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Neutrosophic Local Function and Generated Neutrosophic Topology

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Abstract: In this paper we introduce the notion of ideals on neutrosophic set which is considered as a generalization of fuzzy and fuzzy intuitionistic ideals studies in [9,11] , the important topological neutrosophic ideals has been given in [4]. The concept of neutrosophic local function is also introduced for a neutrosophic topological space. These concepts are discussed with a view to find new neutrosophic topology from the original one in [8]. The basic structure, especially a basis for such generated neutrosophic topologies and several relations between different topological neutrosophic ideals and neutrosophic topologies are also studied here. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Intuitionistic Fuzzy Ideal; Fuzzy Ideal; Topological neutrosophic ideal; and Neutrosophic Topology.

1. Introduction

The neutrosophic set concept was introduced by Smarandache [12, 13]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9, 10]. The fuzzy set was introduced by Zadeh [14] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama at el [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α-cut and topological neutrosophic ideals, which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [12, 13], Hanafy and Salama at el. [4, 5, 6, 7, 8, 9, 10].

3. Topological Neutrosophic Ideals [4].

Definition 3.1: Let X is non-empty set and L a non–empty family of NSs. We will call L is a topological neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
- $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A topological neutrosophic ideal L is called a σ -topological neutrosophic ideal if

$\{A_j\}_{j \in N} \leq L$, implies $\underset{j \in J}{\vee} A_j \in L$ (countable additivity).

The smallest and largest topological neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X. Also, $N.L_f$, $N.L_c$ are denoting the topological neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NS : B \subseteq A\}$ is an NL on X. This is called the principal NL of all NSs of denoted by NL$\langle A \rangle$.

Remark 3.2.

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $O_N \in L$.

Example 3.3.

Any Initiutionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$.

Example 3.4.

Let $X = \{a, b, c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family $L = \{O_N, A, B, D\}$ of NSs is an NL on X.

Example.3.5

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

x	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.3.3: Let L1 and L2 be two NL on X. Then L2 is said to be finer than L1 or L1 is coarser than L2 if L1 $\leq$ L2. If also L1 $\neq$ L2. Then L2 is said to be strictly finer than L1 or L1 is strictly coarser than L2. Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L1 is coarser than L2 this relation is induced the inclusion in NSs. The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.3.1: Let $\{L_j : j \in J\}$ be any non - empty family of topological neutrosophic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are topological neutrosophic ideal on X, In fact L is the smallest upper bound of the set of the Lj in the ordered set of all topological neutrosophic ideals on X.

Remark.3.2: The topological neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all topological neutrosophic ideals on X.

Proposition.3.3: A neutrosophic set A in topological neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof: (Necessity)Suppose A is a base of L. Then clearly every member of L contained in A. (Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.3.4: For a topological neutrosophic ideal L1 with base A, is finer than a fuzzy ideal L2 with base B iff every member of B contained in A.

Proof: Immediate consequence of Definitions

Corollary.3.1: Two topological neutrosophic ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.3.1: Let $\eta = \{\langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J\}$ be a non empty collection of neutrosophic subsets of X. Then there exists a topological neutrosophic ideal L (η) = {A ∈ NSs: A ⊆ ∨ Aj} on X for some finite collection {Aj: j = 1,2, ..., n ⊆ η}.

Proof: *Clear.*

Remark.3.3: The topological neutrosophic ideal L (η) defined above is said to be generated by η and η is called sub base of L(η).

Corollary.3.2: Let L1 be an topological neutrosophic ideal on X and A ∈ NSs, then there is a topological neutrosophic ideal L2 which is finer than L1 and such that A ∈ L2 if $A \vee B \in L2$ *for each* $B \in L1$.

Corollary.3.3: *Let* $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in L_1$ *and* $B = \langle x, \mu_B, \sigma_B, \nu_B \rangle \in L_2$*, where* L_1 *and* L_2 *are topological neutrosophic ideals on the set X. then the neutrosophic set* $A*B = \langle \mu_{A*B}(x), \sigma_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ *on X where* $\mu_{A*B}(x) = \vee\{\mu_A(x) \wedge \mu_B(x) : x \in X\}, \sigma_{A*B}(x)$ *may be* $= \vee\{\sigma_A(x) \wedge \sigma_B(x)\}$ *or* $\wedge\{\sigma_A(x) \vee \sigma_B(x)\}$ *and* $\nu_{A*B}(x) = \wedge\{\nu_A(x) \vee \nu_B(x) : x \in X\}$.

4. Neutrosophic local Functions

Definition.4.1. Let (X, τ) be a neutrosophic topological spaces (NTS for short) and L be neutrsophic ideal (NL, for short) on X. Let A be any NS of X. Then the neutrosophic local function $NA^*(L, \tau)$ of A is the union of all neutrosophic points(NP, for short) $C(\alpha, \beta, \gamma)$ such that if

$U \in N(C(\alpha,\beta,\gamma))$ and $NA^*(L,\tau) = \vee\{C(\alpha,\beta,\gamma) \in X : A \wedge U \notin L$ for every U nbd of $C(\alpha,\beta,\gamma)\}$, $NA^*(L,\tau)$ is called a neutrosophic local function of A with respect to τ and L which it will be denoted by $NA^*(L,\tau)$, or simply $NA^*(\mathrm{L})$.

Example .4.1. One may easily verify that.

If L=$\{0_N\}$, then $NA^*(L,\tau) = Ncl(A)$, for any neutrosophic set $A \in NSs$ on X.

If $\mathrm{L} = \{\text{all NSs on X}\}$ then $NA^*(L,\tau) = 0_N$, for any $A \in NSs$ on X .

Theorem.4.1. Let (X,τ) be a NTS and L_1, L_2 be two topological neutrosophic ideals on X. Then for any neutrosophic sets A, B of X. then the following statements are verified

i) $A \subseteq B \Rightarrow NA^*(L,\tau) \subseteq NB^*(L,\tau)$,

ii) $L_1 \subseteq L_2 \Rightarrow NA^*(L_2,\tau) \subseteq NA^*(L_1,\tau)$.

iii) $NA^* = Ncl(A^*) \subseteq Ncl(A)$.

iv) $NA^{**} \subseteq NA^*$.

v) $N(A \vee B)^* = NA^* \vee NB^*$.,

vi) $N(A \wedge B)^*(L) \leq NA^*(L) \wedge NB^*(L)$.

vii) $\ell \in L \Rightarrow N(A \vee \ell)^* = NA^*$.

viii) $NA^*(L,\tau)$ is neutrosophic closed set .

Proof.

i) Since $A \subseteq B$, let $p = C(\alpha,\beta,\gamma) \in NA^*(L_1)$ then $A \wedge U \notin L$ for every $U \in N(p)$. By hypothesis we get $B \wedge U \notin L$, then $p = C(\alpha,\beta,\gamma) \in NB^*(L_1)$.

ii) Clearly. $L_1 \subseteq L_2$ implies $NA^*(L_2,\tau) \subseteq NA^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p = C(\alpha,\beta,\gamma) \in NA^*$ but $C(\alpha,\beta,\gamma)$ may not be contained in $NA^*(L_2)$.

iii) Since $\{O_N\} \subseteq L$ for any NL on X, therefore by (ii) and Example 3.1, $NA^*(L) \subseteq NA^*(\{O_N\}) = Ncl(A)$ for any NS A on X. Suppose $p_1 = C_1(\alpha,\beta,\gamma) \in Ncl(NA^*(L_1))$. So for every $U \in N(p_1)$, $NA^* \wedge U \neq O_N$, there exists $p_2 = C_2(\alpha,\beta) \in A^*(L_1) \wedge U)$ such that for every V *nbd* of $p_2 \in N(p_2), A \wedge U \notin L$. Since $U \wedge V \in N(p_2)$ then $A \wedge (U \cap V) \notin L$ which leads to $A \wedge U \notin L$, for every $U \in N(C(\alpha,\beta))$ therefore $p_1 = C(\alpha,\beta) \in (A^*(L\ ))$ and so $Ncl(NA^*) \leq NA^*$ While, the other inclusion follows directly. Hence $NA^* = Ncl(NA^*)$.But the inequality $NA^* \leq Ncl(NA^*)$.

iv) The inclusion $NA^* \vee NB^* \leq N(A \vee B)^*$ follows directly by (i). To show the other implication, let $p = C(\alpha,\beta,\gamma) \in N(A \vee B)^*$ then for every $U \in N(p)$, $(A \vee B) \wedge U \notin L, i.e,$ $(A \wedge U) \vee (B \wedge U) \notin L.$ then, we have two cases $A \wedge U \notin L$ and $B \wedge U \in L$ or the converse, this means that exist $U_1, U_2 \in N(C(\alpha,\beta,\gamma)$ such that $A \wedge U_1 \notin L$, $B \wedge U_1 \notin L$, $A \wedge U_2 \notin L$ and $B \wedge U_2 \notin L$. Then $A \wedge (U_1 \wedge U_2) \in L$ and $B \wedge (U_1 \wedge U_2) \in L$ this gives $(A \vee B) \wedge (U_1 \wedge U_2) \in L$, $U_1 \wedge U_2 \in N(C(\alpha,\beta,\gamma)$ which contradicts the hypothesis. Hence the equality holds in various cases.

v) By (iii), we have $NA^{**} = Ncl(NA^*)^* \leq Ncl(NA^*) = NA^*$ Let (X,τ) be a GIFTS and L be GIFL on X . Let us define the neutrosophic closure operator $cl^*(A) = A \cup A^*$ for any GIFS A of X. Clearly, let $Ncl^*(A)$ is a neutrosophic operator. Let $N\tau^*(L)$ be NT generated by Ncl^* .i.e $N\tau^*(L) = \{A : Ncl^*(A^c) = A^c\}$ Now $L = \{O_N\} \Rightarrow Ncl^*(A) = A \cup NA^* = A \cup Ncl(A)$ for every neutrosophic set A. So, $N\tau^*(\{O_N\}) = \tau$. Again $L = \{all$ NSs on X$\}$ $\Rightarrow$ $Ncl^*(A) = A,$ because $NA^* = O_N$, for every neutrosophic set A so $N\tau^*(L)$ is the neutrosophic discrete topology on X. So we can conclude by Theorem 4.1.(ii). $N\tau^*(\{O_N\}) = N\tau^*(L)$ i.e. $N\tau \subseteq N\tau^*$, for any neutrosophic ideal L_1 on X. In particular, we have for two topological neutrosophic ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow N\tau^*(L_1) \subseteq N\tau^*(L_2)$.

Theorem.4.2. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any topological neutrosophic ideal L on X, $\tau_1 \leq \tau_2$ implies $NA^*(L,\tau_2) \subseteq NA^*(L,\tau_1)$, for every A $\in L$ then $N\tau^*_1 \subseteq N\tau^*_2$

Proof. Clear.

A basis $N\beta(L,\tau)$ for $N\tau^*(L)$ can be described as follows:

$N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Then we have the following theorem

Theorem 4.3. $N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Forms a basis for the generated NT of the NT (X,τ) with topological neutrosophic ideal L on X.

Proof. Straight forward. The relationship between τ and $N\tau^*$ (L) established throughout the following result which have an immediately proof .

Theorem 4.4. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any topological neutrosophic ideal L on X, $\tau_1 \subseteq \tau_2$ implies $N\tau^*_1 \subseteq N\tau^*_2$.

Theorem 4.5 : Let (X,τ) be a NTS and L_1, L_2 be two neutrosophic ideals on X . Then for any neutrosophic set A in X, we have

i) $NA^*(L_1 \vee L_2, \tau) = NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$.

ii) $N\tau^*(L_1 \vee L_2) = (N\tau^*(L_1))^*(L_2) \wedge N(\tau^*(L_2))^*(L_1)$

Proof Let $p = C(\alpha,\beta) \notin (L_1 \vee L_2, \tau)$, this means that there exists $U_p \in N(P)$ such that $A \wedge U_p \in (L_1 \vee L_2)$ i.e. There exists $\ell_1 \in L_1$ and $\ell_2 \in L_2$ such that $A \wedge U_p \in (\ell_1 \vee \ell_2)$ because of the heredity of L_1, and assuming $\ell_1 \wedge \ell_2 = O_N$ .Thus we have $(A \wedge U_p) - \ell_1 = \ell_2$ and $(A \wedge U_p) - \ell_2 = \ell_1$ therefore $(U_p - \ell_1) \wedge A = \ell_2 \in L_2$ and $(U_p - \ell_2) \wedge A = \ell_1 \in L_1$. Hence $p = C(\alpha,\beta,\gamma) \notin NA^*(L_2, N\tau^*(L_1))$, or $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$, because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $NA^*(L_1 \vee L_2, \tau) \geq NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$ .To show the second inclusion, let us assume $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$. This implies that there exist $U_p \in N(P)$ and $\ell_2 \in L_2$ such that $(U_p - \ell_2) \wedge A \in L_1$. By the heredity of L_2, if we assume that $\ell_2 \leq A$ and define $\ell_1 = (U_p - \ell_2) \wedge A$. Then we have $A \wedge U_p \in (\ell_1 \vee \ell_2) \in L_1 \vee L_2$. Thus, $NA^*(L_1 \vee L_2, \tau) \leq NA^*(L_1, \tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$ and similarly, we can get $A^*(L_1 \vee L_2, \tau) \leq A^*(L_2, \tau^*(L_1))$. This gives the other inclusion, which complete the proof.

Corollary 4.1. Let (X,τ) be a NTS with topological neutrosophic ideal L on X. Then

i) $NA^{*}(L,\tau)=NA^{*}(L,\tau^{*})$ and $N\tau^{*}(L)=N(N\tau^{*}(L))^{*}(L)$.

ii) $N\tau^{*}(L_1\vee L_2)=\left(N\tau^{*}(L_1)\right)\vee\left(N\tau^{*}(L_2)\right)$

Proof. Follows by applying the previous statement.

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