Recent Advances in Nonlinear Analysis and Optimization with Applications

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Edited by

Savin Treanță

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Preface

This book focuses on recent advances in nonlinear analysis and optimization with important applications drawn from various fields, such as artificial intelligence, genetic algorithms, optimization problems under uncertainty, and fuzzy logic. Specifically, it is devoted to nonlinear problems associated with optimization which have some connection with applications. The ideas and techniques developed here will serve to stimulate further research in this dynamic field, and, in this way, the book will become a valuable reference for researchers, engineers, and students in the field of mathematics, management science, operations research, optimal control science, and economics.

The book is structured into nine chapters. In **Chapter** 1, a class of E-differentiable E-invex multiobjective programming problems is considered. For E-differentiable multicriteria optimization problems, two E-Lagrange functions and their E-saddle points are defined. Also, the E-saddle point criteria are established for the considered E-differentiable multiobjective programming problems with both inequality and equality constraints under E-invexity hypotheses. In **Chapter** 2, a modified interval-valued variational control problem involving first-order partial differential equations (PDEs) and inequality constraints is investigated. Specifically, under some generalized convexity assumptions, LU-optimality conditions are formulated and proved for the considered interval-valued variational control problem. Also, to illustrate the main results and their effectiveness, an application is provided. Moreover, there are studied the connections between the LU-optimal solutions of the interval-valued variational control problem and the saddle-points associated with the interval-valued Lagrange functional corresponding to the modified interval-valued variational control problem. **Chapter** 3 presents a study on the partial differential equations (PDE) and partial differential inequations (PDI) constrained multi-time variational optimization problem (MVOP) of a curvilinear functional by converting it into an equivalent unconstrained multi-time variational optimization problem ($\text{MVOP}_\infty\rho$) with the help of the exact minimax penalty function method. Also, the saddle point criteria for (MVOP) and the relationships between a saddle point for (MVOP) and a minimizer of ($\text{MVOP}_\infty\rho$) are established under convexity assumption. Further, the theoretical results developed in this study are accompanied by suitable examples. **Chapter** 4, presents an adapted negative selection algorithm related to the security of swarm systems. In **Chapter** 5, the exactness property of the absolute value exact penalty function method used for solving a new class of nonconvex nonsmooth constrained optimization problems with both inequality and equality constraints is analyzed. The threshold of the penalty parameter is given such that, for all penalty parameters, there is the equivalence between the sets of optimal solutions for the nonsmooth constrained optimization problem and its associated penalized optimization problem with the absolute value exact penalty function as the objective function. This result is established for nonsmooth constrained optimization problems involving locally Lipschitz b-invex functions. **Chapter** 6 addresses a multi-objective optimization problem wherein all the objectives and constraints are interval-valued functions. Necessary and sufficient optimality conditions for the problem are established. Additionally, the weak and strong duality relationship between the primal and the corresponding dual problem is deliberated. Furthermore, counterexamples are provided to justify the theoretical developments in the paper. The main goal of **Chapter** 7 is to introduce strongly pseudomonotone and strongly quasimonotone maps of higher order in terms of set-valued maps. Solutions associated with the strong Minty type variational inequality are obtained with the help of these maps. Also, an existence theorem is established to obtain the solution for a given complementarity problem over a certain cone in which the underlying map is a strongly pseudomonotone map of a higher order. In **Chapter** 8, a Newton method is proposed to locate non-dominated solutions for a fuzzy optimization problem under the consideration of gH-differentiability, which extends and improves a recent existing method, which generalizes others existing in the recent literature. The efficiency of the considered Newton method is shown and illustrated through practical examples. **Chapter** 9 deals with a semi-infinite programming problem with real-valued Lipschitz continuous nonconvex nonsmooth objective function and an infinite number of inequality and equality constraints. Sufficient optimality conditions are derived for a feasible point under generalized convexity assumptions in terms of Michel-Penot subdifferentials. Also, Wolfe and Mond-Weir type dual models are formulated for the primal nonsmooth semi-infinite optimization problem and weak, strong, and strict converse duality results are established under generalized convexity assumptions.

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E-saddle point criteria for a class of E-differentiable E-invex multiobjective programming problems

Najeeb Abdulaleem

Department of Mathematics, Hadhramout University
P.O. BOX: (50511-50512), Al-Mahrah, Yemen.
Faculty of Mathematics and Computer Science, University of Łódź
Banacha 22, 90–238 Łódź, Poland.
nabbas985@gmail.com

Abstract. In this work, a class of E-differentiable E-invex multiobjective programming problems is considered. For E-differentiable multicriteria optimization problems, two E-Lagrange functions and their E-saddle points are defined. Then the E-saddle point criteria are established for the considered E-differentiable multiobjective programming problems with both inequality and equality constraints under E-invexity hypotheses.

Keywords: E-invex set; E-invex function; E-differentiable function; E-saddle point; E-Lagrange function; E-saddle point criteria.

1 Introduction

In this work, we consider the following (not necessarily differentiable) multiobjective programming problem (MOP) with both inequality and equality constraints:

$$\begin{aligned} &\text{minimize } f(x) = (f_1(x), ..., f_p(x)) \\ &\text{subject to } g_j(x) \leqq 0, \ \ j \in J = \{1, ..., m\}, \\ &\qquad h_t(x) = 0, \ \ t \in T = \{1, ..., q\}, \\ &\qquad x \in R^n, \end{aligned} \tag{MOP}$$

where $f_i : R^n \to R$, $i \in I = \{1, ..., p\}$, $g_j : R^n \to R$, $j \in J$, $h_t : R^n \to R$, $t \in T$, are real-valued functions defined on R^n. We shall write $g := (g_1, ..., g_m) : R^n \to R^m$ and $h := (h_1, ..., h_q) : R^n \to R^q$ for convenience. Let

$$\Omega := \{x \in X : g_j(x) \leqq 0, \ \ j \in J, \, h_t(x) = 0, \ \ t \in T\}$$

be the set of all feasible solutions of (MOP). Further, we denote by $J(x)$ the set of inequality constraint indices that are active at a feasible solution x, that is, $J(x) = \{j \in J : g_j(x) = 0\}$.

Let R^n be the n-dimensional Euclidean space and R^n_+ be its nonnegative orthant. The following convention for equalities and inequalities will be used in this work. For any vectors $x = (x_1, x_2, ..., x_n)^T$ and $y = (y_1, y_2, ..., y_n)^T$ in R^n, we define:

(i) $x = y$ if and only if $x_i = y_i$ for all $i = 1, 2, \cdots, n$;
(ii) $x > y$ if and only if $x_i > y_i$ for all $i = 1, 2, \cdots, n$;
(iii) $x \geqq y$ if and only if $x_i \geqq y_i$ for all $i = 1, 2, \cdots, n$;
(iv) $x \geq y$ if and only if $x_i \geqq y_i$ for all $i = 1, 2, \cdots, n$ but $x \neq y$;