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Estimating and Planning Constant Stress Accelerated Life Test for Generalized Logistic Distribution under Type-II Censoring

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Key Words and Phrases: *Accelerated life test, Constant Stress, Type-II censoring, Maximum likelihood Estimation, Fisher Information Matrix, Optimum Test Plan, Generalized Logistic Distribution.*

Abstract

This paper presents estimation and optimal design of constant stress accelerated life test (CSALT) under type-II censoring. The maximum likelihood (ML) method is applied to obtain the ML estimators of the parameters of generalized logistic distributio. The scale parameter of the lifetime distribution is assumed to be an inverse power law function of the stress level. The Fisher information matrix, the asymptomatic variance-covariance matrix, the confidence bounds, the predictive value of the scale parameter and the reliability function under the usual conditions are obtained. Moreover, the optimal values of the fraction of failure units are specified according to the D-optimality criterion. Finally, numerical studies are introduced to illustrate the results.

1 Introduction

Life data analysis involves analyzing lifetime data of a device, system, or component obtained under normal operating conditions in order to quantify their life characteristics. In many situations, and for many reasons, such data is very difficult, if

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not impossible, to obtain. The reasons for this difficulty can include, the long life times of today's devices and the developments of new technologies need more accurate estimation of reliability of a device or system in a short time. A common way of tackling this problem is to expose the device to sufficient overstress (e.g., temperature, voltage, humidity, and so on), or forcing them to fail more quickly than they would under normal use conditions to accelerate their failures. Therefore, the failure data are analyzed in terms of a suitable physical statistical model to obtain desired information on a device or performance under normal use conditions. This approach is called accelerated life testing (ALT).

The most common ALT loading is constant stress, step stress, and progressive stress (for more details, see Nelson (1990)). In constant stress accelerated life test (CSALT), the stress is kept at a constant level of stress throughout the life of the test, i.e., each unit is run at a constant high stress level until the occurrence of failure or the observation is censored. Practically, most devices such as lamps, semiconductors and microelectronics are run at a constant stress.

Many authors have studied statistical inference of CSALT, for example, see Singpurwalla (1971), McCool (1980), Watkins (1991), Abdel Ghaly et al. (1998), and El-Dessouky (2001). Additional to the statistical inference studies, optimum CSALT plans were studied for different lifetime distributions based on different censoring scheme; for example, Nelson and Kielpinski (1976) studied optimum ALT plans for normal and lognormal life distributions. Nelson (1990) reviewed statistically optimal and compromise plans for the single stress ALT planning problem. Yang (1994) proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Chang et al. (2010) dealt with Weibull distribution. However, no studies have been made on estimating or planning (optimal design) CSALT using the Generalized Logistic distribution.

Generalized Logistic (GL) distribution is used as the name for several different families of probability distributions (see, Johnson et al. (1995)).The main feature of this distribution is expected to be useful in many more practical situations, for example, in extreme value event evaluation, in hydrological risk analysis, in a quanta response data, to model the data with a unimodal density, and in analysis of survival data, (for more details, see Mathai and Provost (2004), Tolikas (2008), Alkasasbeh and Raqab (2009), Tolikas and Gettinby (2009), Garcia-Marin et al. (2011), Shabri et al. (2011), and Tolikas (2011)).

This paper is organized as follows: The underlying distribution and the test method are described in section 2. Section 3 introduces the ML estimators of the model parameters with their properties. Optimum failure test plan is developed in Section 4. The numerical results are presented in Section 5.

2 The Model

2.1 The Generalized Logistic Distribution as Lifetime Model

The probability density function (pdf) of a three-parameter generalized logistic distribution (Molenberghs and Verbeke (2011)) is given by

$$f(x) = \alpha\gamma e^{\alpha x}(1 + \frac{\gamma}{\theta}e^{\alpha x})^{-(\theta+1)}, \quad -\infty < x < \infty, \quad \alpha, \gamma, \theta > 0. \tag{2.1}$$

The reliability function takes the form

$$R(x) = (1 + \frac{\gamma}{\theta}e^{\alpha x})^{-\theta}, \quad -\infty < x < \infty, \quad \alpha, \gamma, \theta > 0. \tag{2.2}$$

and the corresponding failure rate is given by

$$h(x) = \alpha\gamma e^{\alpha x}(1 + \frac{\gamma}{\theta}e^{\alpha x})^{-1}, \quad -\infty < x < \infty, \quad \alpha, \gamma, \theta > 0. \tag{2.3}$$

2.2 Assumptions

We assume the following assumptions for the CSALT procedure

- A total of N units are divided into n_1, n_2, ... , n_k units where $\sum_{j=1}^{k} n_j = N$.
- There are k levels of high stress $V_j, \ \ j = 1, ..., k$ in the experiment, and V_u is the stress under usual conditions, where $V_u < V_1 < \ldots < V_k$.
- Each $n_j, \ j = 1, \ldots, k$ units in the experiment are run at a pre-specified constant stress $V_j, \ \ j = 1, ..., k$.
- It is assumed that the stress affected only on the scale parameter of the underlying distribution.
- The failure times $x_{ij}, \ i = 1, ..., r_j$ and $j = 1, ..., k$ at stress levels $V_j, \ j = 1, \ldots, k$ are the 3-parameter generalized logistic distribution with probability density function

$$f(x_{ij}, \alpha_j, \gamma, \theta) = \alpha_j\gamma e^{\alpha_j x_{ij}}(1 + \frac{\gamma}{\theta}e^{\alpha_j x_{ij}})^{-(\theta+1)}, -\infty < x_{ij} < \infty,$$
$$\alpha_j, \gamma, \theta > 0, \ i = 1, ..., r_j, \ j = 1, ..., k. \tag{2.4}$$

- The scale parameter $\alpha_j,\ \ j = 1, \ldots, k$, of the underlying lifetime distribution (2.4) is assumed to have an inverse power law function on stress levels, i.e.,

$$\alpha_j = CS_j^P,\ C, P > 0,\ where\ S_j = \frac{V^*}{V_j},$$
$$V^* = \prod_{j=1}^{k} V_j^{b_j},\ and\ b_j = \frac{r_j}{\sum_{j=1}^{k} r_j} \tag{2.5}$$

  where C is the constant of proportionality, and P is the power of the applied stress.

3 Maximum Likelihood (ML) Estimation

Considering the common assumptions (2.2), and assuming that the experiment is terminated at a specified number of failure units r_j $(r_j<n_j)$, $j = 1, ..., k$, the likelihood function will be as the following form

$$L = \prod_{j=1}^{k}\{\frac{n_j!}{(n_j - r_j)!}[\prod_{i=1}^{r_j} CS_j^P \gamma e^{CS_j^P x_{ij}}(1 + \frac{\gamma}{\theta}e^{CS_j^P x_{ij}})^{-(\theta+1)}]$$
$$(1 + \frac{\gamma}{\theta}e^{CS_j^P x_{r_j j}})^{-\theta(n_j - r_j)}\}. \tag{3.1}$$

The log-likelihood function

$$\ln L = A + lnC\sum_{j=1}^{k} r_j + ln\gamma\sum_{j=1}^{k} r_j + C\sum_{j=1}^{k}\sum_{i=1}^{r_j} S_j^P x_{ij} - (\theta + 1)$$
$$\sum_{j=1}^{k}\sum_{i=1}^{r_j} ln(1 + \frac{\gamma}{\theta}e^{CS_j^P x_{ij}}) - \theta\sum_{j=1}^{k}(n_j - r_j)ln(1 + \frac{\gamma}{\theta}e^{CS_j^P x_{r_j j}}), \tag{3.2}$$

where, $\sum_{j=1}^{k} r_j lnS_j = 0,\ \ A = \sum_{j=1}^{k} ln[\frac{n_j!}{(n_j - r_j)!}]$.

3.1 ML Estimation of the Parameters

The first derivatives of the log-likelihood function (3.2) with respect to the unknown parameters C, P, γ and θ are

$$\frac{\partial \ln L}{\partial C} = \frac{\sum_{j=1}^{k} r_j}{C} + \sum_{j=1}^{k}\sum_{i=1}^{r_j} S_j^P x_{ij} - (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j} \xi_{ij} - \theta\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}. \tag{3.3}$$

$$\frac{\partial \ln L}{\partial P} = C\{\sum_{j=1}^{k}\sum_{i=1}^{r_j} \sigma_{ij} - (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j} lnS_j\xi_{ij} - \theta\sum_{j=1}^{k}(n_j - r_j)lnS_j\xi_{r_j j}\}. \tag{3.4}$$

$$\frac{\partial \ln L}{\partial \gamma} = \frac{1}{\gamma}\{\sum_{j=1}^{k} r_j - (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j} \nu_{ij} - \theta\sum_{j=1}^{k}(n_j - r_j)\nu_{r_j j}\}. \tag{3.5}$$

$$\frac{\partial \ln L}{\partial \theta} = \sum_{j=1}^{k}\sum_{i=1}^{r_j} \delta_{ij}(\frac{\theta+1}{\theta}\ \nu_{ij} - \pi_{ij}) + \sum_{j=1}^{k}(n_j - r_j)(\nu_{r_j j} - \pi_{r_j j}), \tag{3.6}$$

where, $\xi_{ij}= S_j^P x_{ij}\ \nu_{ij}$, $\nu_{ij}= (1+\frac{\theta}{\gamma}e^{-CS_j^P x_{ij}})^{-1}$, $\xi_{r_j j}= S_j^P x_{r_j j}\ \nu_{r_j j}$, $\nu_{r_j j}= (1+\frac{\theta}{\gamma}e^{-CS_j^P x_{r_j j}})^{-1}$, $\sigma_{ij} = S_j^P x_{ij} lnS_j$, $\pi_{ij}=-ln(1-\nu_{ij})$, and $\pi_{r_j j}=-ln(1-\nu_{r_j j})$.

The numerical solution of the equations (3.3) to (3.6) are placed in section (5.1). The second partial derivatives of the log-likelihood function (3.2) with respect to the model parameters C, P, γ and θ are as follows

$$\frac{\partial^2 \ln L}{\partial C^2} = -\{\frac{\sum_{j=1}^{k} r_j}{C^2} + \frac{\theta(\theta+1)}{\gamma}\sum_{j=1}^{k}\sum_{i=1}^{r_j} \xi_{ij}^2 e^{-CS_j^P x_{ij}} + \frac{\theta^2}{\gamma}\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}^2 e^{-CS_j^P x_{r_j j}}\}. \tag{3.7}$$

$$\frac{\partial^2 \ln L}{\partial P^2} = -C\{\sum_{j=1}^{k}\sum_{i=1}^{r_j} lnS_j[(\theta+1)\xi_{ij}(C\sigma_{ij}(1-\nu_{ij}) + lnS_j) - \sigma_{ij}] + \theta\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j} lnS_j(C\sigma_{r_j j}(1-\nu_{r_j j}) + lnS_j)\}. \tag{3.8}$$

$$\frac{\partial^2 \ln L}{\partial \gamma^2} = \frac{-1}{\gamma^2}\{\sum_{j=1}^{k} r_j\ - (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j} \nu_{ij}^2 - \theta\sum_{j=1}^{k}(n_j - r_j)\nu_{r_j j}^2\}. \tag{3.9}$$

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \frac{-1}{\theta}\{\frac{1}{\theta}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\nu_{ij}[(1-\theta)+(1+\theta)(1-\nu_{ij})] - \sum_{j=1}^{k}(n_j - r_j)\nu_{r_j j}^2\}. \quad (3.10)$$

$$\frac{\partial^2 \ln L}{\partial C \partial P} = -\{\sum_{j=1}^{k}\sum_{i=1}^{r_j}[(\theta+1)\xi_{ij}(C\sigma_{ij}(1-\nu_{ij}) + lnS_j) - \sigma_{ij}] + \theta\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}(C\sigma_{r_j j}(1-\nu_{r_j j}) + lnS_j)\}. \quad (3.11)$$

$$\frac{\partial^2 \ln L}{\partial C \partial \gamma} = \frac{-1}{\gamma}\{(\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j}\xi_{ij}(1-\nu_{ij}) + \theta\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}(1-\nu_{r_j j})\}. \quad (3.12)$$

$$\frac{\partial^2 \ln L}{\partial C \partial \theta} = -\{\sum_{j=1}^{k}\sum_{i=1}^{r_j}\xi_{ij}(1 - \frac{(\theta+1)}{\theta}(1-\nu_{ij})) + \sum_{j=1}^{k}(n_j - r_j)\nu_{r_j j}\xi_{r_j j}\}. \quad (3.13)$$

$$\frac{\partial^2 \ln L}{\partial P \partial \gamma} = \frac{-C}{\gamma}\{(\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j}\xi_{ij}(1-\nu_{ij})lnS_j + \theta\sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}(1-\nu_{r_j j})lnS_j\}. \quad (3.14)$$

$$\frac{\partial^2 \ln L}{\partial p \partial \theta} = -C\{\sum_{j=1}^{k}\sum_{i=1}^{r_j}\xi_{ij}(1 - \frac{(\theta+1)}{\theta}(1-\nu_{ij}))lnS_j + \sum_{j=1}^{k}(n_j - r_j)\xi_{r_j j}\nu_{r_j j}lnS_j\}. \quad (3.15)$$

$$\frac{\partial^2 \ln L}{\partial \gamma \partial \theta} = \frac{-1}{\gamma}\{\sum_{j=1}^{k}\sum_{i=1}^{r_j}\nu_{ij}(1 - \frac{(\theta+1)}{\theta}(1-\nu_{ij})) + \sum_{j=1}^{k}(n_j - r_j)\nu_{r_j j}^2\}. \quad (3.16)$$

where $\sigma_{r_j j} = S_j^P x_{r_j j} lnS_j$

Therefore, the elements of the Fisher information matrix for the MLE can be obtained as the expectations of the negative of the second partial derivatives, i.e.,

$$F = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ & f_{22} & f_{23} & f_{24} \\ & & f_{33} & f_{34} \\ & & & f_{44} \end{pmatrix} = -E\begin{pmatrix} \frac{\partial^2 lnL}{\partial c^2} & \frac{\partial^2 lnL}{\partial c \partial p} & \frac{\partial^2 lnL}{\partial c \partial \gamma} & \frac{\partial^2 lnL}{\partial c \partial \theta} \\ & \frac{\partial^2 lnL}{\partial p^2} & \frac{\partial^2 lnL}{\partial p \partial \gamma} & \frac{\partial^2 lnL}{\partial p \partial \theta} \\ & & \frac{\partial^2 lnL}{\partial \gamma^2} & \frac{\partial^2 lnL}{\partial \gamma \partial \theta} \\ & & & \frac{\partial^2 lnL}{\partial \theta^2} \end{pmatrix}. \quad (3.17)$$

The asymptotic variance-covariance matrix for the MLE is defined as the inverse of the Fisher's information matrix (3.18), i.e.,

$$\Sigma = \hat{F}^{-1} \tag{3.18}$$

3.2 Prediction of the Scale Parameter and the Reliability Function

To predict the value of the scale parameter α_u under the usual condition stress V_u, the invariance property of MLE is used, i.e.,

$$\hat{\alpha} = \hat{C} S_u^{\hat{P}}, \ where \ S_u = \frac{V^*}{V_u},$$
$$V^* = \prod_{j=1}^{k} V_j^{b_j}, \ and \ b_j = \frac{r_j}{\sum_{j=1}^{k} r_j} \tag{3.19}$$

The MLE of the reliability function at the lifetime x_0 under the usual condition stress V_u, is given by

$$\hat{R}_u(x_0) = (1 + \frac{\gamma}{\theta} \ e^{\hat{\alpha}_u x_0})^{-\theta} \tag{3.20}$$

4 Optimum Test Plan

A test plan is usually designed before conducting an accelerated life test. Test plan helps in accurately estimating reliability at operating conditions while minimizing test time and costs. A test plan should be used to decide on the appropriate censoring time and the number of test units that need to be allocated to the different stress levels.

This section describes optimum test plan that minimize the determinant of Fisher information matrix for specifying the best choice of the values of the fraction of the failure units that fail at each stress level $V_j, j = 1, ..., k$ according to the D-Optimality criterion (Gouno (2007)). Let $\lambda_j = \frac{r_j}{N}$ be the fraction of the failure units that may fail at each stress level $V_j, j = 1, ..., k$, then the optimal values of $\lambda_j, j = 1, ..., k$ at

each stress level $V_j, j = 1, ..., k$ can be obtained by solving the following equation

$$\frac{\partial |F|}{\partial \lambda_j} = 0, \quad j = 1, ..., k \tag{4.1}$$

The determinant of F and the derivation of the equation (4.1) are placed in Appendix. The numerical solution of the equation (4.1) is obtained in order to get the optimum value of the fraction and the number of the failure units that fail at each stress level $V_j, j = 1, 2$ as will be shown in section (5.2).

5 Simulation Studies

This section presents the numerical solutions to obtain the MLE of the unknown parameters C, P, γ, and θ, their mean squared errors (MSE), relative absolute biases (RAB), Lower Bound (LB), Upper Bound (UB), the estimated of scale parameter α, and reliability function $R(x_0)$ under normal use conditions V_u. Also, it presents the numerical solution to determine the best choice values of the fraction of the failure units.

5.1 MLE under Type-II Censoring

The numerical solution is performed according to the following steps

- For given values of C, P and stress level $V_j, j = 1, 2, 3$, the values of $\alpha_j, j = 1, 2, 3$ are calculated according to the equation (2.5).
- Generate a random sample of size n from the 3-parameter generalized logistic distribution and obtained the random variables $x_{ij}, \{i = 1, ..., r_j, \ j = 1, ..., k\}$ for given values of $n_j, r_j, j = 1, ..., k$, and different values of $(\alpha_0, \gamma_0, \theta_0)$.
- Based on the values of $n_j, r_j, V_j, x_{ij}, \{i = 1, ..., r_j, \ j = 1, ..., k \}$and V_u, the MLE, and their MSE, RAB, LB, and UB, in additional to, $\hat{\alpha_u}$ and $\hat{R}_u(x_0)$, are obtained.
- The steps are repeated exceed 150 times until getting the MLE as shown in table (5.1).

The numerical results which are placed in tables (5.1) to (5.4) are based on $n_1 = 40, n_2 = 20, n_3 = 5, r_1 = 36, r_2 = 19, r_3 = 4, V_1 = 0.75, V_2 = 1.5, V_3 = 2.25$ and $V_u = 0.5$.

Table 5.1: The MLE, RAB and MSE

C_0	P_0	γ_0	θ_0	$Parameter$	MLE	RAB	MSE
0.8	1.0	1.0	1.0	C	0.8060	0.0075	0.00004
				P	1.3202	0.3202	0.1025
				γ	0.8634	0.1366	0.0187
				θ	1.2887	0.2887	0.0834
				α_1	1.1940	0.1083	0.0136
				α_2	0.4782	0.1123	0.0037
				α_3	0.2800	0.2204	0.0063
1.0		0.8		C	0.9930	0.0070	0.0001
				P	1.2585	0.2585	0.0668
				γ	0.6820	0.1475	0.0139
				θ	1.3508	0.3508	0.1231
				α_1	1.4443	0.0724	0.0095
				α_2	0.6037	0.1035	0.0049
				α_3	0.3624	0.1927	0.0075
		1.0	0.7	C	0.9662	0.0338	0.0011
				P	1.2929	0.2929	0.0858
				γ	0.7971	0.2029	0.0412
				θ	0.9534	0.3620	0.0642
				α_1	1.4198	0.0542	0.0053
				α_2	0.5795	0.1395	0.0088
				α_3	0.3431	0.2358	0.0112
			1.0	C	1.0068	0.0068	0.0001
				P	1.3174	0.3174	0.1007
				γ	0.8627	0.1373	0.0189
				θ	1.2918	0.2918	0.0852
				α_1	1.4903	0.1066	0.0206
				α_2	0.5980	0.1120	0.0057
				α_3	0.3505	0.2192	0.0097
	1.2		0.7	C	0.9662	0.0338	0.0011
				P	1.4929	0.2441	0.0858
				γ	0.7971	0.2029	0.0412
				θ	0.9534	0.3620	0.0642
				α_1	1.5069	0.0542	0.0060
				α_2	0.5354	0.1395	0.0075
				α_3	0.2923	0.2358	0.0081

Table 5.1: The MLE, RAB and MSE (Cont.)

C_0	P_0	γ_0	θ_0	$Parameter$	MLE	RAB	MSE
		1.3	1.0	C	1.0169	0.0169	0.0003
				P	1.5674	0.3062	0.1345
				γ	1.1362	0.1260	0.0268
				θ	1.2572	0.2572	0.0662
				α_1	1.6215	0.1344	0.0369
				α_2	0.5471	0.1206	0.0056
				α_3	0.2898	0.2423	0.0086

Table 5.2: The Confidence Intervals

C_0	P_0	γ_0	θ_0	$Parameter$	$Variance$	$L.B$	$U.B$
0.8	1.0	1.0	1.0	C	0.0132	0.6170	0.9950
				P	0.0840	0.8434	1.7970
				γ	0.0676	0.4357	1.2911
				θ	0.1085	0.7468	1.8306
1.0		0.8		C	0.0205	0.7575	1.2285
				P	0.08530	0.7781	1.7389
				γ	0.0400	0.3530	1.0110
				θ	0.1084	1.1725	1.5291
		1.0	0.7	C	0.0256	0.7028	1.2296
				P	0.0878	0.8056	1.7803
				γ	0.0103	0.2700	1.3242
				θ	0.0500	0.5857	1.3211
			1.0	C	0.0207	0.7701	1.2435
				P	.0840	0.8406	1.7942
				γ	0.0676	0.4350	1.2904
				θ	0.1085	0.7499	1.8337
	1.2		0.7	C	0.0256	0.7030	1.2294
				P	0.0878	1.0055	1.9803
				γ	0.1027	0.2699	1.3243
				θ	0.0500	0.5856	1.3212
		1.3	1.0	C	0.0209	0.7791	1.2547
				P	0.0800	1.1021	2.0327
				γ	0.1295	0.5442	1.7282
				θ	0.1084	0.7156	1.7988

Table 5.3: The Asymptotic Variance-Covariance Matrix

C_0	P_0	γ_0	θ_0	$Parameter$	C	P	γ	θ
0.8	1.0	1.0	1.0	C	0.0132	0.0098	0.0107	-0.0245
				P		0.0840	0.0107	-0.0302
				γ			0.0676	-0.0464
				θ				0.1085
1.0		0.8		C	0.0205	0.0118	0.0068	-0.0303
				P		0.0853	0.0065	-0.0307
				γ			0.0400	-0.0314
				θ				0.1084
		1.0	0.7	C	0.0256	0.0160	0.0243	-0.0257
				P		0.0878	0.0190	-0.0232
				γ			0.0103	-0.0438
				θ				0.0500
			1.0	C	0.0207	0.0122	0.0134	-0.0306
				P		0.0840	0.0107	-0.0302
				γ			0.0676	-0.0464
				θ				0.1085
	1.2		0.7	C	0.0256	0.0160	0.0243	-0.0257
				P		0.0878	0.0189	-0.0232
				γ			0.1027	-0.0438
				θ				0.0500
		1.3	1.0	C	0.0209	0.0124	0.0250	-0.0309
				P		0.0800	0.0178	-0.0286
				γ			0.1295	-0.0713
				θ				0.1084

Table 5.4: Estimates α and $R(x_0)$ under normal conditions

C_0	P_0	γ_0	θ_0	$\hat{\alpha}_u$	x_0	$\hat{R}_u(x_0)$
0.8	1.0	1.0	1.0	2.1548	0.009	0.4952
					0.2	0.3939
					1.0	0.1039
					2.0	0.0133
1.0		0.8		2.6935	0.02	0.5422
					0.4	0.2985
					1.2	0.0470
					2.5	0.0015
		1.0	0.7	2.0201	0.02	0.5284
					0.4	0.3659
					1.2	0.1369
					2.5	0.0226
			1.0	2.6935	0.007	0.4953
					0.03	0.4798
					0.1	0.4331
					1.0	0.0634
	1.2		0.7	2.6935	0.01	0.5301
					0.2	0.3961
					1.0	0.0768
					2.0	0.0078
		1.3	1.0	3.2838	0.02	0.4187
					0.4	0.1714
					1.2	0.0147
					2.5	0.0002

From the results of the tables (5.1) to (5.4), we observe the MSE of the scale parameter $\alpha_j,\ j = 1, 2, 3$ decreases as the stress value $V_j,\ j = 1, 2, 3$ increases. Also, it is decreases when the values of C_0 and the values of γ_0 increases at the same values of P_0 and θ_0. In addition, the reliability decreases when the mission time x_0 increases, and it is reduced when the values of C_0, P_0 increases and the values of γ_0, θ_0 increases at the same mission time. Moreover, there is an inverse proportional relationship between $\hat{\alpha_u}$ and $\hat{R_u}(x_0)$ at the same mission time.

5.2 Optimum Test Plan of Failure Fractions

To illustrate the procedure of optimum test design, a numerical example is given as follows. Suppose that a simple constant stress test was run to estimate the optimum

value of the fraction of the failure units. The stress levels to test units are $V_1 = 1$ and $V_2 = 2$. The optimum fraction of the failure units is determined by solving equation (4.2). Tables (5.5) to (5.9) presented the optimal fraction of the failure units,the number of failure units and the Generalized Asymptotic Variance (GAV) at different values of N .

Table 5.5: The Optimum Values of λ_1^*, r_1^* and GAV at ($C = 1$, $P = 1$, $\gamma = 1$, θ =1)

N	r_1	r_2	λ_1	λ_2	λ_1^*	r_1^*	GAV
105	50	46	0.476	0.438	0.143	15	0.000007550
225	130	73	0.578	0.324	0.279	63	0.000000070
300	150	128	0.500	0.427	0.350	105	0.000000050
400	221	166	0.553	0.415	0.205	82	0.000000004
500	321	165	0.642	0.330	0.549	274	0.000000002

Table 5.6: The Optimum Values of λ_1^*, r_1^* and GAV at ($C = 1$, $P = 1$, γ =1 , θ =1.4)

N	r_1	r_2	λ_1	λ_2	λ_1^*	r_1^*	GAV
105	55	46	0.524	0.438	0.563	59	0.00001340
225	130	73	0.578	0.324	0.335	75	0.00000012
300	150	128	0.500	0.427	0.401	120	0.00000008
400	221	166	0.553	0.415	0.287	115	0.00000001
500	321	165	0.642	0.330	0.549	274	0.00000001

Table 5.7: The Optimum Values of λ_1^*, r_1^* and GAV at ($C = 1$, $P = 1.5$, $\gamma = 1$, $\theta = 1.2$)

N	r_1	r_2	λ_1	λ_2	λ_1^*	r_1^*	GAV
105	55	46	0.524	0.438	0.562	59	0.0000078000
225	130	73	0.578	0.324	0.316	71	0.0000001000
300	150	128	0.500	0.427	0.329	99	0.0000000600
400	221	166	0.553	0.415	0.260	104	0.0000000100
500	321	165	0.642	0.330	0.542	271	0.0000000096

Table 5.8: The Optimum Values of λ_1^*, r_1^* and GAV at ($C = 0.8$, $P = 1.5$, $\gamma = 1$, $\theta = 1.4$)

N	r_1	r_2	λ_1	λ_2	λ_1^*	r_1^*	GAV
105	55	46	0.524	0.438	0.563	59	0.000008570
225	130	73	0.578	0.324	0.335	74	0.000000080
300	150	128	0.500	0.427	0.398	120	0.000000050
400	221	166	0.553	0.415	0.287	115	0.000000010
500	321	165	0.642	0.330	0.546	273	0.000000009

Table 5.9: The Optimum Values of λ_1^*, r_1^* and GAV at ($C = 1.2$, $P = 1.4$, $\gamma = 1.2$, θ =0.7)

N	r_1	r_2	λ_1	λ_2	λ_1^*	r_1^*	GAV
105	55	46	0.524	0.438	0.509	54	0.000010070
225	130	73	0.578	0.324	0.247	56	0.000000110
300	150	128	0.500	0.427	0.370	11	0.000000060
400	221	166	0.553	0.415	0.145	58	0.000000010
500	321	165	0.642	0.330	0.568	284	0.000000007

From the results of tables (5.5) to (5.9), we observe that less than half of the units will fail under the stress V_1 , and the others will fail under the stress V_2 , because of λ_1^* is taken the values between 0.1 and 0.4. In table (5.9), when $N = 225$, there are 56 units will fail under the stress V_1 , 73 units will fail under the stress V_2 , and 96 will be censored. Also, we observe that GAV decreasing as the sample size increasing.

6 Conclusion

This paper presents the Maximum Likelihood method of the parameter estimation under type-II, censoring. The data failure times at each stress level are assumed to follow the 3-parameter generalized logistic distribution with scale parameter that is an inverse power law function. The ML estimation, Fisher information matrix, the asymptomatic variance-covariance matrix, the predictive value of the scale parameter and the reliability function under the usual conditions stress were obtained for various combinations of the model parameters. In additional, the corresponding optimum value of the fraction of the failure units are obtained numerically by the D-optimality criterion. GL distribution have been extensively used in many different areas and it is very useful in a wide variety of applications. Since, standard Logistic, four-parameters extended GL , four-parameters extended GL type-I, two parameter GL , type-I GL , Generalized Log-logistic,standard Log-logistic, Logistic Exponential, Exponentiated Exponential (for x>0), Generalized Burr, Burr III, Burr XII distributions are special

cases from the GL distribution then their results of the MLE and optimum test plan become particular cases of the results obtained here.

A *Appendix*

The determinant of F is

$$\begin{aligned}|F| &= (f_{33}f_{44} - f_{34}^2)(f_{11}f_{22} - f_{12}^2) - (f_{23}f_{44} - f_{24}f_{34})(f_{11}f_{23} - f_{12}f_{13}) \\ &+(f_{23}f_{34} - f_{24}f_{33})(f_{11}f_{24} - f_{12}f_{14}) - (f_{13}f_{44} - f_{14}f_{34})(f_{13}f_{22} - f_{12}f_{23}) \\ &+(f_{13}f_{34} - f_{33}f_{14})(f_{14}f_{22} - f_{12}f_{24}) - (f_{13}f_{24} - f_{23}f_{14})(f_{14}f_{23} - f_{13}f_{24}) \end{aligned} \tag{A.1}$$

The derivative of $|F|$ for obtaining the optimum value of the fraction of the failure units λ_j, $j = 1, ..., k$ is given as follows

$$\begin{aligned}\frac{\partial|F|}{\partial\lambda_j} &= (f_{33}f_{44} - f_{34}^2)(f'_{11}f_{22} + f_{11}f'_{22} - 2f_{12}f'_{12}) + (f'_{33}f_{44} + f_{33}f'_{44} - 2f'_{34}f_{34}) \\ &(f_{11}f_{22} - f_{12}^2) - (f'_{23}f_{44} + f_{23}f'_{44} - f'_{24}f_{34} - f_{24}f'_{34})(f_{11}f_{23} - f_{12}f_{13}) - \\ &(f_{23}f_{44} - f_{24}f_{34})(f'_{11}f_{23} + f_{11}f'_{23} - f'_{12}f_{13} - f_{12}f'_{13}) + (f_{23}f_{34} - f_{24}f_{33}) \\ &(f'_{11}f_{24} + f_{11}f'_{24} - f'_{12}f_{14} - f_{12}f'_{14}) + (f'_{23}f_{34} + f_{23}f'_{34} - f'_{24}f_{33} - f_{24}f'_{33}) \\ &(f_{11}f_{24} - f_{12}f_{14}) - (f_{13}f_{44} - f_{14}f_{34})(f'_{13}f_{22} + f_{13}f'_{22} - f'_{12}f_{23} - f_{12}f'_{23}) - \\ &(f'_{13}f_{44} + f_{13}f'_{44} - f'_{14}f_{34} - f_{14}f'_{34})(f_{13}f_{22} - f_{12}f_{23}) + (f_{13}f_{34} - f_{33}f_{14}) \\ &(f'_{14}f_{22} + f_{14}f'_{22} - f'_{12}f_{24} - f_{12}f'_{24}) + (f'_{13}f_{34} + f_{13}f'_{34} - f'_{33}f_{14} - f_{33}f'_{14}) \\ &(f_{14}f_{22} - f_{12}f_{24}) - (f_{13}f_{24} - f_{23}f_{14})(f'_{14}f_{23} + f_{14}f'_{23} - f'_{13}f_{24} - f_{13}f'_{24}) \\ &-(f'_{13}f_{24} + f_{13}f'_{24} - f'_{23}f_{14} - f_{23}f'_{14})(f_{14}f_{23} - f_{13}f_{24}), \end{aligned} \tag{A.2}$$

where,

$$f'_{11} = N(\frac{1}{C^2} - \frac{\theta^2}{\gamma}e^{-CS^{P_j}x_{r_jj}}\xi_{r_jj}^2). \tag{A.3}$$

$$f'_{22} = -NClnS_j\xi_{r_jj}(lnS_j + C(1 - \nu_{r_j)j}\sigma_{r_jj}). \tag{A.4}$$

$$f'_{33} = \frac{N}{\gamma^2}(1 + \theta\nu_{r_jj}^2). \tag{A.5}$$

$$f'_{44} = \frac{N}{\theta}\nu_{r_jj}^2. \tag{A.6}$$

$$f'_{12} = -N\theta\xi_{r_jj}(lnS_j + C(1-\nu_{r_jj})\sigma_{r_jj}). \tag{A.7}$$

$$f'_{13} = \frac{-N\theta}{\gamma}\xi_{r_jj}(1-\nu_{r_jj}). \tag{A.8}$$

$$f'_{14} = -N\xi_{r_jj}\nu_{r_jj}. \tag{A.9}$$

$$f'_{23} = \frac{-NC\theta}{\gamma}lnS_j\xi_{r_jj}(1-\nu_{r_jj}). \tag{A.10}$$

$$f'_{24} = -NC\nu^2_{r_jj}\sigma_{r_jj}. \tag{A.11}$$

$$f'_{34} = \frac{-N}{\gamma}\nu^2_{r_jj} \tag{A.12}$$

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