THE OVERSTABILITY MODE OF ROTATING CONVECTION IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

The fact of convection is one of the most remarkable problems in fluid dynamics. In this paper we shall study the case of over-stability mode of linear stability of rotating electrically conducting viscous layer heated from below lying in a uniform magnetic field based on the Boussinesq approximation. We shall follow the same analysis used in previous paper when we analyses the case of stationary convection of linear stability at the onset of rotating convection in the presence of magnetic field, so we restrict our study to the case when the direction of magnetic field and rotation are parallel; the discussion is focused on the case of large Taylor number T and Chandrasekhar number Q. Generally, we have seen early in a rotating and magnetic convection that thermal stability sets mostly as over stability [11], when we combine rotation and magnetic field the manner of the instability behaves in a complicated way depending on four dimensionless numbers T, Q, $\Pr$ and $\Pr_m$ [11]. We shall study the over-stability case seeking the nature of the dependence of these dimensionless numbers.

Keywords: Chandrasekhar Number Q, Convection, Rotating Convection, Magnetic Field, Rayleigh Number Ra, Over-Stability convection, Taylor Number T, Prandtl number $\Pr$, Prandtl magnetic number $\Pr_m$.

1. INTRODUCTION

This paper considers as a future study of a previous paper which is "The linear stability at the onset of rotating convection in the presence of magnetic field" [11]. In this paper, we shall study the over-stability mode of linear stability convection in the presence of magnetic field, following Rayleigh's ideas. We will analyze the over-stable mode of rotating convection in the presence of magnetic following the same analysis process used in analyzing the stationary convection, the governing equations required are: momentum equation added Lorentz force and Coriolis force for incompressible fluid (1), the heat equation (2), the induction equation with constant magnetic diffusivity (3) and the continuity equation (4).

$$\frac{D\underline{u}}{Dt} + 2\underline{\Omega} \times \underline{u} = -\frac{1}{\rho_0}\nabla p + \frac{\rho g}{\rho_0}\hat{\underline{z}} + \nu\nabla^2\underline{u} + \frac{1}{\rho}\underline{J} \times \underline{B} \tag{1}$$

$$\frac{\partial T}{\partial t} + (\underline{u} \cdot \times \nabla)T = \kappa\nabla^2 \mathrm{T} \tag{2}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \eta\nabla^2\underline{B} \tag{3}$$

$$\nabla \cdot \underline{u} = \nabla \cdot \underline{B} = 0 \tag{4}$$

With basic states

$$\Theta = T_0 - \beta z, \quad \underline{u} = \underline{0}, \quad P = P_0 - g\rho_0\left(z + \frac{\alpha\beta z^2}{2}\right), \quad \underline{B} = \underline{B}_0.$$

2.THE BASIC PROBLEM AND THE PROCESS OF SOLUTION

The steps used in finding solution to the governing equations at the case of over-stable mode are exactly the same steps used in finding solutions in the case of stationary convection, so due to the limit time of the research we shall call the dispersion equation (32) in our previous paper "The linear stability at the onset of rotating convection in the presence of magnetic field" [11]. We shall study the over-stability case seeking the nature of the dependence of four dimensionless numbers T, Q, Pr and Pr_m in two special cases, when $\Pr = 0.025$ and $\Pr_m = 0$ and Pm = 0 and when $\Pr = 0.6185$ and $\Pr_m = 0.1$. Returning back to the dispersion equation (5),

$$\left(s-\left(D^2-a^2\right)\right)\left[\begin{array}{l}\left(D^2-a^2\right)\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)^2 \\ +T\Pr^2 D^2\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)^2\end{array}\right]W$$
$$=-a^2\Pr Ra\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)$$
$$\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)W \qquad (5)$$

Seeking a solution to the case of free boundaries adjoining a non-conducting medium of the form:

$$W = W_0 \sin(\pi z)$$

Substituting this solution into equation (5), we obtain

$$\left(s+\left(\pi^2+a^2\right)\right)\left[\begin{array}{l}\left(\pi^2+a^2\right)\left(\left(s+\Pr\left(\pi^2+a^2\right)\right)\left(s+\frac{\Pr}{\Pr_m}\left(\pi^2+a^2\right)\right)+\frac{\Pr^2 Q}{\Pr_m}\pi^2\right)^2 \\ +T\Pr^2\pi^2\left(s+\frac{\Pr}{\Pr_m}\left(\pi^2+a^2\right)\right)^2\end{array}\right]$$
$$=a^2\Pr Ra\left(s+\frac{\Pr}{\Pr_m}\left(\pi^2+a^2\right)\right)$$
$$\left(\left(s+\Pr\left(\pi^2+a^2\right)\right)\left(s+\frac{\Pr}{\Pr_m}\left(\pi^2+a^2\right)\right)+\frac{\Pr^2 Q}{\Pr_m}\pi^2\right) \qquad (6)$$

At the onset of oscillatory instability, $s = \pm i\omega$, where $\omega \in R$, substitute for s in equation (6), (i.e. take + sign for s), so we obtain:

$$\left(i\omega + \left(\pi^2 + a^2\right)\right)\left[\begin{array}{l}\left(\pi^2 + a^2\right)\left(\left(i\omega + \Pr\left(\pi^2 + a^2\right)\right)\left(i\omega + \frac{\Pr}{\Pr_m}\left(\pi^2 + a^2\right)\right) + \frac{\Pr^2 Q}{\Pr_m}\pi^2\right)^2 \\ + T\Pr^2 \pi^2 \left(i\omega + \frac{\Pr}{\Pr_m}\left(\pi^2 + a^2\right)\right)^2\end{array}\right]$$

$$= a^2 \Pr Ra\left(i\omega + \frac{\Pr}{\Pr_m}\left(\pi^2 + a^2\right)\right)$$

$$\left(\left(i\omega + \Pr\left(\pi^2 + a^2\right)\right)\left(i\omega + \frac{\Pr}{\Pr_m}\left(\pi^2 + a^2\right)\right) + \frac{\Pr^2 Q}{\Pr_m}\pi^2\right) \tag{7}$$

Define

$$x = \frac{a^2}{\pi^2}, \quad \omega_1 = \frac{\omega}{\pi^2}, \quad R = \frac{Ra}{\pi^4}, \quad T_1 = \frac{T}{\pi^4}, \quad Q_1 = \frac{Q}{\pi^2}$$

Then equation (7) becomes:

$$\left(i\omega_1 + 1 + x\right)\left[\begin{array}{l}(1+x)\left(\left(i\omega_1 + \Pr(1+x)\right)\left(i\omega_1 + \frac{\Pr}{\Pr_m}(1+x)\right) + \frac{\Pr^2 Q_1}{\Pr_m}\right)^2 \\ + T_1 \Pr^2\left(i\omega_1 + \frac{\Pr}{\Pr_m}(1+x)\right)^2\end{array}\right]$$

$$= x\Pr R\left(i\omega_1 + \frac{\Pr}{\Pr_m}(1+x)\right)\left(\left(i\omega_1 + \Pr(1+x)\right)\left(i\omega_1 + \frac{\Pr}{\Pr_m}(1+x)\right) + \frac{\Pr^2 Q_1}{\Pr_m}\right) \tag{8}$$

This simplifies to :

$$R = \frac{1+x}{x}\left(\begin{array}{l} \frac{1}{\Pr}(i\omega_1 + 1 + x)(i\omega_1 + \Pr(1+x)) + \Pr Q_1 \frac{(i\omega_1 + 1 + x)}{(i\omega_1 \Pr_m + \Pr(1+x))} + \\ \frac{T_1 \Pr}{1+x}\left(\frac{(i\omega_1 + 1 + x)(i\omega_1 \Pr_m + \Pr(1+x))}{(i\omega_1 + \Pr((1+x))(i\omega_1 \Pr_m + \Pr(1+x)) + \Pr^2 Q_1}\right) \end{array}\right) \quad (9)$$

Taking the real and imaginary parts of equation (9), we obtain:

Real part:

$$R = \frac{1+x}{x}\left(\begin{array}{l} (1+x)^2 - \frac{\omega_1^{\ 2}}{\Pr} + Q_1\left(\frac{\omega_1^2 \Pr \Pr_m + \Pr^2 (1+x)^2}{\omega_1^2 \Pr_m^2 + \Pr^2 (1+x)^2}\right) + \\ \frac{T_1 \Pr}{1+x}\left(\frac{\begin{array}{c}\left(\Pr^2 (1+x)^2 - \Pr_m \omega_1^2 + \Pr^2 Q_1\right)\left(\Pr(1+x)^2 - \Pr_m \omega_1^2\right) + \\ \omega_1^2 (1+x)^2 (\Pr + \Pr_m)(\Pr + \Pr \Pr_m)\end{array}}{\omega_1^2 (1+x)^2 (\Pr + \Pr \Pr_m)^2 + \left(\Pr^2 (1+x)^2 - \Pr_m \omega_1^2 + \Pr^2 Q_1\right)^2}\right) \end{array}\right) \quad (10)$$

Imaginary part:

$$\frac{T_1\,\mathrm{Pr}^2}{1+x}\left(\frac{\left(\mathrm{Pr}^2(\mathrm{Pr}-1)(1+x)^2+\mathrm{Pr}^2(\mathrm{Pr}+\mathrm{Pr}_m)Q_1+\mathrm{Pr}_m^2\,\omega_1^2(\mathrm{Pr}-1)\right)}{\omega_1^2(1+x)^2(\mathrm{Pr}+\mathrm{Pr}\,\mathrm{Pr}_m)^2+\left(\mathrm{Pr}^2(1+x)^2-\mathrm{Pr}_m\,\omega_1^2+\mathrm{Pr}^2\,Q_1\right)^2}\right)+$$
$$Q_1\,\mathrm{Pr}^2\,\frac{(\mathrm{Pr}-\mathrm{Pr}_m)}{\mathrm{Pr}_m^2\,\omega_1^2+\mathrm{Pr}^2(1+x)^2}+(1+\mathrm{Pr})=0 \qquad (11)$$

For assigned values for T_1 and Q_1, equations (10) and (11) define R as a function of x, the minimum value of this function determine the critical value of Rayleigh number for oscillatory convection, comparing this value with critical Rayleigh value at the onset of stationary convection determines the manner in which instability first set, which will depend on the minimum of the two critical Rayleigh number.

Looking at equation (11), if $\mathrm{Pr}>1$ and $\mathrm{Pr}>\mathrm{Pr}_m$, the left hand side will be positive, so equation (11) never be satisfied, so ω_1^2 must be zero, so no possibility of over-stability at this case. Chandrasekhar did only the case for mercury when $\mathrm{Pr}=0.025$ and Pr_m close to zero, in the next section; we shall discuss when over-stability sets for special case of liquid metals such as mercury. Moreover, we shall study the over-stability mode when $\mathrm{Pr}=0.6185$ and $\mathrm{Pr}_m=0.1$, which considers a special case where the critical values of Rayleigh numbers are equals at stationary and oscillatory convection for $Q_1=100$ and $T_1=10^6$.

3. AN APPROXIMATE SOLUTION TO LIQUID METALS

Equation (10) and equation (11) can be simplified when applied on liquid metals such as mercury; this simplification mainly depends on the value of

$\Pr_m$ which is very small, for example for mercury, the value of Prandtl and Prandtl magnetic number are:

$\Pr = 0.025, \quad \Pr_m = 1.5\times 10^{-7}$

According to this work, we can neglect $\Pr_m$ in comparison with Pr, therefore equation (10) and (11) become:

$$R=\frac{1+x}{x}\left(\begin{array}{l}(1+x)^2-\frac{\omega_1{}^2}{\Pr}+Q_1+\\ T_1\Pr(1+x)\left(\frac{\omega_1^2+\Pr(1+x)^2+\Pr Q_1}{\omega_1^2(1+x)^2+\Pr^2\left((1+x)^2+\Pr Q_1\right)^2}\right)\end{array}\right) \tag{12}$$

$$(1+\Pr)(1+x)+\frac{Q_1\Pr}{1+x}=T_1\Pr^2\left(\frac{(1-\Pr)(1+x)^2-\Pr Q_1}{\omega_1^2(1+x)^2+\Pr^2\left[(1+x)^2+Q_1\right]^2}\right) \tag{13}$$

Equation (13) gives an explicit formula for

$$\omega_1^2=\frac{T_1\Pr^2}{(1+x)}\left(\frac{(1-\Pr)(1+x)^2-\Pr Q_1}{(1+x)^2(1+\Pr)+\Pr Q_1}\right)-\Pr^2\left((1+x)^2+\frac{Q_1}{(1+x)}\right) \tag{14}$$

Equations (12) and equation (13) can be combined to give

$$R=2\frac{1+x}{x}\left[\frac{(1+x)^2+Q_1}{(1+x)^2(1-\Pr)-\Pr Q_1}\left((1+x)^2+\omega_1^2\right)\right] \tag{14}$$

Equations (12) and (13) can be solved numerically using Maple program to determine the critical numbers for the onset of over-stability for various values of T_1 and Q_1. Numerical solutions showed that Ra_c increases monotonically as Q_1 increases (i.e. see tables 1-3).

TABLE 1. THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T = 10^4$

Q_1	Wave number *a*	Ra_c	ω_1^2
10	4.56	7052.97	1.79
50	5.06	14744.12	1.17
100	6.63	35402.31	0.57
150	7.20	50155.6	0.21

TABLE 2. THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T = 10^6$

Q_1	Wave number *a*	Ra_c	ω_1^2
10	8.35	36497.34	73.10
100	9.78	70718.21	52.97
500	11.54	1.893×10^5	34.90
1000	12.64	3.271×10^5	25.63

TABLE 3. THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T = 10^{10}$

Q_1	Wave number *a*	Ra_c	ω_1^2
100	37.7	1.254×10^7	40975.90
1000	38.4	1.305×10^7	39504.63
10000	41.9	1.720×10^7	33209.84
100000	50.1	4.576×10^7	21300.92

4. PREFERRED MODE OF INSTABILTIY WHEN $\Pr = 0.025$ AND $\Pr_m = 0$

Now, we shall investigate which mode of instability preferred for liquid metals. Figure (1-3) show that for $Q_1 = 100$, oscillatory convection is always preferred to stationary convection.

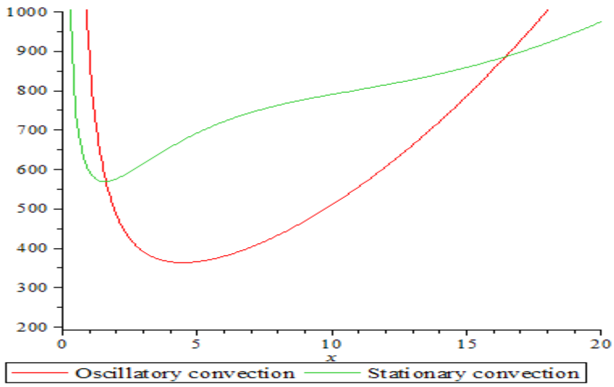

Figure 1. Stationary convection and oscillatory convection for $Q_1 = 100,\ T_1 = 10^4$

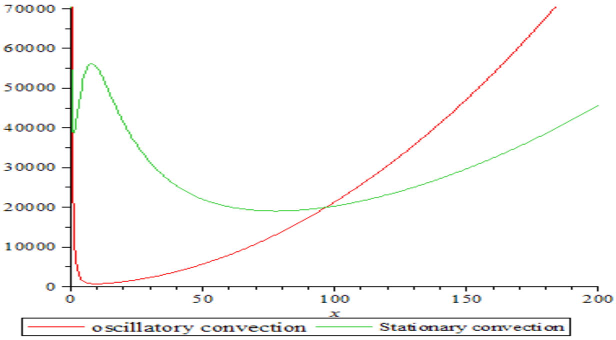

Figure 2. Stationary and oscillatory convection for $Q_1 = 100,\ T_1 = 10^6$

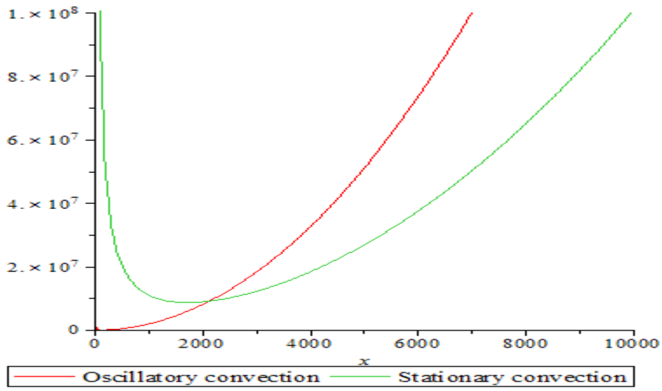

Figure 3. Stationary and oscillatory convection for $Q_1 = 100,\ T_1 = 10^8$

Comparing values of Ra_c at stationary and oscillatory convection from tables (3)-(6), it is clear that the oscillatory mode is the preferred mode until Q_1 reaches a specific value where Ra_o becomes larger than Ra_s, then the preferred mode of instability is the stationary mode.

For example when $T_1 = 10^6$ and at $Q_1 = 1661.22$, we have $Ra_o = Ra_s$ which is marginal stable, thus for $Q_1 < 1661.22$, the preferred mode of instability is oscillatory mode where $Ra_o < Ra_s$, and if $Q_1 > 1661.22$, then the stationary mode is the preferred mode. Similarly for $T_1 = 10^8$ and $T_1 = 10^{10}$ (See figures 4, 5 and 6).

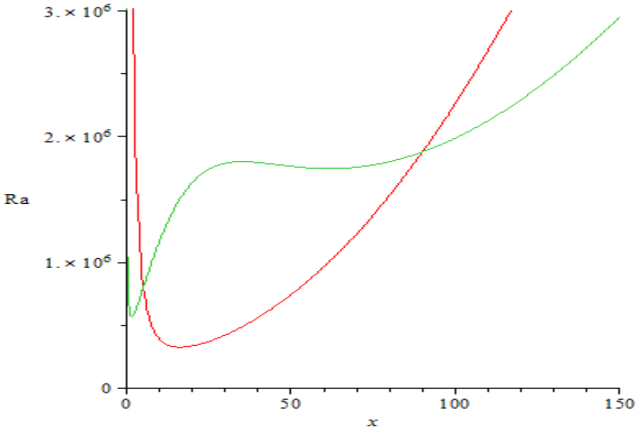

Figure 4. Plot of Ra_o (Red line) and Ra_s (green line) and for $Q_1 = 1000$, over-stability is preferred.

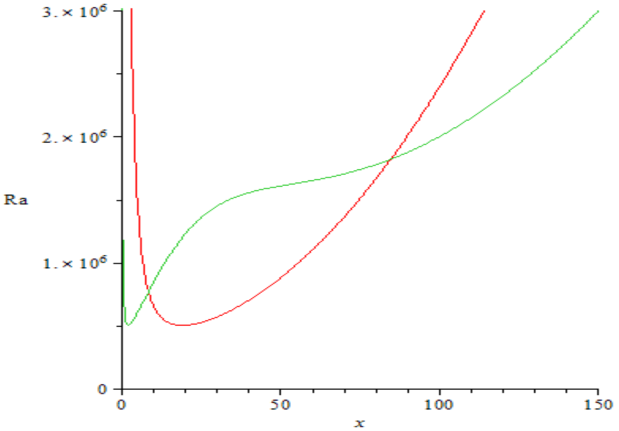

Figure 5. Plot of Ra_o (Red line) and Ra_s (green line) and for $Q_1 = 1661.22$, marginal stability occurred.

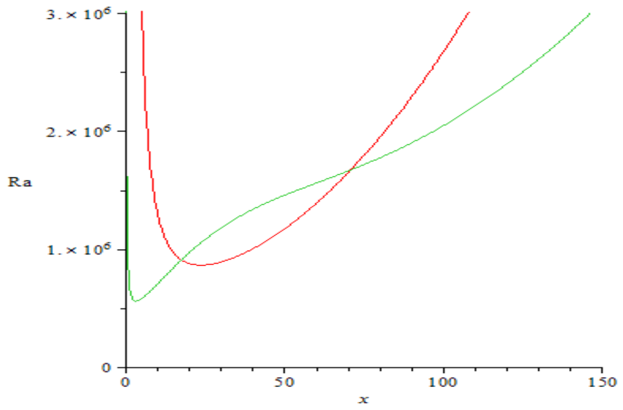

Figure 6. Plot of Ra_o (Red line) and Ra_s (green line) and for $Q_1 = 2000$, stationary convection is preferred.

TABLE 4. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^6$ AND VARIOUS VALUES OF Q_1

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
10^2	7.072×10^4	1.885×10^6	3.760×10^6
10^3	1.893×10^5	5.664×10^5	1.747×10^6
1661.22	5.064×10^5	5.064×10^5	-
10^4	1.720×10^7	1.2×10^6	-
10^6	4.576×10^7	-	9.928×10^6
10^8	1.023×10^{32}	-	9.781×10^9

TABLE 5. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^8$ AND VARIOUS VALUES OF Q_1

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
10^2	6.492×10^5	3.976×10^7	-
10^3	1.020×10^6	3.976×10^7	3.900×10^7
10^4	3.642×10^7	-	5.676×10^6
15396	5.087×10^6	5.087×10^6	-
5×10^4	1.422×10^7	7.250×10^6	-

TABLE 6. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^{10}$ AND VARIOUS VALUES OF Q_1

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
10^2	1.254×10^7	8.550×10^8	-
10^3	1.305×10^7	8.549×10^8	-
10^4	1.720×10^7	-	3.914×10^8
124718.7	5.273×10^7	5.274×10^7	1.407×10^8
10^6	2.837×10^8	-	1.189×10^8

5. AN APPROXIMATE SOLUTION TO THE DISPERSION RELATION FOR $\Pr = 0.6185$ AND $\Pr_m = 0.1$

In this section we shall investigate an approximate solution to the dispersion relation for the over-stability mode for $\Pr = 0.6185$ and $\Pr_m = 0.1$, the Maple program runs for any values of $\Pr$ and $\Pr_m$, but due to the time limit of the research we shall only consider the case when $\Pr = 0.6185$ and $\Pr_m = 0.1$, because it gives the same critical Rayleigh number at stationary and oscillatory modes for $Q_1 = 100$ and $T_1 = 10^6$, discussing the manner of the critical Rayleigh values for large T_1 and various value of Q_1, moreover we shall discuss the preferred mode of instability.

Numerical solutions of equations (10) and (11) for $\Pr = 0.6185$ and $\Pr_m = 0.1$ only exists when Q_1 is small for $T_1 = 10^5$ & 10^6 and Ra_c increases monotonically with Q_1, tables (7)-(9) give the critical values of the wave-number a_c and Ra_c for various values of T_1 and Q_1.

TABLE 7. THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T=10^5$, $\Pr=0.6185$ AND $\Pr_m=0.1$

Q_1	Wave number a	Ra_c	ω_1^2
10	19.246	4.005×10^5	676.594
30	19.844	4.166×10^5	474.920
50	20.374	4.307×10^5	292.098
80	21.084	4.481×10^5	41.763

TABLE 8. THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T=10^6$, $\Pr=0.6185$ AND $\Pr_m=0.1$

Q_1	Wave number a	Ra_c	ω_1^2
50	42.097	1.813×10^6	3213.058
100	42.827	1.856×10^6	2683.908
200	44.141	1.928×10^6	1715.104
400	46.382	2.044×10^6	11.169

TABLE 9. . THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T=10^8$, $\Pr=0.6185$ AND $\Pr_m=0.1$

Q_1	Wave number a	Ra_c	ω_1^2
50	193.687	3.747×10^7	82748.247
100	193.869	3.751×10^7	82144.707
500	195.297	3.787×10^7	77404.278
1000	197.013	3.831×10^7	71681.474

6. PREFERRED MODE OF INSTABILITY WHEN $\Pr=0.6185$ AND $\Pr_m=0.1$

Now, we shall investigate which mode of instability preferred for $\Pr=0.6185$ and $\Pr_m=0.1$.Comparing the critical values of Rayleigh number at stationary and oscillatory convection from table (10), when

$T_1 = 10^5$, $Q_1 = 100$ is the marginal state, and for $Q_1 < 100$ the preferred mode of instability is the oscillatory mode where $Ra_o < Ra_s$, while if $Q_1 > 100$, the stationary mode is the preferred mode. The same situation applied for $T_1 = 10^6$ and $T_1 = 10^8$ (i.e. see tables (11) and (12)).

TABLE 10. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^5$, $\Pr = 0.6185$ AND $\Pr_m = 0.1$

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
10	4.005×10^5	-	4.067×10^5
17	4.064×10^5	-	4.060×10^5
50	4.307×10^5	7.261×10^5	4.028×10^5
80	4.481×10^5	4.781×10^5	3.998×10^5

TABLE 11.. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^6$, $\Pr = 0.6185$ AND $\Pr_m = 0.1$

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
50	1.813×10^6	7.174×10^6	1.860×10^6
100	1.854×10^6	3.760×10^6	1.855×10^6
200	1.929×10^6	1.948×10^6	1.845×10^6
400	2.044×10^6	1.041×10^6	1.824×10^6

TABLE 12. A CRITICAL VALUES OF Ra_o AND Ra_s FOR $T = 10^8$, $\Pr = 0.6185$ AND $\Pr_m = 0.1$

Q_1	Ra_o	Ra_{s1}	Ra_{s2}
50	3.747×10^7	7.164×10^7	3.976×10^7
100	3.751×10^7	3.740×10^8	3.976×10^7
1000	3.831×10^7	3.900×10^7	3.976×10^7

1017	3.832×10^{7}	3.836×10^{7}	3.967×10^{7}
5000	4.132×10^{7}	8.733×10^{6}	-

7. SUMMARY AND CONCLUSION

At over-stability convection, a numerical solution is obtained for liquid metals such as mercury, and it showed that Ra_c increases monotonically with Q_1 and T_1. For a given value of T_1, instability will set in as over-stability until Q_1 reaches a specific value where $Ra_o = Ra_s$, then for Q_1 less than this value, oscillatory mode is preferred, otherwise stationary mode is preferred.[1]

When $\Pr = 0.6185$ and $\Pr_m = 0.1$, over-stability only exists for Q_1 small contrary to liquid metals. However, Ra_c increases monotonically with Q_1 and the preferred mode of instability is oscillatory mode until Q_1 reaches a specific value where $Ra_o = Ra_s$, then for Q_1 less than this value, oscillatory mode is preferred, otherwise stationary mode is preferred.

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