_H}(\alpha,\beta,\lambda)$ are $\{h_n\}$ and $\{g_n\}$.

Proof: The form (15), we get

$$f(z) = \sum_{n=1}^{\infty} (T_n h_n(z) + S_n g_n(z))$$
$$= \sum_{n=1}^{\infty} (T_n + S_n)z - \sum_{n=2}^{\infty} \frac{(1-\alpha)}{1-\alpha n|\Phi_n(s,a;b,\lambda)|}T_n z^n + (-1)^{n-1}\sum_{n=1}^{\infty} \frac{((1-\alpha))}{1-\alpha n|\Phi_n(s,a;b,\lambda)|}S_n\overline{z}^n$$
$$= z - \sum_{n=2}^{\infty}\left(\frac{(1-\alpha)}{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}\right)T_n z^n + (-1)^k\sum_{n=1}^{\infty}\left(\frac{(1-\alpha)}{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}\right)S_n\overline{z}^n.$$

Therefore

$$\sum_{n=2}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}|a_n| + \sum_{n=1}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}|b_n|$$
$$\sum_{n=2}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}\left(\frac{(1-\alpha)}{1-\alpha n|\Phi_n(s,a;b,\lambda)|}a_n\right) + \sum_{n=1}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}\left(\frac{(1-\alpha)}{1-\alpha n|\Phi_n(s,a;b,\lambda)|}b_n\right)$$
$$= \sum_{i=1}^{\infty} T_n + \sum_{i=1}^{\infty} S_n = 1 - T_n \le 1.$$

And so $f \in \overline{B_H}(\alpha,\beta,\lambda)$.

Conversely, assume that $f \in \overline{B_H}(\alpha,\beta,\lambda)$.

Letting

$T_1 = 1 - \sum_{n=2}^{\infty} T_n + \sum_{n=1}^{\infty} S_n$.

Putting

$$T_n = \frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}|a_n|, \quad n = 2,3,\dots$$
$$S_n = \frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}|b_n|, \quad n = 1,2,\dots$$

we obtain

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n + (-1)^n \sum_{n=1}^{\infty} b_n \overline{z}^n$$
$$= z - \sum_{n=2}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}T_n z^n + (-1)^n\sum_{n=1}^{\infty}\frac{(1-\alpha n)|\Phi_n(s,a;b,\lambda)|}{1-\alpha}S_n\overline{z}^n.$$
$$= z - \sum_{n=2}^{\infty}[z - h_n(z)]T_n - \sum_{n=1}^{\infty}[z - g_n(z)]S_n$$
$$= 1 - \left[\sum_{n=2}^{\infty} T_n - \sum_{n=1}^{\infty} S_n\right]z + \sum_{n=2}^{\infty} T_n h_n(z) + \sum_{n=1}^{\infty} S_n g_n(z)$$
$$= \sum_{i=1}^{\infty} (T_n h_n(z) + S_n g_n(z)).$$

This completes the proof.

Conclusion

We have shown that a new class to functions of harmonic univalent type, interesting results concerning the harmonic univalent functions defined by the Dziok-Srivastava operator. Thus, some geometric properties like coefficients conditions, distortion theorem, convolution (Hadamard product), extreme points and convex combination are investigated and examined. Finally, Moreover, many problems still opened, for example, the extension of these results to the case of subclasses for various linear operator [9-11].

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الدوال الاحادية التكافؤ التوافقية معرفة بواسطة المؤثر Dziok-Srivastava

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الملخص

الغرض من هذا العمل هو تقديم فئة من الدوال التوافقية أحادية التكافؤ التي حددها عامل التشغيل Dziok-Srivastava.
تم دراسة بعض الخصائص الهندسية مثل شروط المعاملات، نظرية التشويه، الالتواء (ضرب هادمارد)، التركيبة المحدبة والنقاط المتطرفة.

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