REVIEW AND MODELING THE METHODS OF RADIUS ESTIMATING TECHNIQUES FOR HORIZONTAL CURVES

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ABSTRACT
The establishment of figures on the ground is an important task of the field surveyor, not only in engineering construction but also in cadastral surveying. It is a relatively easy task to peg out the boundary of a rectangular concrete slab, but considerably more difficult to establish the location of points along an elevated curved highway. Thus Radii measurements are important for many reasons such as: setting curve advisory speeds; predicting vehicle operating speeds; spacing curve delineation treatments such as post-delineators, chevrons, and raised retro reflective pavement markers; performing highway safety audits; and evaluating traffic crashes. Many groups, including transportation agencies, accident investigators, and transportation researchers, would find an accurate, quick, and safe method to estimate the radius of horizontal curves particularly useful. The present work explains the ten methods of estimating radius, which are as follows: 1) Basic ball bank indicator (BBI), 2) Advanced BBI, 3) Chord length, 4) Compass, 5) Field survey, 6) Global Positioning System (GPS) unit, 7) Lateral acceleration, 8) Plan sheet, 9) Speed advisory plate, and 10) Vehicle yaw rate. Each method was programmed as software named RET (Radius Estimating Techniques). It represent of a collection of subroutines for each method. This program is written in Mathematical Computer Aided Design language (MCAD). It is also include a subroutine written in Visual BASIC language for calculating the super elevation of a designed or existing horizontal curves.

Keywords: model, horizontal curve, radius estimation methods, transportation engineering, designs.

1. INTRODUCTION
When a highway changes horizontal direction, making the point where it changes direction a point of intersection between two straight lines is not feasible. The change in direction would be too abrupt for the safety of modern, high-speed vehicles. It is therefore necessary to interpose a curve between the straight lines. The straight lines of a road are called tangents because the lines are tangent to the curves used to change direction. Horizontal curves are, in effect, transitions between two tangents. These deflection changes are necessary in virtually all highway alignments to avoid impacts on a variety of field conditions (e.g., right-of-way, natural features, and man-made features) (Aashto, 2001).

Practically, in all modern highways, the curves are circular curves; that is, curves that form circular arcs. The smaller the radius of a circular curve, the sharper the curve. For modern, high-speed highways, the curves must be flat, rather than sharp. That means they must be large-radius curves (FHWA, 2003).

In highway work, the curves needed for the location or improvement of small secondary roads may be worked out in the field. Usually, the horizontal curves are computed after the route has been selected, the field surveys have been done, and the survey base line and necessary topographic features have been plotted. In urban work, the curves of streets are designed as an integral part of the preliminary and final layouts, which are usually done on a topographic map. In highway work, the road itself is the end result and the purpose of the design. But in urban work, the streets and their curves are of secondary importance; the best use of the building sites is of primary importance (Aashto, 2001).

The aim of this paper is to estimate the radii of the horizontal curves using ten different methods. Radii measurements are important for (Information Site, 2008):
- Setting curve advisory speeds;
- Predicting vehicle operating speeds;
- Spacing curve delineation treatments such as post-delineators (1), chevrons, and raised retro reflective pavement markers;
- Performing highway safety audits; and
- Evaluating traffic crashes.

2. TYPES OF HORIZONTAL CURVES
There are four types of horizontal curves as illustrated in Figure-1. They are described as follows (Garber and Hoel, 2002):

a) Simple
The simple curve is an arc of a circle. The radius of the circle determines the sharpness or flatness of the curve.

b) Compound
Frequently, the terrain will require the use of the compound curve. This curve normally consists of two simple curves joined together and curving in the same direction.
c) **Reverse**

A reverse curve consists of two simple curves joined together, but curving in opposite direction. For safety reasons, the use of this curve should be avoided when possible.

d) **Spiral**

The spiral is a curve that has a varying radius. It is used on railroads and most modern highways. Its purpose is to provide a transition from the tangent to a simple curve or between simple curves in a compound curve.

Super elevation of the carriageway on horizontal curves must be related to the design speed and radius. Super elevation is tilting the roadway to help offset centripetal forces developed as the vehicle goes around a curve. Along with friction they are what keep a vehicle from going off the road. The values of super elevation are determined from the AASHTO Design Guide and are a function of the rate of super elevation and the curve radius (Michigan Tech, 2008).

### 3. METHODS OF ESTIMATING THE RADIUS OF HORIZONTAL CURVES

Many groups, including transportation agencies, accident investigators, and transportation researchers, would find an accurate, quick, and safe method to estimate the radius of horizontal curves particularly useful. The radius estimating methods were as follows (Carlson et al., 2005):

- (a) Basic ball bank indicator (BBI)
- (b) Advanced BBI
- (c) Chord length
- (d) Compass
- (e) Field survey
- (f) Global Positioning System (GPS) unit
- (g) Lateral acceleration
- (h) Plan sheet
- (i) Speed advisory plate
- (j) Vehicle yaw rate

#### 3.1 Ball bank indicator

A BBI is a curved level commonly used to determine the safe speed of horizontal curves. A BBI measures the combination of lateral acceleration, vehicle body roll, and super elevation in the following relationship (Carlson et al., 2005):

\[
BBI = \text{lateral acceleration} - \text{super elevation} + \text{body roll}
\]

Each term in the above equation cannot be individually determined from the BBI reading alone. The BBI reading is simply a relationship of these three components. If it were not for the body-roll angle, the BBI reading in degrees would be a direct measure of lateral acceleration. If body roll is neglected, then the radius can be estimated using the point–mass equation (Garber and Hoel, 2002):

\[
R = \frac{V^2}{15\times(f+e)}
\]

where

- \( R \) = radius (ft),
- \( V \) = speed (mph),
- \( e \) = average full super elevation, and
- \( f \) = side friction factor.

The above equation requires the super elevation of the curve as an input. The super elevation of each test curve was measured at approximately 100- to 200-ft intervals (from the point of curvature (PC) and point of tangency,PT) so that a profile of the super elevation could be achieved. The average super elevation was calculated for each direction of travel on each curve by visually inspecting the super elevation profile and averaging only the measurements that were representative of full super elevation. A digital BBI was used for this study in an attempt to minimize the error in reading older BBIs. Figure-2 shows the digital BBI and two older BBIs that were used for redundancy (Carlson et al., 2005).
3.2 Advanced BBI

In this approach, the researchers hoped to improve the BBI method by measuring the body roll of the vehicle and incorporating the measurements into the radius estimation. To accomplish this improvement, the researchers used roll-rate sensors, which are shown in Figure-3, to measure the body roll of the vehicle’s sprung mass and the roll of the vehicle’s fixed suspension. In theory, a roll-rate sensor positioned in the sprung mass of the vehicle would measure the body roll of the vehicle plus the super elevation, whereas a roll-rate sensor positioned on the fixed suspension of the vehicle would measure only the super elevation. The difference of these two methods would result in the body roll. In addition, this method would alleviate the need for a field crew to leave their vehicle to measure super elevation or estimate super elevation. The added features of this method were envisioned to potentially increase accuracy and safety as compared with the BBI method (Carlson and Mason, 2008).

3.3 Chord length

With the chord method, a technician stretches a string of known length so that each end just touches the lane edge-line of the horizontal curve. It should be noted that the string can be stretched between any two points between the PC and PT of equal radii. However, both ends of the string must be within the limit of the curve. After the string is stretched, an offset distance is measured from the middle of the string to the lane edge-line. With the string length and offset known, the curve radius can be calculated by (Carlson et al., 2005):

\[ R = \frac{C^2 + 4HI}{8H} \]  

where

\[ R = \text{radius (ft)}, \]
\[ C = \text{length of string (ft)}, \]
\[ H = \text{offset distance (ft)} \]

The offset measurements were taken at least twice for both the inside and outside lane edge-lines, giving a minimum total of four radii measurements. These radii were averaged to estimate the curve radius at the centerline.

3.4 Compass

The compass method used the measured length of the curve by using surveying compass instrument, which is shown in Figure-4 and the compass heading of each tangent approach section to calculate the radius of the curve. The length of the curve was considered to be the average of the lengths measured along the inside and outside lane edge-line paint stripes, and the compass heading was recorded for each tangent approach of the curve. The length of the curve was measured along the inner and outer edges of the road, rather than along the centerline, in an attempt to keep the researcher out of the main traffic lanes. The difference of the two compass headings was calculated in degrees, and the radius of the curve was then calculated by (Carlson et al., 2005):

\[ R = \frac{57.3L}{D_c} \]  

where

\[ R = \text{radius (ft)}, \]
\[ L = \text{length of curve (ft)}, \]
\[ D_c = \text{difference in compass headings (degrees)} \]

3.5 Chord offset method

A very common method used by accident investigators is to calculate roadway curve radii from chord-offset measurements taken in the field. The Chord-Offset Method is taught in most of the major accident reconstruction schools.

The Chord-Offset Method usually uses a 100-foot tape held on either end at the precise edge of the roadway, while a carpenter’s rule is used at the middle of the tape to measure the offset distance.
measure the distance between the edge of the tape and the edge of the roadway. These two measurements, the 100-foot chord length and the measured middle offset are then used in the following equation to compute the radius (Carlson et al., 2005):

\[ R = \frac{L^2}{8m} + \frac{m}{2} \]  

(5)

where

- \( R \) = the roadway curve radius, feet
- \( L \) = the chord length, feet
- \( m \) = the measured middle offset, feet

Although the Chord-Offset method is widely used, it has inherent weaknesses that make it a questionable practice, as follows:

a. Unless investigators can precisely layout, tie down, and keep taut a string line for their chord, they will need at least one other person to help with the measurements.

b. The Chord-Offset Method is dependent on having a clearly definable arc on the roadway. Unpaved roadways have variable edges. Most asphalt pavements or painted edge lines and centerlines do not have a true edge throughout. Concrete pavements with curves have a more definable arc, but are not necessarily true.

c. The tape or string must be held taut or will not accurately represent a true chord.

d. Measuring a chord-offset often puts investigators in harm’s way on high-speed and/or high-volume roadways.

e. The Chord-Offset equation is very sensitive to small inaccuracies in measuring the middle offset.

3.6 Field survey

A field survey of each of the selected horizontal curves was completed by using a total station Figure-5. Within the boundaries of the PC and PT of the horizontal curve, at least three and up to five measurements were recorded on both the inside and outside of the curve. That is, all the recorded points were points on the circular arc, not on the adjacent tangent sections.

The survey rod was placed on the outside of the lane edge-line for the collection of all the data points. The data points were recorded on the inner and outer edge of each horizontal curve in an attempt to keep the researcher out of the main traffic lanes. The actual radius of the curve was assumed to be the average of the inner and outer radii. The radius found by using this method was assumed to be the true radius and was used for comparisons against the other methods.

3.7 Global positioning system (GPS) unit

Advancements in GPS technology have significantly reduced the price of GPS equipment see Figure-6 and increased its accuracy to a point that it is now a feasible option to use in making highway-related measurements. In concept, this method is similar to the compass method in that the curve radius can be estimated from a measured deflection angle and the corresponding curve length information that can be acquired from typical GPS data streams.

The Radius-meter immediately processes GPS data by means of a microcontroller and solves for the radius of the curve. This system can operate from any vehicle traveling at any speed, which means that a data collection vehicle can travel along with the normal traffic flow. The display of the Radius-meter is shown in Figure-7 (Carlson et al., 2005).
3.8 Lateral acceleration

The lateral acceleration radius-estimating method is similar to the BBI method in that the measurement was the unbalanced lateral acceleration rate, or the side friction factor. This assumed that body roll was neglected and that superelevation was known or could be estimated. The lateral acceleration rate was digitally measured every 0.01 second using the VC3000. These measurements were stored in a file along with the vehicle’s speed and distance traveled. The lateral acceleration, in highway design terminology, is related to the radius in the following manner:

\[
R = \frac{V^2}{15 \times (L_f + e)}
\]  

(6)

where

R = the curve radius (ft),
V = vehicle speed (mph),
L_f = lateral force (percentage of gravity, g), and
e = superelevation, as a decimal.

3.9 Plan sheet

The plan sheet method determines the radius of a curve by using information provided on as-built plan sheets, which are accessible at local transportation offices. Plan sheets contain information such as location of PC and PT, deflection angle, and tangent length for all horizontal curves. The information provided on plan sheets is usually the as-built information. From this information, each curve radius was calculated.

The required information found on the plan sheets was the location of the start of the curve (PC), the end of the curve (PT), and the degrees of turn of the curve (\(\Delta\)). This information was input into the following curve radius-estimation equation, and the radius of the curve was calculated as follows (Garber and Hoel, 2002):

\[
R = \frac{5729.578}{100 \times \left( \frac{\Delta}{PT - PC} \right)}
\]  

(7)

where

R = curve radius (ft),
\(\Delta\) = curve deflection angle (degrees),
PT = curve point of tangent (ft), and
PC = curve point of curve (ft).

The plane table and alidade, which is illustrated in Figure-8, were once the most common tools used to produce detailed site plan maps in the field. The Egyptians are said to have been the first to use a plane Table to make large-scale accurate survey maps to represent natural features and man-made structures.

Plane table mapping is rarely done today—plane table surveying has, for most purposes, been replaced by aerial photogrammetry and total stations, but the final map is still similar. Plane table surveys were performed in the Corps well into the 1980s, and perhaps into the 1990s in some districts. A plane table survey system is described as follows: A blank map upon which control points and grid ticks have been plotted is mounted on the plane table. The table is mounted on a low tripod with a specially made head. The head swivels so that it can be leveled, locked in the level position, and then be rotated so that the base map can be oriented. The base map is a scaled plot of the ground control stations.

3.10 Advisory speed plate

Advisory speeds Figure-9 for curves have been determined in the field by making several trial runs through the curve at different speeds in a test-vehicle equipped with a ball-bank indicator. The ball-bank reading indicates overturning forces on the vehicle and is a combined measure of the centrifugal force, vehicle roll and superelevation. The generally accepted criteria for setting advisory speeds are ball-bank readings of 14 deg for speeds below 20 mph, 12 deg for speeds between 20 and 35 mph, and 10 deg for speeds of 35 mph or greater. These criteria are based on tests conducted in the 1930s that were intended to represent the 85th - to 90th -percentile curve speed. Another method used to determine the advisory speed is using the nomograph in the Traffic Control Devices Handbook (TCDH) which utilizes the
following standard curve formula (Garber and Hoel, 2002):

\[ V^2 = 15 R (e+f) \]  

(8)

where

- \( V \) = speed in mph,
- \( R \) = radius of curve in feet,
- \( e \) = rate of superelevation in feet per foot and
- \( f \) = coefficient of side friction (friction factor).

Figure-9. Advisory speed plate (Ibraheem, 2008).

A friction factor of 0.16 is assumed in the nomograph that corresponds to the speed at which discomfort begins for an average rider in a 1930 vintage car. This factor may not be valid for modern vehicles. The side friction factors recommended in the design criteria of the American Association of State Highway and Transportation Officials (AASHTO, 2001) vary from 0.17 at low speed to 0.10 at the highest speed. However, these values also are based on tests conducted in the 1930s and represent the limit at which a rider will notice a "side pitch" and begin to feel some discomfort. The ball-bank readings of 14 deg, 12 deg, and 10 deg were found in these earlier studies to correspond to side friction values of 0.21, 0.18, and 0.15, respectively.

The friction factors used in current criteria do not reflect the maximum safe speed but rather an average comfortable speed. Modern cars on dry pavement are capable of generating friction coefficients of 0.65 and higher before skidding. Friction coefficients of 0.40 and higher are typical on wet pavements. Thus, a curve designed for 70 mph could be driven well over 100 mph before it skidded out.

### 3.11 Modified yaw rate

The modified yaw rate method is a measure of the deflection angle of the curve. With the same roll rate sensors as before, only this time mounted in an inverted position, the deflection angle of the curve was measured as the test vehicle traversed the highway. The data acquisition unit (in this case a VC3000) simultaneously recorded the yaw rate (degrees per second) and the distance traveled along the curve. Essentially, the modified yaw rate method is an automated version of the compass method; however, the compass method required additional time and exposed the field crews to potential hazards on and along the road (Carlson et al., 2005).

The curve was traveled at the posted advisory speed, and the radius of the curve was calculated using the following equation (Garber and Hoel, 2002):

\[ R = \frac{57.3 \times L}{\Delta} \]

(9)

where

- \( R \) = the curve radius (ft),
- \( L \) = the roadway curve length (ft), and
- \( \Delta \) = the change in roadway direction (degrees).

For the same reasons as noted above in the description of the advanced BBI method (i.e., too much noise in the yaw rate transducer results), this method was not pursued past the initial stages.

### 4. MODELING THE METHODS OF RADIUS ESTIMATING TECHNIQUES

The radius of a horizontal curve can be found using the computer program RET (Radius Estimating Techniques) shown in Figure-10. It represents a collection of subroutines for each method. This program is written in Mathematical Computer Aided Design language (MathCAD). It also includes a subroutine written in Visual BASIC language for calculating the super elevation of a designed or existing horizontal curves. The following sections show the procedure of programming each method.

Figure-10. The main menu of RET program.

#### 4.1 Ball bank indicator

Figure-11 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program.
The symbols (La, Roll, BBI, V, f, Su, R) represent the (Lateral acceleration, body roll, Ball Bank Indicator, speed, side friction factor, average full superelevation, radius) and they can be calculated according to the equation that appeared in the form and it is referred to in equation (1).

![MathCAD program of BBI](image)

**Figure-11.** MathCAD program of BBI.

### 4.2 Chord length

Figure-12 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols:

\[ R = \text{radius (ft)} \]

\[ C = \text{length of string (ft)} \]

\[ H = \text{offset distance (ft)} \]

They can be calculated according to the equation that appeared in the form.

![MathCAD of chord length](image)

**Figure-12.** MathCAD of chord length.
4.3 Compass

Figure-13 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols represent:

\[ R = \text{radius (ft)} \]

\[ L = \text{length of curve (ft)} \]

\[ D_c = \text{difference in compass headings (degrees)} \]

They can be calculated according to the equation that appeared in the form.

Figure-13. MathCAD of compass.

4.4 Chord offset method

Figure-14 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols (R, L, M) represent the (the roadway curve radius (ft), the chord length (ft)). And they can be calculated according to the equation that appeared in the form.

Figure-14. MathCAD of chord offset method.
4.5 Lateral acceleration

Figure-15 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols \( R, V, L_f, e \) represent:

\[ R = \text{the curve radius (ft).} \]

\[ V = \text{vehicle speed (mph).} \]

\[ L_f = \text{lateral force (percentage of gravity, } g \text{).} \]

\[ e = \text{super elevation, as a decimal.} \]

And they can be calculated according to the equation that appeared in the form.

4.6 Plan sheet method

Figure-16 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols:

\[ R = \text{curve radius (ft).} \]

\[ \Delta = \text{curve deflection angle (degrees).} \]

\[ PT = \text{curve point of tangent (ft).} \]

\[ PC = \text{curve point of curve (ft).} \]

And they can be calculated according to the equation that appeared in the form.
4.7 Advisory speed plate

Figure-17 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols refer to: \( V \) = speed in mph, \( R \) = radius of curve in feet, \( e \) = rate of superelevation in feet per foot and \( f \) = coefficient of side friction (friction factor). And they can be calculated according to the equation that appeared in the form.

![Figure-17. MathCAD of advisory speed plate.](image)

4.8 Modified yaw rate

Figure-18 shows the inputs and the computations, which are required for the cost analysis, and it shows the flowchart of the program. The symbols:

\[ R = \text{the curve radius (ft).} \]
\[ L = \text{the roadway curve length (ft).} \]
\[ \Delta = \text{the change in roadway direction (degrees).} \]

And they can be calculated according to the equation that appeared in the form.

![Figure-18. MathCAD of modified yaw rate.](image)
5. CALCULATING THE SUPER ELEVATION

Most highways will change directions several times over the course of their lengths. These changes may be in a horizontal plane, in a vertical plane, or in both. The engineer is often charged with designing curves that accommodate these transitions, and consequently must have a good understanding of the physics involved. The super elevation of the highway cross-section and the side-friction factor are two of the most crucial components in the design of horizontal curves. The super elevation, as shown in chapter two, is normally discussed in terms of the super elevation rate, which is the rise in the roadway surface elevation as you move from the inside to the outside edge of the road. The super elevation calculator program is used to compute super elevation transition points a elevation differences at a specified station interval. A report of the results can be saved to a file or sent to a printer. Figure-19 shows the page of this subroutine.

The module on horizontal curve minimum radii will bring the effects of the super elevation rate and the side-friction factor together. Both of these concepts contribute to the final alignment of horizontal curves (Michigan Tech, 2008).

6. CONCLUSIONS AND RECOMMENDATIONS

Based on the methods of estimating the radius of any horizontal curve carried out in this work, the following conclusions can be drawn:

a) There are four types of horizontal curves: simple, compound, reverse, and spiral curve.
b) Many groups, including transportation agencies, accident investigators, and transportation researchers, would find an accurate, quick, and safe method to estimate the radius of horizontal curves particularly useful.
c) The radius estimating methods were: Basic ball bank indicator (BBI), Advanced BBI, Chord length, Compass, Field survey, Global Positioning System (GPS) unit, Lateral acceleration, Plan sheet, Speed advisory plate, and Vehicle yaw rate.
d) The radius of a horizontal curve can be found using the computer program RET (Radius Estimating Techniques). It represent of a collection of subroutines for each method. This program is written in Mathematical Computer Aided Design language (MCAD). It is also include a subroutine written in Visual BASIC language for calculating the super elevation of a designed or existing horizontal curves.
e) The precision of some methods was affected by the size of the radius. The only two methods that were not affected by the size of the radius were the plan sheet method and the GPS method. However, the plan sheet method and the GPS method provided the most accurate results.
f) The total cost of each method (measured as cost per estimated radius) depends on the frequency with which it is used. Overall, the surveying method was
always the most expensive, and the advisory plate method always the most economical. If the method will be used more than 16 times, the GPS method ends up being more cost effective than the plan sheet method. If the GPS method is going to be used more than 35 times, it becomes the second most cost effective method.

g) The advisory speed method is the easiest to use; however, not all highway curves have advisory speed plaques. The compass method and the GPS method were also relatively easy to use.

h) From a safety point of view, the safest methods minimize the amount of time maintenance crews, accident investigators, or researchers are on the roadway or roadside. The methods that were evaluated in this study that did not require people to be on or near the roadway include BBI, lateral acceleration, advisory speed, plan-sheet, and GPS.

i) For engineers and designers working in an office environment, the plan sheet method is preferred, because plan sheets are easily accessible to these users. However, when road safety audits are performed in the field or when maintenance crews responsible for curve delineation need to know horizontal curve radii, GPS is the preferred method. It is the most accurate and precise field method, and it is easy to implement using a device like the Radius-meter.

j) Although the compass method is not the most accurate or precise, it is easy to use and has a low cost. It does require a technician to be on the roadside and therefore is not the safest method. To achieve significant improvements in accuracy, precision, and safety, infrequent users should consider the GPS method.

k) Transportation researchers desire reliable and robust data. They also can have difficulty accessing plan sheets when performing research outside of their normal range. Therefore, the GPS method would likely best suit their needs. This method performed the best overall, with minor precision and accuracy losses as compared to surveying (considered to be exact). It also offered significant cost, ease of use, and safety benefits over surveying.

l) It is recommended to measure more than 20 horizontal curves on different highways in Baghdad to evaluate fully each method accuracy, precision, cost, ease of use, and safety. The results can be analyzed statistically within 95% level of confidence.

m) All the above methods require that users undergo a significant amount of training before they can measure the radius of a curve.

n) The safety of the technicians performing the measurements was another important consideration. So it is recommended to study this factor.

REFERENCES


