Studying Combined Effects Lubrication and Elastic Deformation on Knee Joint Using Mathematical Model A Theses Submitted to In Partial Fulfilment of the Requirements for the Degree...

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Studying Combined Effects Lubrication and Elastic Deformation on Knee Joint Using Mathematical Model

A Theses Submitted to The Council of the Faculty of Education for Girls- University of Kufa

In Partial Fulfilment of the Requirements for the Degree of Master in Mathematics

By

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بسم الله الرحمن الرحيم

﴿نَرْفَعُ دَرَجَاتٍ مِّن نَّشَاء
وَفَوْقَ كُلِّ ذِي عِلْمٍ عَلِيمٌ﴾

صَدَقَ اللهُ الْعَلِيُّ الْعَظِيمُ

سورة يوسف / الآية / ٧٦

Dedication

To who gave me everything I have…

To my first school in life…

My dear father in my heart, God prolong my father's life…

To the light that illuminates my life…

To the source of love and tenderness…

To my dear, who prays to me with success, she takes care of me step by step...

My dear mother...

May God reward it with goodness…

To them, I dedicate this humble work to bring happiness to their hearts...

To those who share the burden of life with me

My brothers and all friends...

And to everyone who believes that the seeds of successful change are in ourselves and minds before they are in other things...

Thanks and Gratitude

Who did not thank the creature did not thank the Creator. After praising Allah, which worthy of his sacred self and peace be upon the Messenger of God Muhammad (peace be upon him and his family).

I would like to extend my sincere thanks and appreciation to my honorable professor, Dr. (Enas Yahya Abdullah), who contributed her abundant work and precious time to my assistance, until I was able, with Allah's help, to direct the research on this image. She understood the circumstances and was a mentor scientist who does not skimp on her knowledge or her advice and guidance.

I am also pleased and honored to extend my sincere thanks and appreciation to the members of the teaching staff in the Department of Mathematics, College of Education for Girls, ask Allah Almighty to prolong their ages and expand their knowledge and reward them.Also, i thanks my wonderful teacher, Mr. Mudher Abd Allah, for helping hand without delay or hesitation.

I thank who gave me all the helpness I was needed in all the problems that confronted me in writing the letter, Mr. Muhannad Al-Daami, and I thank my family, who stood on my side, in overcoming all the difficulties I faced.

I do not forget my friends and colleagues for their support and assistance.

Researcher

Abstract

The main aim of this study is determined the squeeze film characteristics by using mathematical modules to obtain analytical manifestation for pressure hydrodynamic of pressure, load carrying capacity of synovial human knee joint during daily activity (walk-run), friction force, reduction film thickness to minimum film with time of approach femur to tibia and determined peak load on knee joint with different lubrication, mathematical modules concentrate on flexibility of knee joint (external parameter) and elastic deformation of articular cartilage (internal parameter), and determine their impact in improving joint performance. The research highlighted wear of cartilage layers and the wear rate in each layer according to the friction force between articular and rate of elastic deformation.

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List of Symbols

Symbols	The defintion
P	Pressure
P^*	Dimensionless pressure
τ	Shear stress parameter
μ	Dynamic viscosity
ρ	Density
P_e	Peclet number
SF	Synovial fluid
HA	Hyaluronic acid
OA	Osteoarthritis
R	The effective radius of curvature
S	Stride length
HL	Hydrodynamic lubrication
SQL	Squeeze film lubrication
EHL	Elastohydrodynamic lubrication
WL	Weeping lubrication
BL	Boundary lubrication
ML	Mixed lubrication
M	Mass
E	Effective modulus of elasticity
$\overline{w}$	Non-dimensional load
w	Load
Ø	Permeabitity of the osteoarthritis
L_p	Length of molecular
T_p	Thickness of molecular
T	Time
t^*	Dimensionless time
e	Eccentricity
f	Flexibility
Q	Total discharge
h^*	Dimensionaless film thickness
h_m	The Minimum film thickness
$h^*{}_m$	Dimensionaless minimum film thickness

Symbols	The defintion
l	Couple stress length
l^*	Dimensionaless couple stress length
r	Radial coordinate
r^*	Dimensionless radial coordinate
β	The ratio of the microstructure size to the pore size
R_a	Surface roughness
h	The film thickness of gab between two articular
h^*	Dimensionless film thickness of gab between two articular
H	The thickness of the porous
H^*	Dimensionless thickness of the porous
W	Weight
w	Load carry capacity
w^*	Dimensionless load carry capacity
P	Peak load

1. Introduction

The knee joint is one of the most important joints in the human body and the most complex. It is one of the largest joints in the body which helps us on accomplish three functions: large movement, continuous gait and running. It is referred that knee joint able to carry the body weight and cannot be separated between these processes its solid construction, exquisite engineering of cartilage and tendons. This makes the joint very sensitive and vulnerable to injury. Also the knee joint is one of our most flexible joints that allow a greater range of motions (Flexion – Extension – Abduction- Adduction) than all other joints. **[2]**

Lubrication synovial knee joint is a necessary process in order to reduce friction and wear between the surfaces of cartilage during daily activities, lubrication process depended on synovial fluid **[3]**. It has three main functions: lubrication, absorption and nutrition of the cartilage of the joint. In a healthy knee joint synovial fluid appears non-Newtonian, in result the relationship between viscosity and shear rate.

Lubrication mechanisms provide a protective film which allows for two surfaces to be separated and smoothed. In the absence of lubrication, surface would connect separate each one with another, thus it would get damaged in the joint. Lubrication mechanisms are differed by gait cycle (stance phases_swing phase). Lubrication mechanisms have been adopted in the lubrication of joints in general and knee joint in particular fluid film including (hydrodynamic,squeeze-EHL-weeping) boundary lubrication and mixed lubrication.

Friction is developed due to the shearing of the lubricant, the coefficient of friction that is a positive relationship to the lubricant viscosity and speed, but it is inversely proportional to the applied pressure. The main function of the lubricant is reduce the amount interaction between articular surface and to carry part of the synovial fluid acts as a lubricant in the joint. With the increasing number of the gait cycle, loads appear important of each (boundary – mixed) lubrication.

In fluid film lubrication, a relatively thick film of the lubricant separates the opposing bearing surfaces. Thickness of fluid film is greater than the surface roughness (irregularities), and wear due to surface contact is eliminated. Fluid film lubrication can be generated externally or internally, i.e. by external pressurized fluid or by self-generated hydrodynamic films due to relative surface motion. Fluid flows from the center of the bearing to the outer edges.

In hydrodynamic lubrication, a fluid film is developed by the motion of the lubricant between the surface articular cartilage. The frictional forces in hydrodynamic lubrication are affected by speed motion and lubricant viscosity.

The squeeze lubrication describes the pressure generated by the flow of liquid oiler between the head of the femur and tibia the impact on the viability of the joint to bear weights double as a result of the movement also describes the time compression of the liquid and out of the gap. It is one of the most important types of lubrication for being directly linked to stance phase.

Loads inflicted on the articular cartilage lead to a reduction in the thickness of the membrane and a decrease in the synovial fluid that contains molecules. This leads to a deformtion of the surface of the cartilage. Because of elastic cartilage, this process will produce lubrication type called "elastohydrodynmic lubrication".

The articular cartilage behaviour viscoelastic that dependent on a non-linear stress–strain and the rate of loading. Whereas there are many studies of cartilage are discussed low rates of loading, where strain rates less than 1 s^{-1}, when high rates of loading, strain rates becomes 10 to 1000 s^{-1} . At these high strain-rates, inertia precludes significant water movement and it is commonly assumed that the tissue becomes more elastic **[6]** i.e. having a smaller time-dependency, and lower hysteresis. Hysteresis means that the stress–strain path during unloading does not follow that developed during loading. The area under the loading curve is a measure of the energy stored in the deformation and that under the unloading curve the energy returned**.[29]**

The Young's modulus and Poisson's ratios describe the response of a material to deformation. Although originally defined for small deformations of elastic materials, they can be adapted to describe large deformations. At slow rates of loading, measured Young's modulus of cartilage as approximately 1–10 MPa.**[38]**.

The mechanical behaviour of cartilage is complex, since the tissue structure is a combination of partly porous, viscous and elastic components. This results in deformation rate-dpendent stiffness, i.e. how much energy is needed to deform the cartilage (dynamic modulus) and energy dissipation (loss angle) properties**.[34]**

2. Literature Review

The first significant analytical study and application of the theory of finite elasticity on the deformation of elastic membranes has been carried out by **Adkins and Rivlin [1](1952)**, using the neo-Hookean and Mooney forms of the strain energy function**.**

Dintenfass in [9](1963), discussed elastic properties of synovial fluid and articular cartilage, the synovial fluid is normal and as long as the articular cartilage remains elastic, the articulate surfaces are never extremely close to each other. Consequently, the boundary lubrication does not take place. However, if with pathological changes in synovial fluid, a boundary-type of lubrication may exist. He found the similarity and suggested that EHL is responsible for the lubrication of human joints, which is supported by later studies and researchers.

Haut D.,el at [18](2002), State that mechanical properties of each component in the knee, specifically the femur and tibia, In addition, elastic properties and models have been used to reproduce the mechanical behaviour of articular cartilage.

Wujz and Herzog [44](2002)

Indicate that energy losses in the case of articular cartilage tissue impact loading are smaller in comparison with the low loading rates. It implies that the material reacts to impact loads in a more elastic manner and its compression deformation changes are to some extent reversible.

Albeart E.Yousif et-al [5](2012)

In this study a three dimensional model of the human femoral bone was developed for us CT (computer Tomography) image and the data associated with the hip contact forces for normal walking and standing up during our cycle has been employed on the femur bone in order to investigate behavior of the femur during these activities. The finite element (stresses) are obtained and compared with previous studies with reasonable agreement. The results of the analysis are helpful for the orthopedic surgeon to understand the biomechanical behavior of the femur bone. They are also important for surgeon in femur surgeries and bone prosthesis.

In chapter one, elementary concepts, basic definitions and describe the anatomy, physiology, engineering and gait cycle of the human knee joint including its bones (bearing material), articular cartilage (porosity – elastic), synovial fluid (lubricating), disease synovial knee joint which are used in our study are present.

Chapter two studies the squeeze film lubrication with mischance lubrication (hydrodynamic–elasto hydrodynamic–weeping-boundary–mixed). It will also calculate the value of film parameters and film thickness depended on film parameters that will determine type lubrication, discussing the relationship between mechanisms for lubrication with gait cycle on the one hand and the friction force on the other hand.

Chapter three introduces the model discussion through (flexibilty synovial knee joint–permeability of articular cartilage–surface roughness) problems. The formulations of micro polar fluid equation and continuity equation for the problem under consideration are obtained.
The effect of pressure distribution is obtained by modified Reynolds equation. Load carry capacity and the friction force are obtained, depending on pressure film , time approach spherical to plate calculation by load carry capacity, well we calculated peak load on knee joint. We have discusised depeneding on the squeeze film characteristics.

Chapter four, introduces the model discussion through the effect of elastic deformation of articular cartlige on the performance of the human synovial knee joint and reduces friction force. The effect of pressure distribution is obtained by modified the Reynolds equation, the results and discussions of this problem are considered in this chapter.

Chapter One

Elementary Concepts and Basic Definitions

Introduction

In this chapter, we present some elementary concepts and basic definition that will be used in our study later on.

1.1 Fluid

It is any substance that deforms continuously when subjected to shear stress no matter how it is small.

1.2 Type of Fluid Mechanics

1. **Fluid Statics**: It deals with the fluid elements which are at rest relative to each other i.e
   - no motion of fluid layer relative to an adjacent layer.
   - no stress in the fluid.
   - when the fluid velocity is zero then the pressure variation is due only to the weight of the fluid.
2. **Kinematics:** It deals with the effect of motion i.e. translation, rotation and deformation on the fluid elements.
3. **Fluid Dynamics:** It deals with the effect of applied force on the fluid elements. In fluid dynamics, density and viscosity are the predominant properties.

1.3 Basic Definition of Fluid Mechanics

In this part, we introduce definitions related to the fluid mechanic.

1.3.1 Pressure [19]

Shear, tensile and compressive forces are the three kinds of forces which may act on a body. Fluids are very weak in tension and move continuously under the action of shear force. However, fluid is capable of standing high compressive stress. The normal compressive force per unit area is defined as the pressure and denoted by P where.

$$P = \frac{Force}{Area} \tag{1.1}$$

1.3.2 <u>Shear Stress</u> [10]

It is defined as the force per unit area, mathematically

$$\tau = \frac{F}{A} \tag{1.2}$$

Where τ , F and A represent shear stress, the applied force and the cross-sectional area of material with an area parallel to the applied force vector respectively.

1.3.3 <u>Shear Strain</u> [10]

Also known as shear deformation of solid bodies, it is displaced parallel planes in the body; quantitatively it is the displacement of any plane relative to a second plane divided by the perpendicular distance between planes the force causing such deformation.

1.3.4 <u>Viscosity</u> [12], [25]

Viscosity is the resistance of a fluid to move its internal friction. A fluid at a static state is by definition unable to resist even the slightness amount of shear stress. Application of shear force results in a continual and permanent distortion known as flow.

1.3.5 <u>Dynamic Viscosity</u> [12]

A dynamic viscosity (μ) is defined as the tangential force required per unit area to sustain a unit velocity gradient.

1.3.6 <u>Newton's Law of Viscosity</u> [12]

Newton 's law of viscosity states that the shear stress (τ) on a fluid element a layer is proportional to the shear strain or gradient:

$$\tau \propto \frac{du}{dy} \tag{1.3}$$

This may be written as :

$$\tau = \mu \frac{du}{dy} \tag{1.4}$$

Where $\frac{du}{dy}$ is called velocity gradient and μ is a constant of proportionality? The dimensions may be found as follows:

$$\mu = \frac{\tau}{du/dy} = \frac{Stress}{Velosity\ /\ Dis\tan ce} \tag{1.5}$$

1.3.7 <u>Classification of Fluid</u> [19]

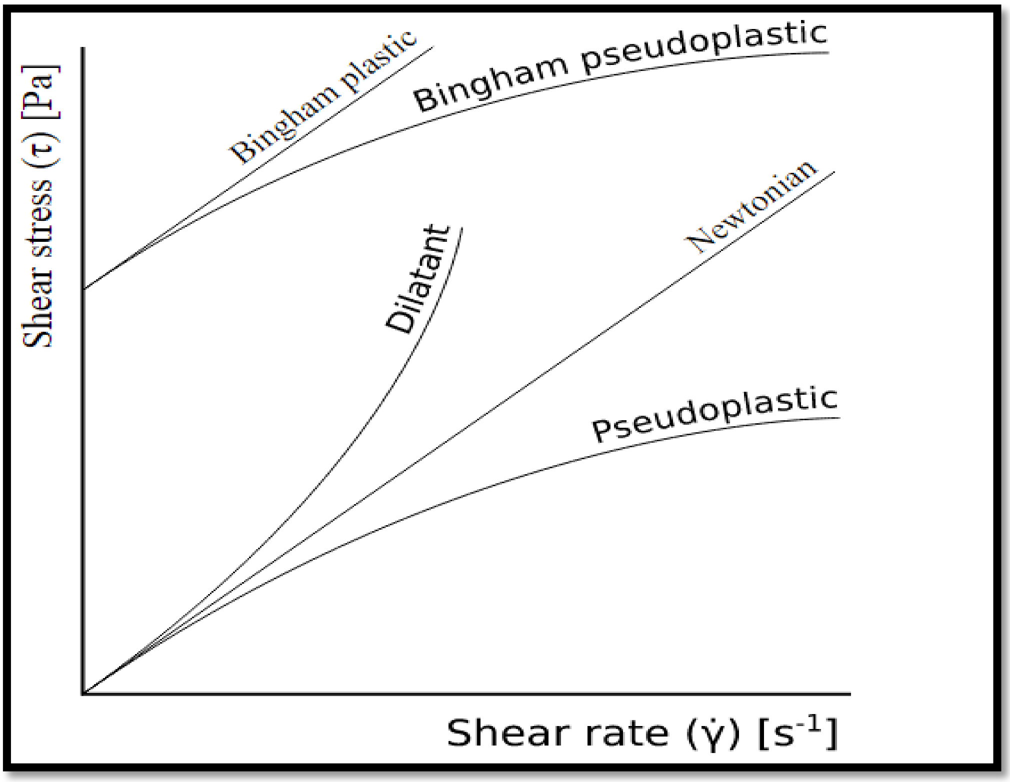
Fluids may be classified according to the relation between the stress and rate of strain as follows:

- **Newtonian Fluid**

The Newtonian fluid is a straight line passing through the origin. The slope of the line is the viscosity, which is the ratio of shear stress to shear rate. For a Newtonian fluid, the viscosity is independent of shear rate and may depend only on pressure and temperature.

- **Non- Newtonian Fluid**

All fluids for which the viscosity varies with shear rate are not-Newtonian fluid for which the viscosity defined as the ratio of shear stress to shear rate, is often called the apparent viscosity to emphasize the distinction from Newtonian behavior.



Fig(1.1): Classification of fluids with shear stress as a function of shear rate **[19]**.

1.3.8 <u>Some Types of Fluid Flow</u> [26]

A fluid flow consists of a flow of a number of small particles grouped together. These particles may group themselves in a variety of ways and the type of flow depends on how these groups behave. The following are important types of fluid flow:

1. **Steady and Unsteady Flow**

A flow is considered to be steady when conditions at any point in the fluid flow do not change with time, i.e.

$$\frac{\partial}{\partial t}(V, p, \rho, ..., ect) = 0 \qquad (1.6)$$

Where V, p, ρ represents, velocity, pressure, and density respectively Otherwise, the flow is unsteady.

2. Compressible and Incompressible Flow

A flow is considered to be compressible if the mass density of fluid ρ changes from point to point or ρ is variable. In the case of incompressible flow, the change of mass density in the fluid is neglected or density is assumed to be constant.

1.3.9 Continuity Equation [40]

The continuity equation simply expresses the law of conservation of mass (mass per unit time entering the tube must flow out at same rate) mathematical form:

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}+\rho(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})=0 \qquad (1.7)$$

Where, ρ is density and (u, v, w) are the velocity components in (x, y, z) directions, respectively.

If the fluid is incompressible, then ρ will be constant, and the continuity equation may be written as:

In three – dimension:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1.8)$$

In two– dimension:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1.9)$$

In one – dimension:

$$\frac{\partial u}{\partial x}=0 \qquad (1.10)$$

1.3.10 <u>The Navier –Stokes Equation</u> [28]

The equations of motion of a real fluid can be developed from consideration of the force acting on a small element of the fluid including the shear stresses generated by fluid motion and viscosity. These equations are called the "Navier- Stokes" equation and given by:

In the x-direction:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=X-\frac{\partial p}{\partial x}+\mu\nabla^2 u \qquad (1.11)$$

In the y-direction:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=Y-\frac{\partial p}{\partial y}+\mu\nabla^2 v \qquad (1.12)$$

In the z-direction:

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=Z-\frac{\partial p}{\partial z}+\mu\nabla^2 w \qquad (1.13)$$

Where (u,v,w) are the velocity component in the (x,y,z) direction respectively, (X,Y,Z) are the body force in the (x,y,z) direction respectively, ρ is the mass density, p is the pressure and μ is the dynamic viscosity, and

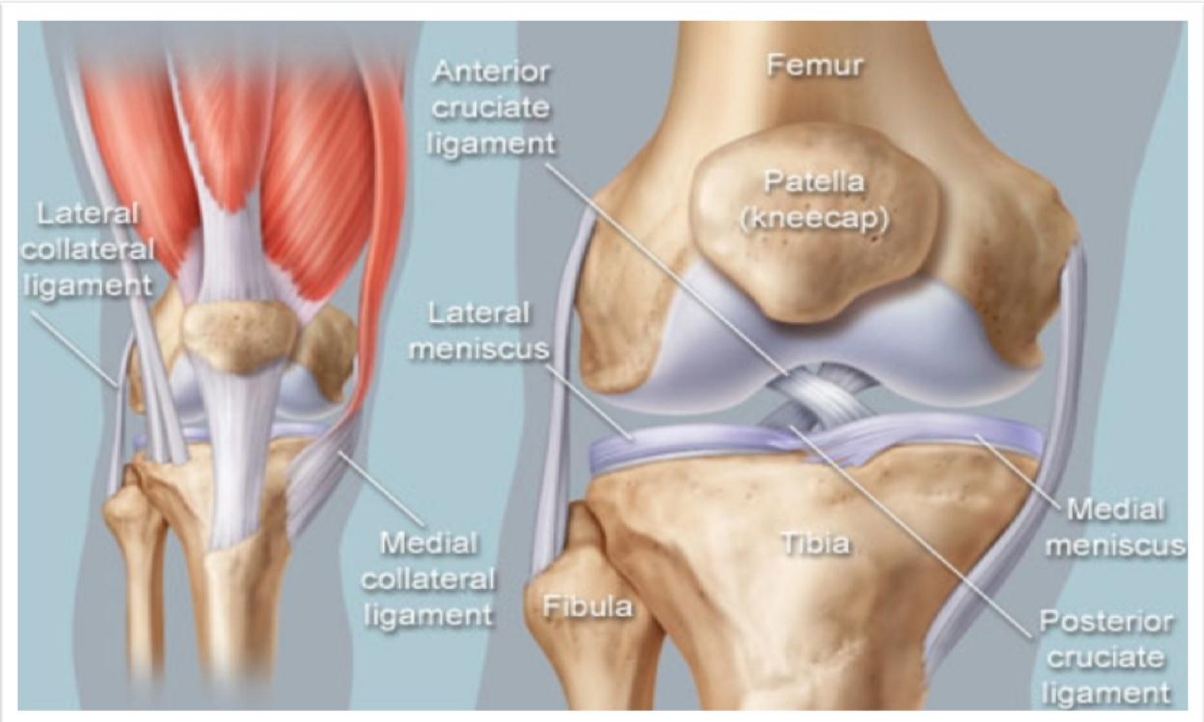
$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial y^2}\text{(Laplace operator)} \qquad (1.14)$$

1.3.11 <u>Peclet Number</u> [17]

The peclet number denoted by (P_e) is a dimensionless group representing the ratio of heat transfer by the motion of fluid to heat transfer by the mal conduction. $P_e=\frac{ul}{k}$ (1.15)

1.4 <u>Synovial Human Knee Joint</u> [5]

The knee is a joint that connects the thigh to the leg, the largest joint in the human body. The knee joint consists of two joints: the joint between the femur and leg, and the joint between the femur and the patella. Soft cartilage covers the surfaces of these joint bones to ensure easy mobility. Between the thighs and the sternum is hemangiomas cartilage that acts as cushions to help absorb shocks during walking and running. The knee is a moving spiral joint (joint) that allows bending and stretching as well as minor internal and external rotation. The knee joint is vulnerable to acute injury or osteoporosis. The lining wall of the capsule lining the inside of the membrane is called a membrane called the synoptic membrane, which produces fluid that helps to soften the movement of the joint and feed cartilage cells.



Fig(1.2): schematic representation articular cartilage of the synovial human knee joint[5].

1.4.1 <u>Importance of The Knee Joint</u> [2]

The knee joint is one of the most important joints in the human body and the most complex It is one of the largest joints in the body where large movement and continuous gait and running are important functions knee joint , also also able to carry the bodyweight during daily active.

1.4.2 <u>Synovial Fluid</u> [32], [21]

It is clear and yellowish substance found in the cavity of freely moving joints and interacting with cartilage to provide lubricating action. It occurs in small quantities yet it acts both as a lubricant for the articular surface and as nutrients for the cartilage. Synovial fluid is secreted by synovial lining cells see fig (1.3). It plays a very important role in synovial joints. It occupies the joint cavity and lines the synovial joint, providing nutrients and removing catabolic products. The thin film of synovial fluid and it covers the surface of the inner layer of the joint capsule and articular cartilage helps to keep the joint surface lubricated and reduces friction as fluid moves in and out of the cartilage as compression is applied, then released. The composition of synovial fluid also contains (HA) and glycoprotein called "lubricant". The (HA) component of synovial fluid is responsible for the viscosity of the fluid and is essential for joint lubrication. (HA) reduces the friction between the synovial folds of the capsule and the articular surfaces.

Normal healthy synovial fluid is highly non-Newtonian viscous fluid present in small amounts at all synovial joints. However, when a joint is injured or diseased the volume of the fluid may increase. The synovial fluid, like all viscous substance, resists shear loads.

The viscosity of the fluid varies inversely with the joint velocity or rate of shear; that is, it becomes less viscous at high rates of shear.

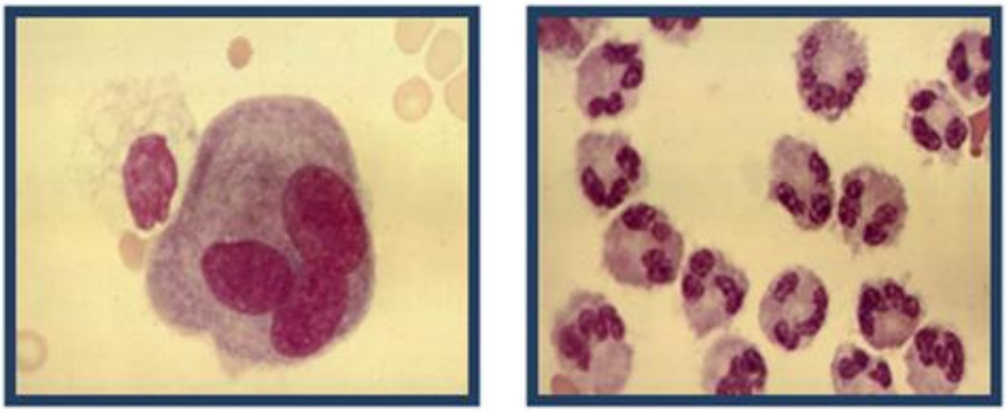
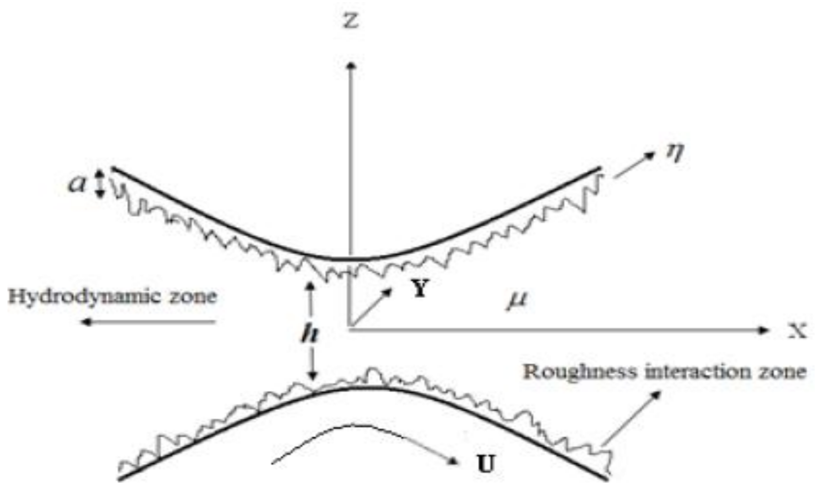


Fig (1.3): show synovial lining cell **[32], [21]**.

1.4.3 <u>The Surface Roughness of the Articular Cartilage</u> [11], [8]

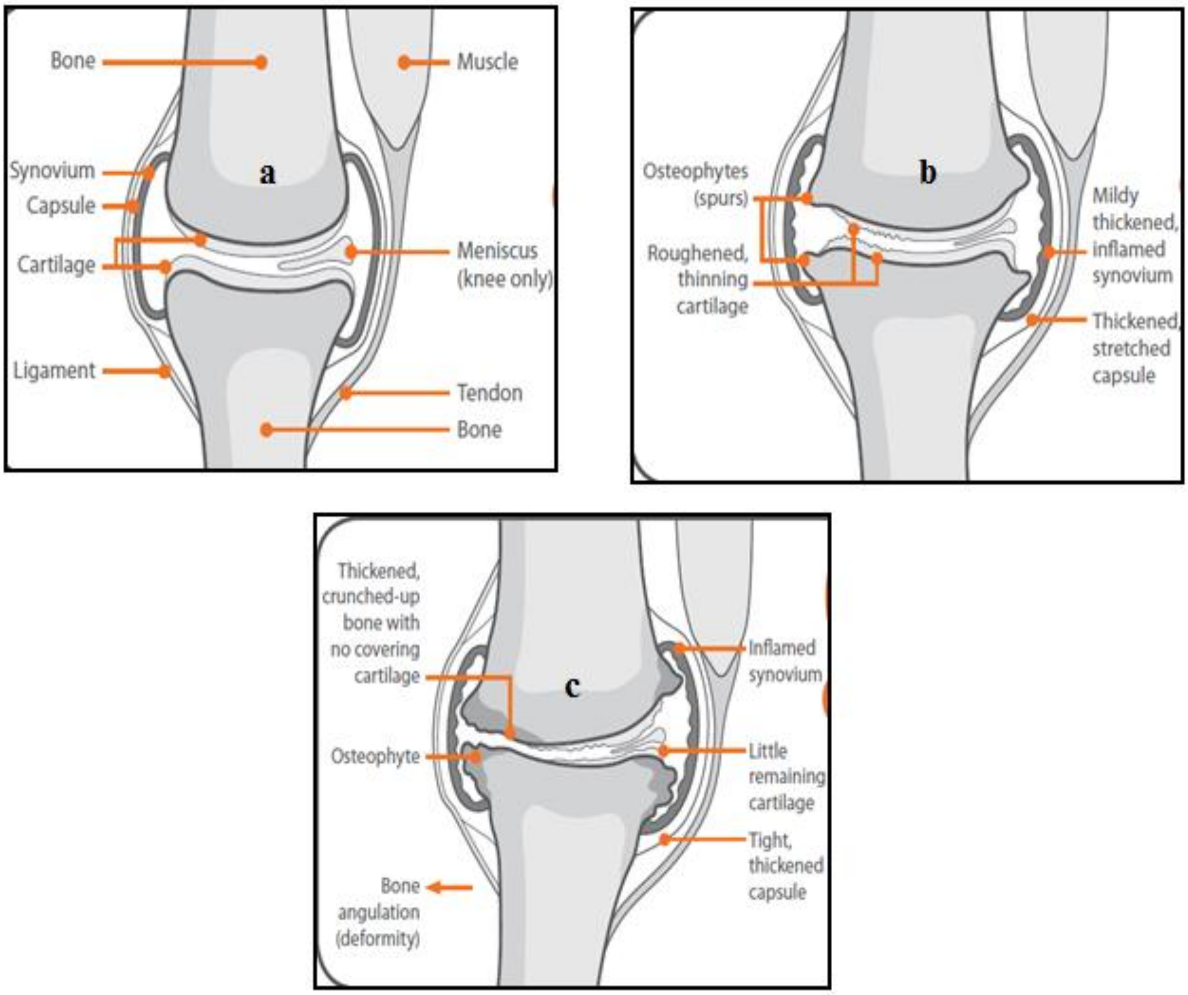
Surface roughness often shortened to roughness is a component of surface texture. It is quantified by the deviations in the direction of the normal vector of a real surface from its ideal form. If these deviations are large, the surface is rough; if they are small, the surface is smooth. Roughness plays an important role in determining how a real object will interact with its environment, see fig (1.4). Rough surfaces usually wear more quickly and have higher friction coefficients than smooth surfaces. Value of roughness is less or equal film thickness in fluid film lubrication and greater than film thickness in (boundary – mixed) lubrication.



Fig(1.4) Lubrication between two rough surfaces **[11], [8]**.

1.4.4 <u>Knee Joint Osteoarthritis</u> [22]

Osteoarthritis (OA) occurs when inflammation and injury to a joint cause a breaking down of cartilage tissue. In turn, that breakdown causes pain, swelling, and deformity. Cartilage is a firm, rubbery material that covers the ends of bones in normal joints. It is primarily made up of water and proteins. The primary function of cartilage is to reduce friction in the joints and serve as a "shock absorber." The shock-absorbing quality of normal cartilage comes from its ability to change shape when usually, occurs slowly over many years. Fig (1.5) explains the synovial knee joint in healthy and osteoarthritis.



Fig(1.5): show that (a) normal synovial joint (b) a synovial joint with mild osteoarthritis(c) a joint that has deformed by osteoarthritis **[22]**.

1.5 Engineering Representation of a Human Knee Joint

In this part, we introduce the engineering characteristics of the work synovial human knee joint .

1.5.1 The Bearing Material-Articular Cartilage

Articular cartilage is the smooth gristle which lines the articulating surfaces of the bone in joints. In this position it is called upon to take the wear which results from joint movement and to minimize joint friction while transmitting some of the highest loads developed in the human body.

1.5.2 Speed

The sliding speeds encountered in synovial human points seem to be in the range 0-0.1 m/s. The motion is often a combination of rolling and sliding, particularly in the knee, and it is the entraining velocity, or near-surface speed, that determines hydrodynamic action.

1.5.3 The Effective Radius of Curvature

The lubrication of engineering bearings is often discussed in relation to the effective radius of curvature (R). This term represents the radius of (or principal radii) of an equivalent curved surface near a plane. It is calculated from cylindrical surfaces of radii R_1 and R_2.

$$1/R = 1/R_1 \mp 1/R_2 \qquad (1.16)$$

The negative sign is used if the centers of curvatures lie on the same side of the contact (i.e. conforming joint). The effective radius of the unloaded surfaces is about 0.02-0.10 m. The highly conforming knee joint is attributed with an effective radius of 0.10-1.00 m.

1.6 The Gait Cycle [36]

A gait cycle is most often defined as the time interval between two successive instants of the foot to ground contact or initial contact, for the same foot. The gait cycle is separated into two distinct periods of stance and swing, see fig (1.6). Functional tasks include weight acceptance and single limb support during stance and limb advancement during the swing. The stance period of the gait cycle includes initial contact, loading response, mid- stance, terminal stance, and pre-swing. The swing period of the gait cycle includes initial swing, mid-swing, and terminal swing.

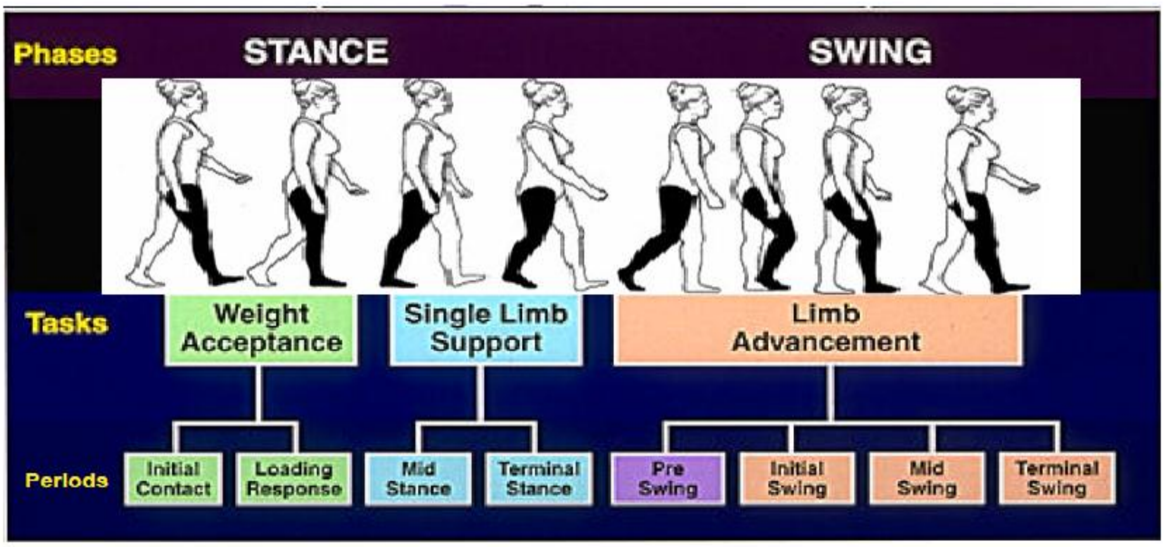


Fig (1.6): show that the (phases-tasks-periods) of the gait cycle **[36]**.

The stance is subdivided into two periods of double-limb support and one period of single-limb support. Double- limb support occurs at the beginning and the end of the stance phase. During the double – limb support, both feet are on the ground. During the single-limb support, one limb is supporting the weight of the body while the other is in an advancing swing.

Table (1.1): the time frame of the gait cycle .

Gait cycle	Phases	Time Frame
Stance phase	1. Initial contact. 2. Loading Response(foot –flat). 3. Mid-stance. 4. Terminal Stance(heel-off). 5. Toe-off.	60%
Swing phase	1. Initial swing (acceleration) 2. Mid-swing 3. Terminal swing (deceleration).	40%

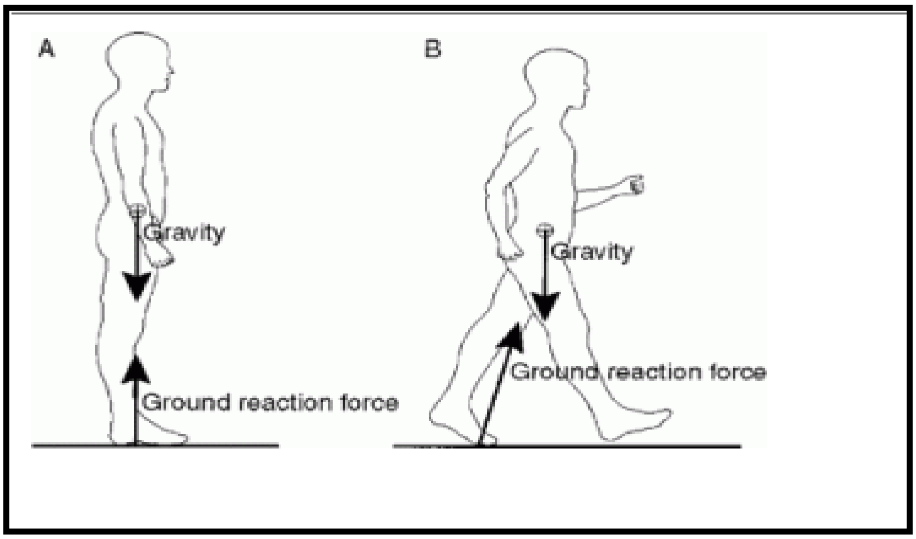
1.6.1 <u>Type of Force in The Gait Cycle</u> [31]

The movement is observed during walking result from the interaction between external forces and internal forces.

- **External Forces**

1. **Ground Reaction:** This shows the force at stance phase where the foot touches the ground, see fig (1.7).It sheds no vertical force on the ground. So, the reaction will be working in the opposite direction to the force of his body vertical. Therefore, these two forces equal the amount opposite direction.
2. **Gravity:** This force results from the withdrawal of the body towards the ground position and size of the force depends on body weight or body mass in which it operates.

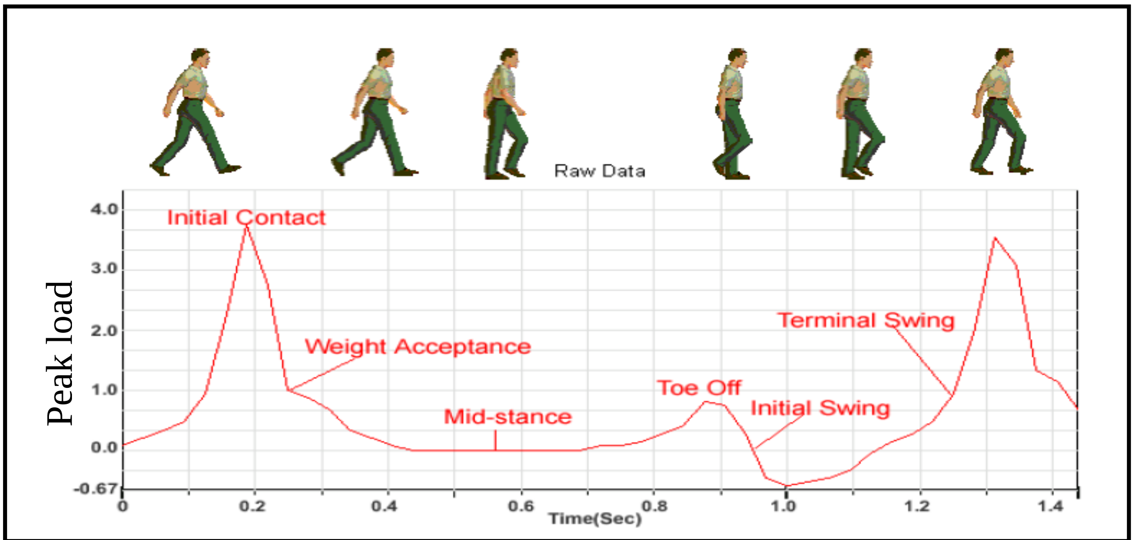
- **Internal Forces:** Muscle strength and strength of ligaments and tendons.



Fig(1.7): (a) During the stand (b) During walking **[31]**.

1.6.2 <u>Cycle Time</u> [33]

The time is required in seconds to complete one walking cycle. To calculate the cycle time, the following relationship is used: cycle time = time/number of steps. Fig (1.8) illustrates the method measure by using the sensor from the right foot.



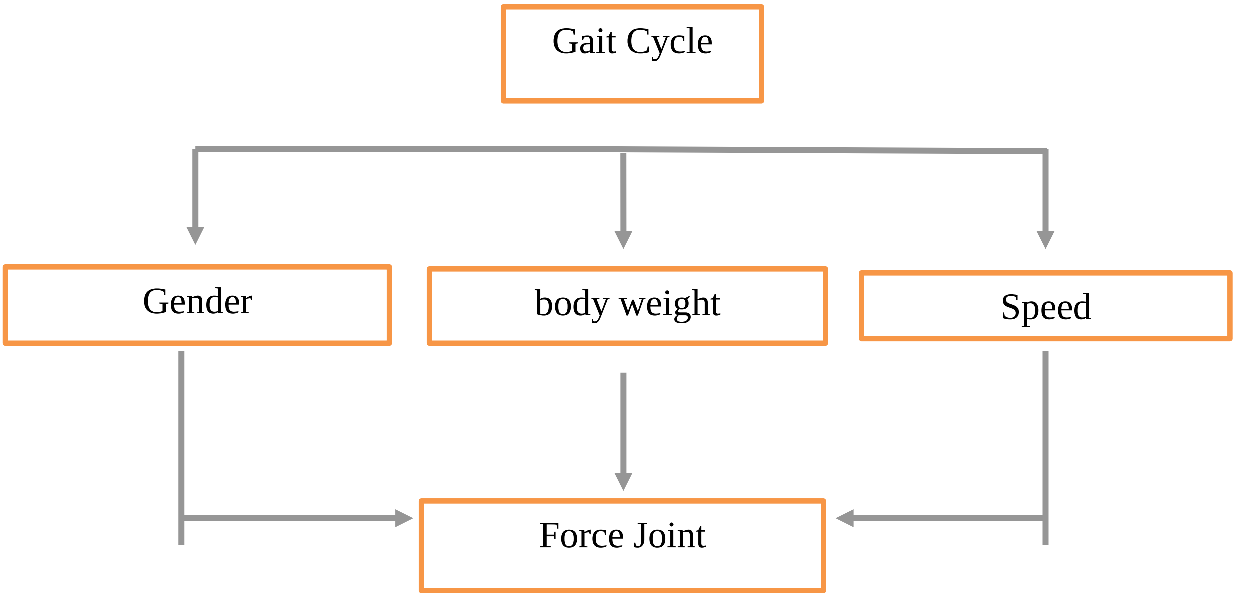
Fig(1.8): The method measure gait cycle**[33]**.

Table (1.2): Peak Loadings on Synovial Joints in the Lower Limb in steady Walking.

Joint	Hip	Knee	Ankle
Peak Load /Body Weight	3-6	3-7	2-5

1.6.3 <u>Force of The Knee Joint</u> [43]

The forces transmitted by synovial joints in the lower limb have been estimated by recording the ground-to-foot reactions on a force plate, by noting the weights and locations of the centers of main body parts, by recording movement by cine photography and by making assumptions about the insertions and lines of action of dominant muscle groups. All these studies reveal that in the walking cycle the loads range from zero to several times body weight. The magnitude of the force in healthy knee joint is related to (Gender, body weight, & speed gait). Figs (1.9) illustrates the relationship between the gait cycle and force at the knee joint.



Fig(1.9): Relations between parameter gait cycle & force knee joint**[43]**.

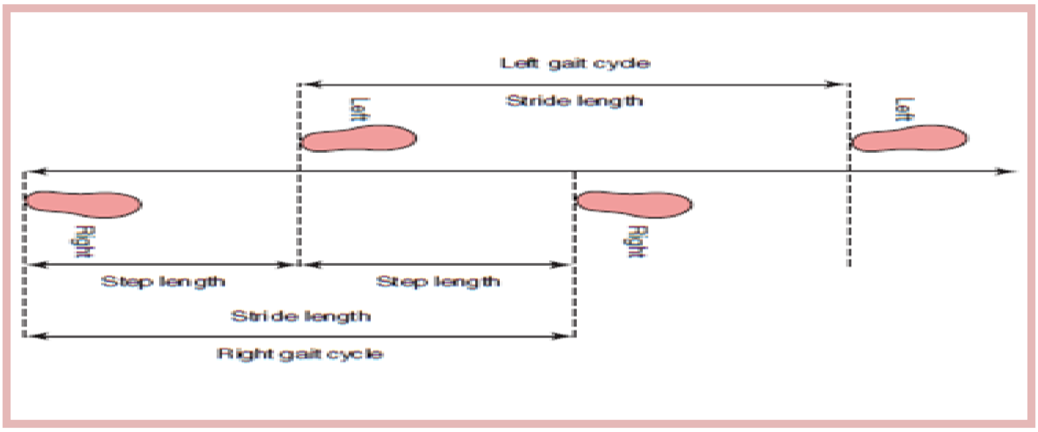
The weight of a person differs through his life stages where studies confirmed when people start walking a person's double weight several times, which leads to increase pressure on human joints (ankle, knee, hip) and this doubles the weight depending on the kind of movement done by the person,see table (1.3). When the leg swings freely in the walking cycle, the knee and hip carry loads in the range zero to one times body weight; when the leg is in contact with the ground (stance phase), the peak loads range from three to seven times body weight. Load peaks occur just often heel strike and just before toe-off. Because of body weight for any person after heel strike across with 4.2 and 4. 8 before toe-off. For males, the maximum force varies between three and eight-time the body weight and females maximum force varies between two and five times the body weight.

Table (1.3): The maximum typical knee joint forces for daily routine and activities:**[39]**

Activity	Multiple of the Body Weight
Standing	0.3 times body weight
Standing on one Limb	2.6 times body weight
Normal Walking	4.2 times body weight after heel strike 4.8-time body weight before the toe of
Fast Walking & Jogging	7.6 times body weight
Walking up stairs	3 times body weight
Walking slowly with the cane	2.2 times body weight

1.6.4 <u>Stride Length</u> [7]

It is the distance between successive points of heel contact of the same foot. A stride includes two steps, right and left, but the stride length is not always equal to length of two steps as there may be unequal steps, stride length greatly varies among individuals because it is affected by length, sex, age and speed as explained in apendix (A). Some studies depend in the calculation of step length on distance and number of steps. These can be expressed in the following relation [stride length = 2×(distance/number of steps)]. In this way, the step length will be: (stride length/2) . Fig (1.10) illustrates methods of measuring stride length and step length.



Fig(1.10): Methods for measuring step length and stride length**[7]**.

Chapter Two

Influence of Gait Cycle on Lubrication of Synovial Human Knee Joint

Introduction

In chapter two, the general classification of the lubrication systems in the synovial human knee joint is given. The discussion focuses on two main points, the calculation of film parameter and film thickness for each of the (hydrodynamic lubrication, squeeze film lubrication, elasto-hydrodynamic lubrication, weeping lubrication and boundary lubrication), and we are interested in the relationship between lubrication systems and gait cycle (stance phase and swing phase) also we introduce types of friction force and parameters effective. Physical properties affecting on the film thickness will be discussed in each lubrication system.

2.1 Lubrication Regimes Studies of The Synovial Joint [30],[9]

Synovial joints function as excellent bearings under biological conditions as well as introduce supports to considerable load on articular cartilage and provide low friction of articular cartilage surface. The lubrication regimes play the main ineffectiveness and maintenance of the joint. Several mechanisms have been adopted in more sources in synovial human joint lubrication, such as (hydrodynamic, squeeze-film, boundary and mixed) lubrication, as seen in fig (2.1). Even though so much work has been done, it is still unclear how synovial joints have such low coefficients of friction and high load carrying capacity during daily activities.

Lubrication of the human knee joint is the focus of our attention in this chapter and that the importance of this joint in the human body. The lubrication in the knee joint depends on the properties of fluid used it. In the human body, it could be found as the lubricant contains hyaluronic acid and called synovial fluid. The properties of this fluid depend on the shear rate and velocity, the film that flows through it lubricantis called "film thickness". Researchers suggest that a film thickness of fluid is formed between the cartilage surfaces and thickness varies depending on the type of lubricating.

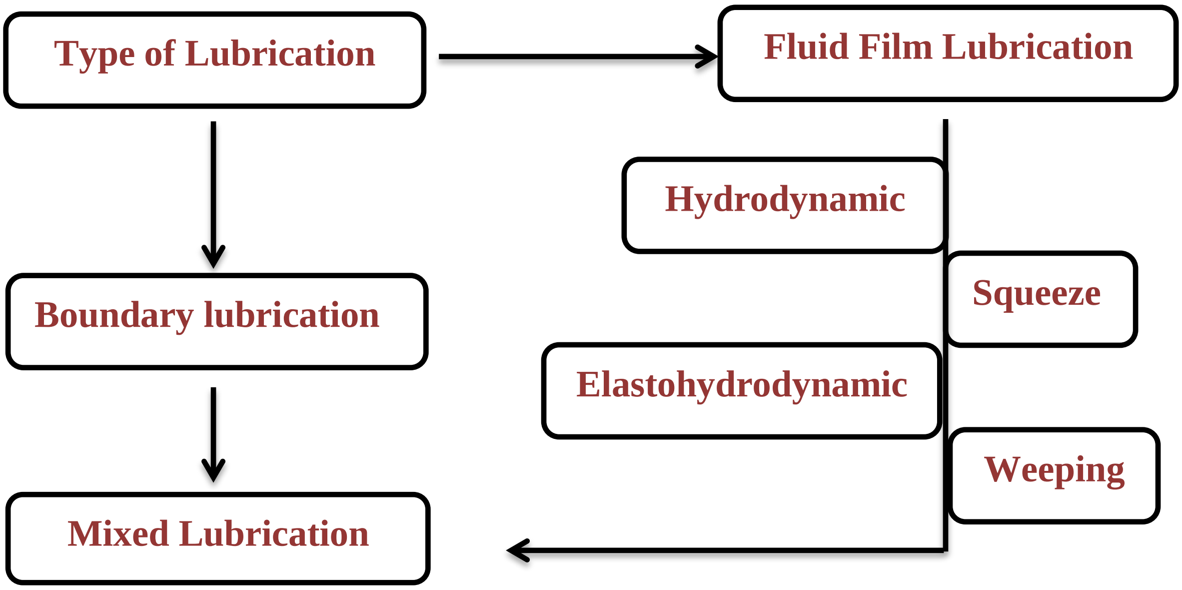
The important factors in lubrication of synovial joints are friction coefficient, which is the decisive factor in a healthy joint. Healthy human synovial joints the estimated coefficient of friction with (μ =0.1-0.001) degradation of either part of the synovial fluid or the articular cartilage system leads to increased friction and wear, and reduction of mobility.

In order to understand the lubricating mechanism of natural joints, researchers have proposed various theories such as boundary lubrication, hydrodynamic lubrication, boosted lubrication, biphasic lubrication, elastohydrodynamic lubrication, micro-elasto-hydrodynamic lubrication, weeping lubrication, squeeze film lubrication. Generally, the main elements in the lubrication operations are: the lubricating properties and protection synovial joints from (wear and friction).

There is a clear relation of knee between the mechanism lubrication and the gait cycle (stance phase-swing phase) where the knee joint during the swing phase in the loading on the joint is very low and the entraining velocity is high. When the heel strikes the ground, the load suddenly rises to a value in excess of 3.2 N while the entraining velocity is very low.

Researchers have showed the squeeze-film time that is greater than the peak loading time at heel strike (squeeze lubrication). But if the squeeze-film time is lower than the peak loading time at boundary lubrication. Whatever the mechanism is, the cartilage is well protected in situation healthy joint. The stance phase of walking involves lower entraining velocities and higher loads which then the EHL or micro-elasto-hydrodynamic lubrication may be used to explain.

Synovial joint lubrication is very complex. It may include several lubrication mechanisms consistent with the various stages of walking, and final conclusions have been supported by experiments. To understand these processes, the basic engineering principles of lubrication should be considered see fig (2.1) where the lubricating mechanism depends on the load and sliding velocity during the walking cycle (stance phase-swing phase).



Fig(2.1): Lubrication processes for articular cartilage in synovial human knee joint.

2.2 <u>Lubrication Characteristics</u> [20]

A number of studies agree that the process of lubrication of the synovial human knee joint possesses a number of properties that are directly dependent on the characteristics of lubricant, which constitutes the main element in the lubrication of joint, as follows:

1. Rheological properties.
2. Viscosity and elasticity.
3. Film geometry.
4. Speed of relative motion of two surfaces.

2.3 Lubrication Regimes

This section introduces three modes of lubrication exist between contacting surfaces articular cartlige also calculation film thickness and film parameters for each model.

2.3.1 Fluid Film Lubrication

Fluid film lubrication involves a thin fluid film that provides separation of joint surfaces. Surface lubricated by a fluid typically has a lower coefficient of friction than boundary lubricated surfaces do and the fact that the coefficient of friction is very low in synovial joints suggests that some sort of fluid film lubrication exists. Several models or fluid film lubrication exist including hydrodynamic, hydrostatic, weeping lubrication, squeeze film lubrication, elastohydrodynamic (a combination of hydrodynamic and squeeze film) and boosted lubrication.

2.3.1.1 Hydrodynamic Lubrication(HL) [24]

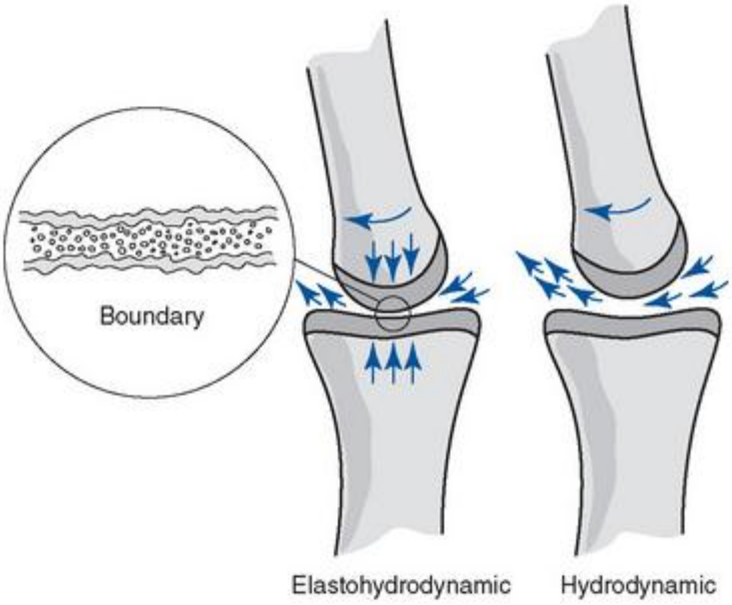
Hydrodynamic lubrication is a form of fluid lubrication in which a wedge of fluid is created when nonparallel opposing surfaces slide on each other. In hydrodynamic lubrication, the thickness of the fluid film is between $10^{-4}$and 10^{-3}cm. This is much larger than the surface roughness thickness. Hydrodynamic lubrication is generally characterized by conformal surfaces. The pressure in the fluid film is generated because of the relative motion of surfaces and wedge action see fig (2.3.(a)). The viscosity of the fluid the geometry and relative motion of the surfaces may be used to generate the sufficient pressure to prevent solid contact. The resulting lifting pressure generated in the wedge of fluid and by the fluid's viscosity keeps the joint surface apart. The magnitude of the pressure developed is not generally large enough to cause significant elastic deformation of the surfaces. The minimum film thickness in a hydrodynamically lubricated bearing is a function of normal applied load, surface velocity, lubricant viscosity, and geometry. If the surfaces are not moving and the pressure is generated by external pumping, the term hydrostatic lubrication is used.

2.3.1.2 Squeeze-Film Lubrication(SQL) [16]

Squeeze-film lubrication occurs when the bearing surfaces are moving perpendicularly towards each other. The pressure is created in the fluid film by the movement of articular surfaces that are perpendicular to one another. As the opposing surfaces move close together, they squeeze the fluid film out of the area of impending contact. The viscosity of the fluid in the gap between the surfaces produces pressure, which tends to force the lubricant out to see fig (2.3.(b)). The resulting pressure created by the fluid’s viscosity keeps the surface separated. This type of lubrication is suitable for high loads maintained for a short duration. This mechanism is capable of carrying high loads for short lengths of time. As the fluid is forced out, so the layer of fluid lubricant becomes thinner and the joint surfaces come into contact. The height of the film can be measured under certain circumstances, and a squeeze-film time determined as the time taken for the film to diminish under load from a given height to zero.

2.3.1.3 Elastohydrodynamic Lubrication (EHL) [16]

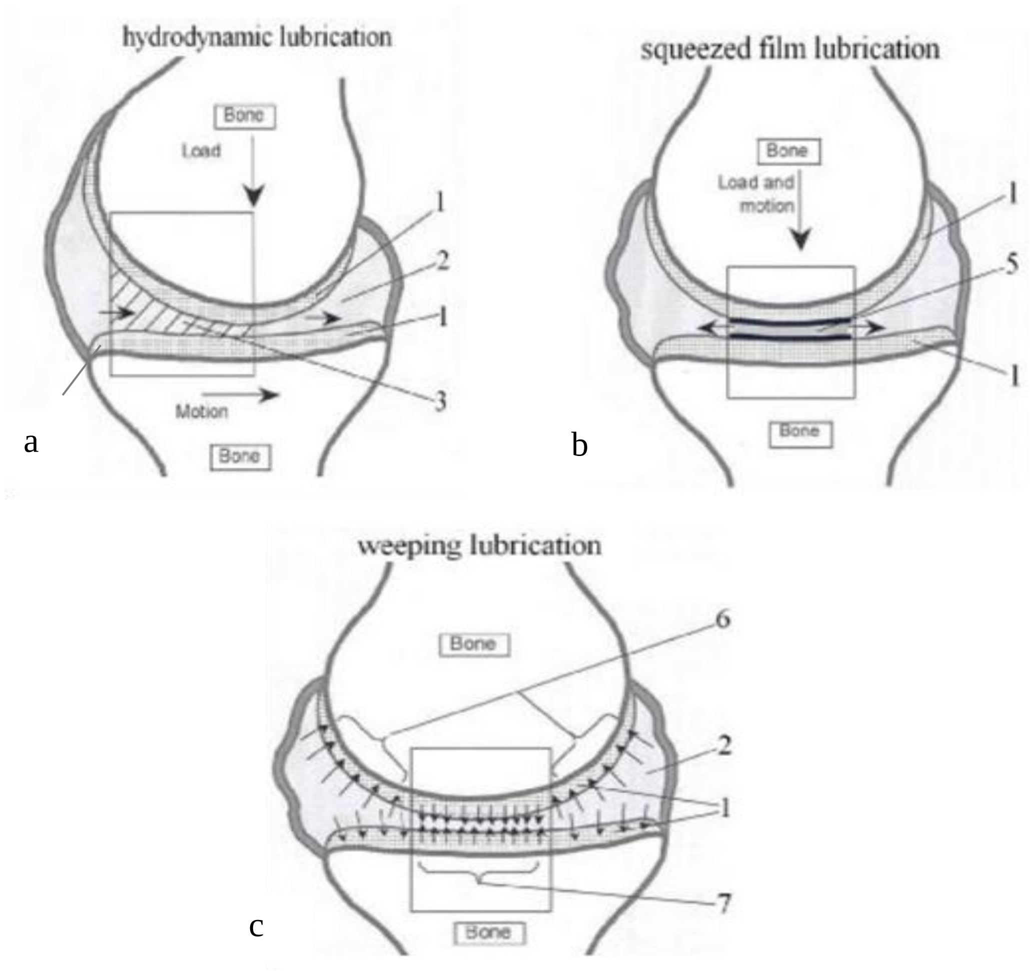
Elastohydrodynamic Lubrication – or EHL –is a type of hydrodynamic lubrication (HL) in which significant elastic deformation of the surfaces takes place and it considerably alters the shape and thickness of the separating lubricant film see fig (2.2). The term underlies the importance of the elastic deflection of the bodies in contact in the development of the total lubricant film. EHL, the same way as HL, is used to decrease friction and wear in tribological contacts. It is achieved by the development of a thin lubricant film between rubbing surfaces, which separates them and decreases friction. EHL has characteristic features, such as constant film thickness and Hertz contact pressure profile within the Hertzian contact area, as shown in the fig below. These features have been extensively used in the construction of approximate solutions of EHL theory.



Fig(2.2): That Lubrication models are hydrodynamic[16].

2.3.1.4 Weeping Lubrication(WL) [4]

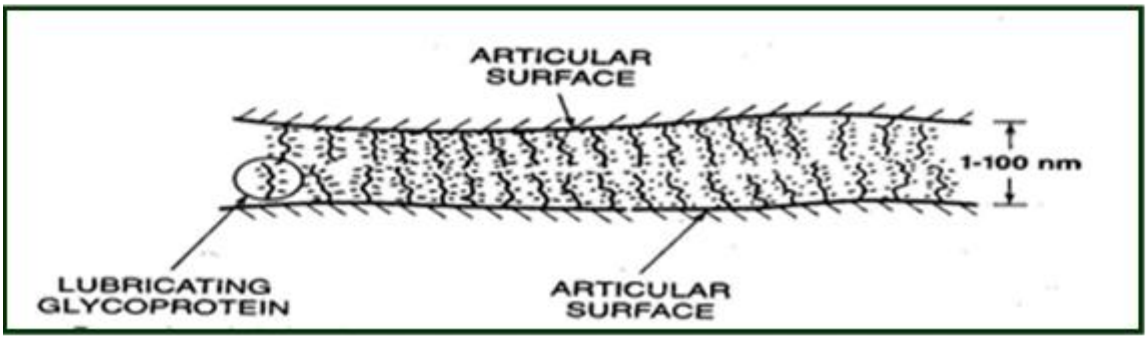
Weeping lubrication is a form of fluid lubrication in which the load bearing surfaces are held apart by a film of lubricant that is maintained under pressure. In engineering, the pressure is usually supplied by an external pump. In the synovial human knee joint, the pump action can be supplied by contractions of muscles around the joint or by compression from weight bearing compression of articular cartilage the causes the cartilage to deform and to “weep” fluid which forms a fluid film over the articular surfaces. The fluid can only move into the joint because the impervious layer of calcified cartilage keeps it from being forced into the subchondral bone. When the load is removed, the fluid flows back into the articular cartilage through osmotic pressure, see fig (2.3.(c)). Weeping lubrication represents a special form of self-acting hydrostatic bearing mechanism, which has been advanced by McCutchen (1967) **[4]**.



Fig(2.3): That hydrodynamic, squeeze and weeping lubrication 1- articular cartilage 2-synovial fluid 3- wedge-shaped gap 4- synovial fluid flow in cartilage matrix 5- the squeezed synovial liquid 6-non-loaded region of cartilage 7- the loaded region of cartilage [14].

2.3.2 <u>Boundary Lubrication(BL)</u> [41]

Boundary lubrication arises when the separation of bearing surfaces is of particular dimension 10^{-7}-10^{-6} cm. It occurs when each load bearing surface is coated with a thin layer of large molecules that form a gel that keeps the opposing surfaces from touching each other. The layers slide over each other more readily than they are sheared off the underlying surface. In synovial human joints, these layers contain the lubricin that adheres to the articular surface. This type of lubrication is considered to be most effective at low loads, see fig (2.4). The load capacity is determined by the particular properties of lubricants and roughness of surfaces, but the viscosity of the fluid does not play an important role.



Fig(2.4): That the mechanics work boundary lubrication of articular cartilge**[41]**.

2.3.3 <u>Mixed Lubrication(ML)</u> [41]

This type of lubrication combines boundary lubrication and fluid-film lubrication; the reason is due to loads on articular cartilage part of which it is carries the pressure fluid generated in the fluid film and the other part to be portable asperity contact in boundary lubrication. The film thickness is less than 10^{-6} m and greater than 10^{-8} m. However, this is between the theoretical limit of the mixed lubrication regime and the full hydrodynamic regime (with film thickness much higher than roughness).

There is a region where surface roughness still have a significant influence on contact performance, even if asperity contact is not present asperity contact see fig (2.5).

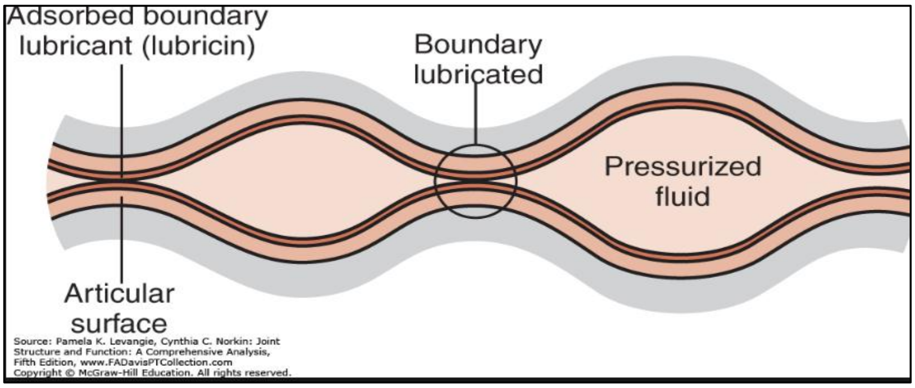


Fig (2.5): Shows that the mechanics work mixed lubrication of articular cartilage of knee joint**[41]**.

2.4 <u>Film Parameter</u>

The film parameter is used to classify the four important lubrication regimes of the synovial human knee joint. This classification depends on the surface roughness of articular cartilage, the viscosity of a synovial fluid, a human mass (male and female), and cycle time in a normal walk, see table (2.1) the describes the range of values for the four lubrication regimes. The general law of film parameter are:

$$\Lambda = \frac{3*\sqrt{{R_a}^2+{R_b}^2}}{m*\eta} n.1.4 \tag{2.1}$$

R_a, R_b the roughness of two articular cartilages, where n is number of cycle time, m is mass and η is viscosity when m, $\eta \neq 0$.

Table (2.1): Film thickness regimes.

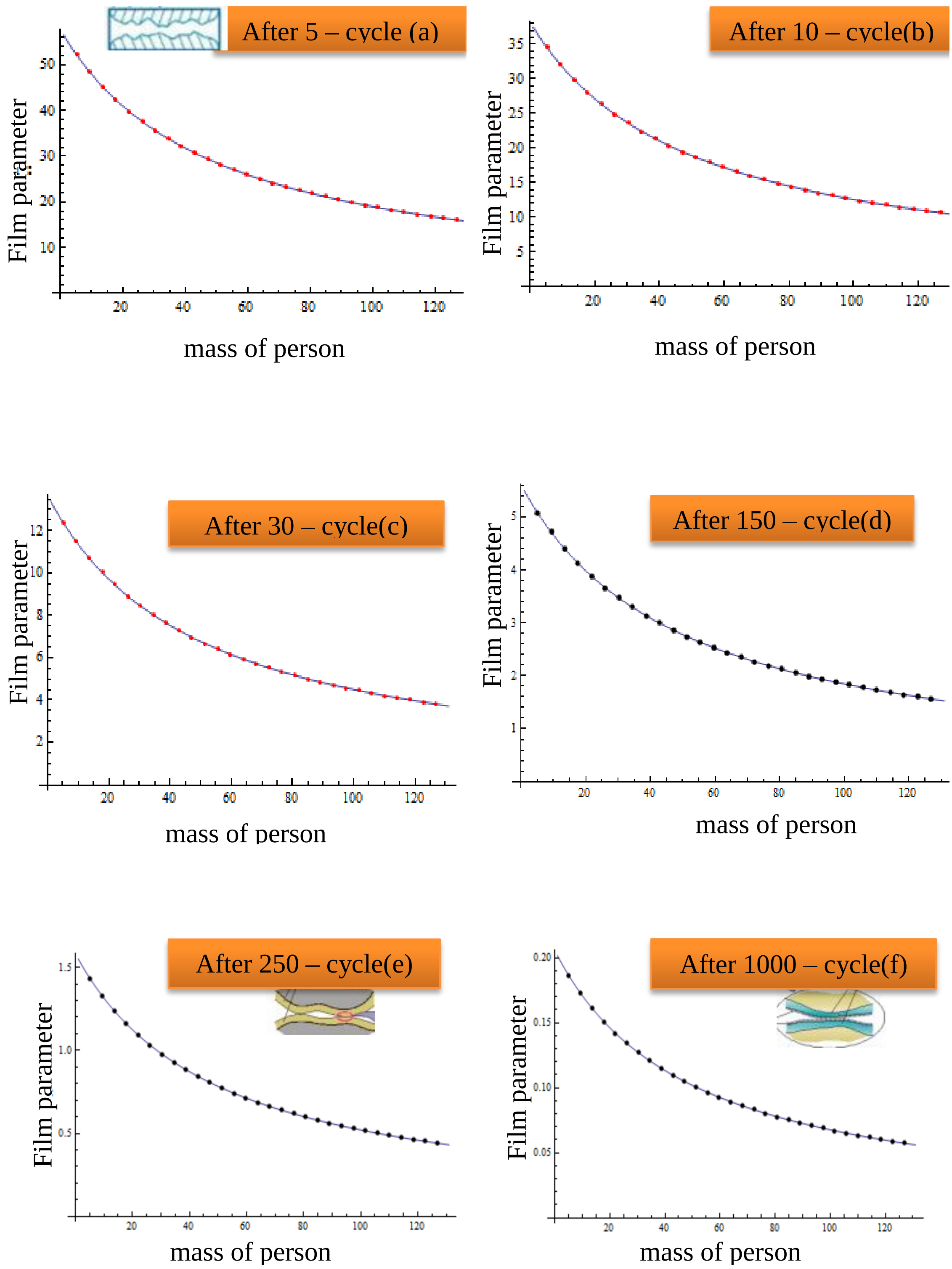
Regimes	Hydrodynamic Lubrication	Squeeze Lubrication	Mixed Lubrication	Boundary Lubrication
Specific film thickness	($\Lambda > 5$)	($1.5 < \Lambda < 5$)	($0.7 < \Lambda < 1.5$)	$\Lambda < 0.7$

2.4.1 Effect of Mass

Human mass (male and female) is very important to determine film parameter of each type of lubricant, increasing the load on synovial human knee joint leads to decrease film parameter through cycle – time. in advanced tges of human life loses a muscle strength , which reduces the burning of fat and calories, especially female which leads to different film parameter, see fig (2.6).

Table (2.2): Relations knee between mass &film parameter.

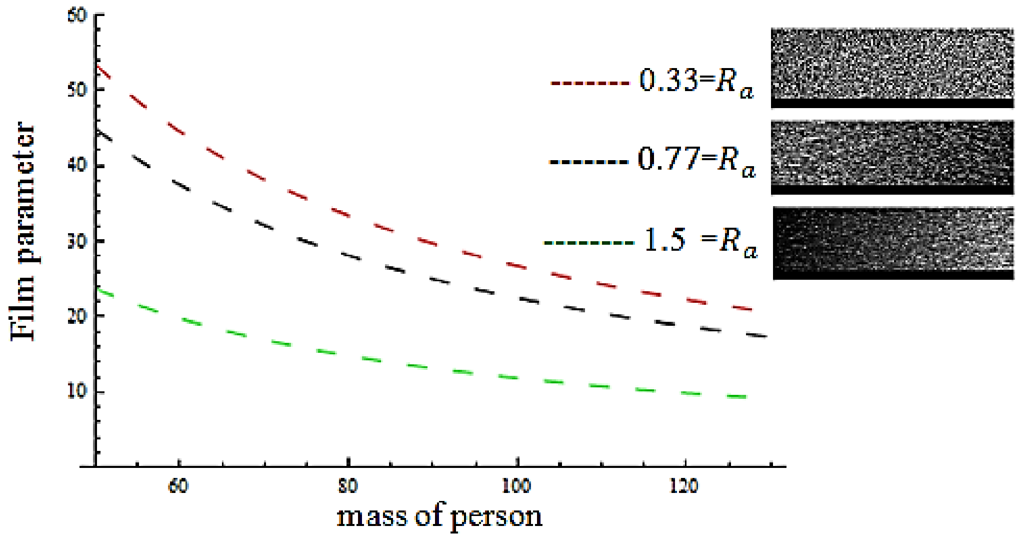
mass (kg)	Film parameter after 1- cycle	Time (s)	Film parameter after 20- cycle	Time (s)	Film parameter after 150-cycle	Time (s)	Film parameter after 250-cycle	Time (s)	Film parameter after 1000 -cycle	Time (s)
	Hydrodynamic lubrication				Squeeze lubrication		Mixed lubrication		Boundary lubrication	
50	59.39	1.4	37.56	28	5.502	210	1.551	350	0.56	1400
60	49.49	1.4	31.30	28	4.535	210	1.292	350	0.46	1400
70	42.42	1.4	26.83	28	3.930	210	1.108	350	0.35	1400
80	37.11	1.4	23.4	28	3.439	210	0.969	350	0.31	1400
90	32.99	1.4	20	28	3.057	210	0.861	350	0.28	1400
100	29.69	1.4	18	28	2.751	210	0.775	350	0.25	1400
110	26.99	1.4	17	28	2.501	210	0.705	350	0.25	1400
120	24.79	1.4	15.65	28	2.297	210	0.646	350	0.23	1400
130	22.84	1.4	14	28	2.116	210	0.596	350	0.21	1400



Fig(2.6): Classification of film parameter in (a, b, c) hydrodynamic (d) squeeze, (e) mixed and (f) boundary lubrication during a normal walk.

2.4.2 The Effect of Surface Roughness

The surface roughness of articular cartilage in knee joint effective effect on the value of film parameter of different regime lubrication. Age is the important cause of surface roughness of articular cartlige in knee joint see fig (2.7), where arthritic erosion increases result to a decrease in synovial fluid. In the young phase, the ratio of roughness is very low between (0.3-0.5)and the proportion of fluid produced from the cells is synovial 90 % .After the age of fifty, the rate surface roughness increases in the rate of(2-5) , especially among female and synovial production that is reduced to 36%.



Fig(2.7): That affect surface roughess on film parameter with vires mass.

2.5 Assumptions of Hydrodynamic Film Lubrication

1. Film thickness is small compared to bearing dimensions.
2. The inertia force effect is neglected.
3. Laminar flow occurs in the pressure-bearing film.
4. Lubricant is a simple Newtonian fluid with viscosity independent of shear rate.
5. Viscosity and density are constant throughout the bearing.
6. Incompressible fluid.
7. Steady-state condition.

2.6 Film Thickness Calculation

The film thickness that separates the surfaces from each other changes thickness during the various events practiced by humans in (stance phase and swing phase) hydrodynamic factors that effect on hydrodynamic lubrication and boundary lubrication including the (η,R_a , W, U, σ) while in squeezes lubrication the lubricating film carries most of the load. The squeeze film time plays a major role in changing the magnitude of film thickness. In an attempt to evaluate the film thickness of synovial fluid using the various laws mechanically fit knee joint movement. The lubrication film thickness will be estimated for each type of lubrication mechanism using the numerical factors, see a table (2.3).

Table (2.3): Parameters affecting film thickness for the knee joint [5].

Parameters	Symbols	Numerical values	Units
Effective modulus of elasticity	E	$10^7 - 10^9$	N/m^2
Effective radius of curvature	R	0.1-1	m^2
Load	W	2500	N
Non-dimensional load	$\overline{W}$	2.5×10^{-4}	-----
Permeability of the osteoarthritis	ϕ	6×10^{-17}	(m^2)
Speed	U	0.075	m/s
Fluid viscosity	η	2×10^{-3}	Pa.s
Length of molecular	L_p	0.5×10^{-3}	Mm
Thickness of molecular	T_p	0.5×10^{-3}	µm

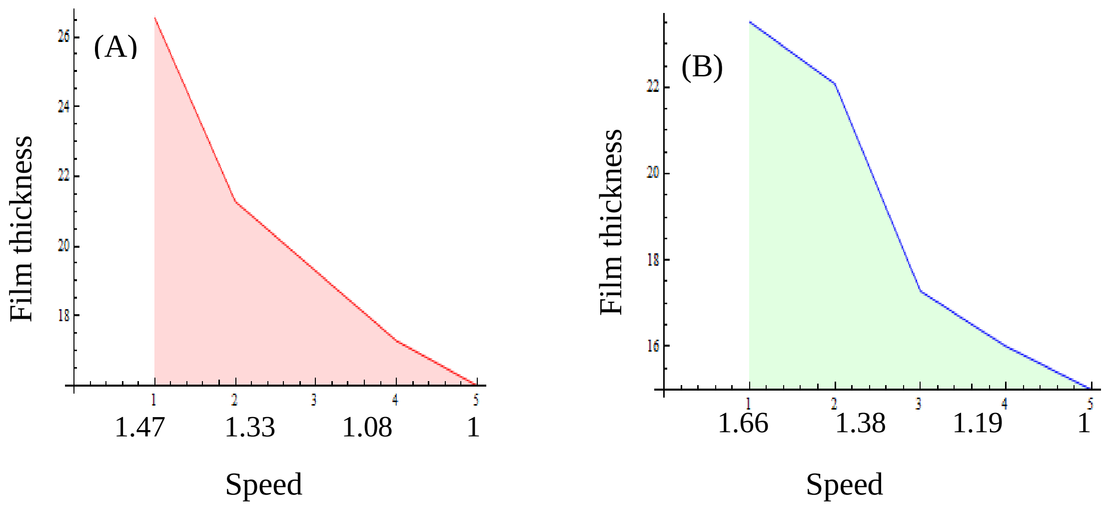
2.7 Film Thickness Calculation in Hydrodynamic Lubrication

Calculation film thickness in hydrodynamic lubrication during swing pleas depends on the speed of walk and viscosity of synovial fluid that lubricates surface articular cartilage additional that curvature of the cartilage surfaces and weights on the synovial knee joint, the law of film thickness will be:

$$h = \frac{U*\eta}{R*W} \qquad (2.2)$$

2.7.1 Effect of Speed of Walk

Walking fast and active helps control weight better than walking at a slow pace; it burns a large number of calories and forms muscle. The man's speed of walk differs from the woman because of the physical structure of each, where there are many structural and sexual differences, where step length and stride length for male are more than for female see fig (2.8) . As a result, the thickness of the membrane is different in hydrodynamic lubrication and the difference is evident in the youth period, where the percentage of membrane thickness for male is 55% while the percentage of membrane thickness for female is 47 %. This difference in speed walk decreases in aging for both sexes due to lack of movement and weight gain.



Fig(2.8): That relation between film thickness and speed of walk for (A) male and (B) female.

Table (2.4): The film thickness for male and female at normal walking speed.

Age Group(yes.)	Gender	Speed (m)	Film Thickness	Gender	Speed (m)	Film Thickness
20-29	F	1.47	23.52	M	1.66	27.56
30-39	F	1.33	22.08	M	1.38	23.28
40-49	F	1.08	17.28	M	1.19	19.28
50-59	F	1	16.33	M	1	17.28
60-69	F	0.96	13.36	M	1.08	16.12

2.7.2 Effect of Viscosity:

Viscosity affects the film thickness of hydrodynamic lubrication ,in swing phase (initial swing (acceleration), mid-swing and terminal swing (deceleration). The pressure in the fluid film is generated because of relative motion of surfaces and wedge action, describeing relative motion of flow synovial fluid between articular cartilage with laminar flow we pointed him at fig (2.3), pressure in swing phase is simple; therefore, gravity force is decreasing. The difference in sex is linked to the thickness of the membrane since the muscle strength in meal is greater so the pressure on the joints is lower and the viscosity of the fluid is greater and the thickness of the membrane is greater in meal than femeal, see fig (2.9). Viscosity is different through swing phase where in initial swing (acceleration) viscosity is high film thickness while terminal swing (deceleration) viscosity below so film thickness, see fig (2.10)

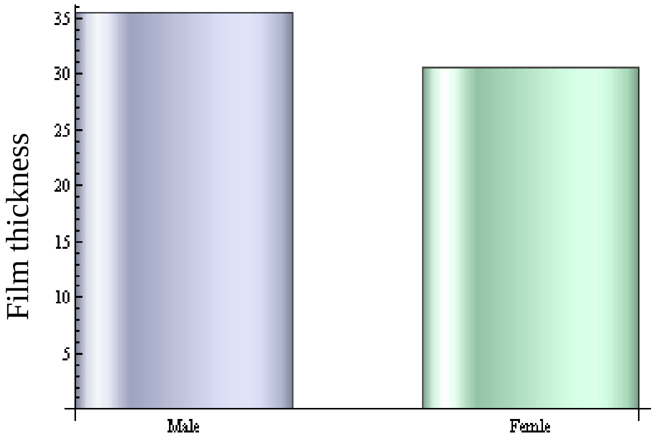


Fig (2.9): That relation between film thickness and Viscosity for male and female.

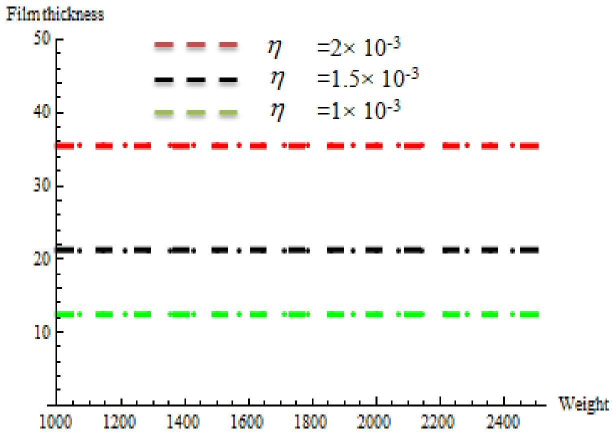


Fig (2.10): That relation between film thickness and weight with different Viscosity.

2.8 Film Thickness Calculation in Squeeze Lubrication

Calculation film thickness in squeeze lubrication during swing phase depends on the time of approaching surfaces from each of other additional to the radius of curvature and vicocity synovial fluid of gab between articular cartilage. The law of film thickness is:

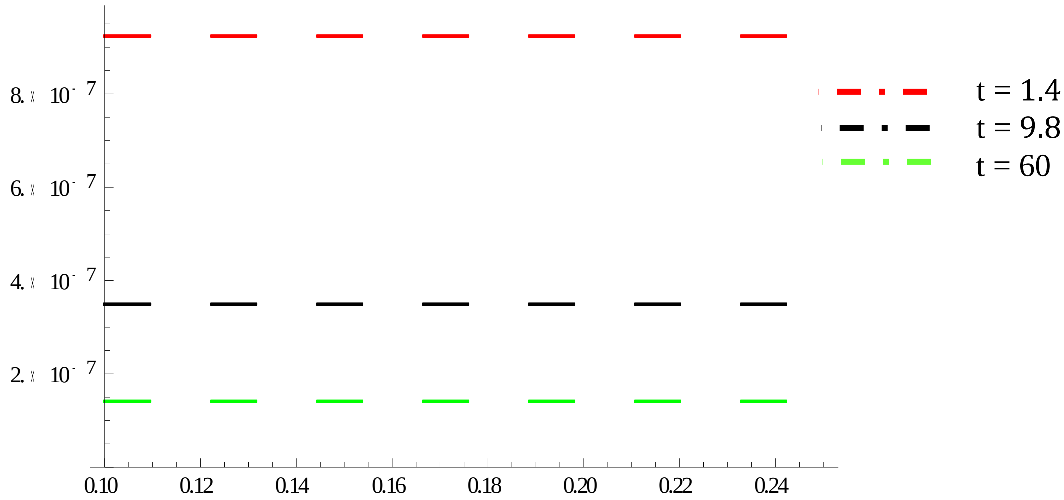
$$h = 2.86 \sqrt{\frac{\eta}{t}} \left(\frac{R}{E}\right)^{\frac{2}{3}} \qquad (2.3)$$

2.8.1 Effect of Time approaching

Time is very important in lubrication technique in stance phase (initial contact) where flow synovial fluid from synovial cell to gap between two articular cartilages, loads on the knee joint lead to squeezing on articulars, see fig (2.11). This compression varies with different walking stages and cycles where in (1– cycle) squeeze reach to 1.51 in initial contact while (8– cycle) squeeze reach to 34.35 in loading response.

Table(2.5): Relations knee between squeeze lubrication and tasks of the gait cycle.

<table>
<tr><th>Time</th><th>Cycle</th><th>Film
Thickness</th><th>Squeeze</th><th>Tasks of the Gait Cycle</th><th>period</th></tr>
<tr><td>1.40</td><td>1</td><td>0.924</td><td>1.51</td><td rowspan="8">Weight acceptance</td><td rowspan="7">initial contact</td></tr>
<tr><td>2.80</td><td>2</td><td>0.653</td><td>4.30</td></tr>
<tr><td>4.20</td><td>3</td><td>0.533</td><td>7.87</td></tr>
<tr><td>5.6</td><td>4</td><td>0.456</td><td>12.28</td></tr>
<tr><td>7</td><td>5</td><td>0.4133</td><td>16.93</td></tr>
<tr><td>8.4</td><td>6</td><td>0.377</td><td>22.28</td></tr>
<tr><td>9.8</td><td>7</td><td>0.349</td><td>28.08</td></tr>
<tr><td>11.2</td><td>8</td><td>0.326</td><td>34.35</td><td>loading response</td></tr>
<tr><td colspan="2"></td><td colspan="4">After 60 (m)</td></tr>
<tr><td colspan="2">Cycle</td><td>Film
Thickness</td><td></td><td>Tasks of the Gait Cycle</td><td>Period</td></tr>
<tr><td colspan="2">43</td><td>0.049</td><td>109</td><td>Weight acceptance</td><td>loading response</td></tr>
<tr><td colspan="2"></td><td colspan="4">After 1 (hour)</td></tr>
<tr><td colspan="2">86</td><td>0.0176</td><td>305</td><td>Weight acceptance</td><td>loading response</td></tr>
</table>



Fig(2.11): shows that relation between film thickness and cycle time in the gait cycle.

2.8.2 Effect of Viscosity

The synovial membrane secretes the synovial fluid, which softens the movement of the joint, facilitates its sliding and protects against shocks. This liquid contains a high proportion of hyaluronic acid, which gives it a vicious and flexible body to perform. Also the inflammation affects the membrane and increases the secretion of synovial fluid, the proportion of hyaluronic acid is responsible for its flexibility, see fig (2.12),when accumulates fluid in the joint the knee loses its role and causing swelling of the joint.

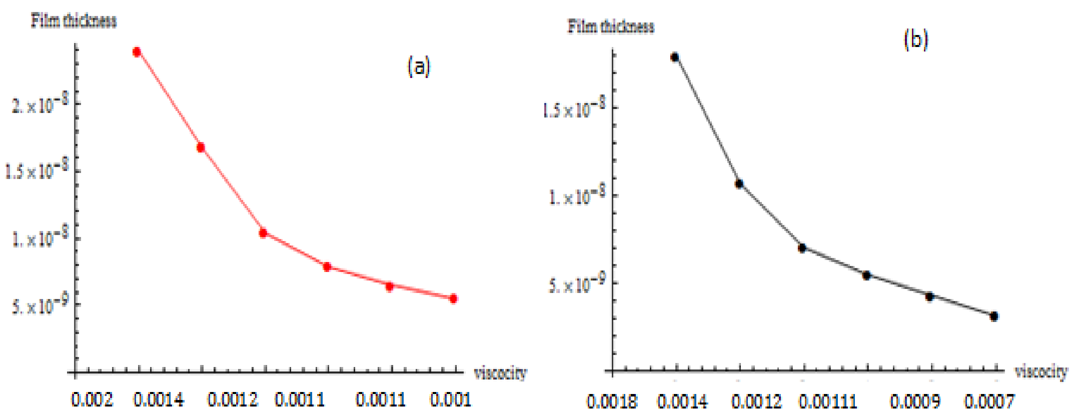


Fig (2.12): The effective viscosity on film thickness after 5 hour of normal walk ,(a) normal knee joint (b) disease knee joint.

2.8.3 Radius of Curvature (R)

The curvature of the articular cartilage is connected to squeeze lubrication and load carries capacity in young age (male-female) curvature appears lower since the thickness of the membrane is high, so load carries capacity of articular cartilage, see fig (2.13). With the age progresses, a curvature of the articular cartilage increases since the joint is unable to bear body weight see fig (2.14). This appears more in female over the age of 55 due to physiological reasons.

Table (2.6): Effect speed on the curvature of articular cartilage.

(Female)		(Male)	
Curvature	Speed	Curvature	Speed
0.16	1.66	0.17	1.47
0.2	1.38	0.211	1.33
0.21	1.19	0.45	1
0.3	1.2	0.54	0.5

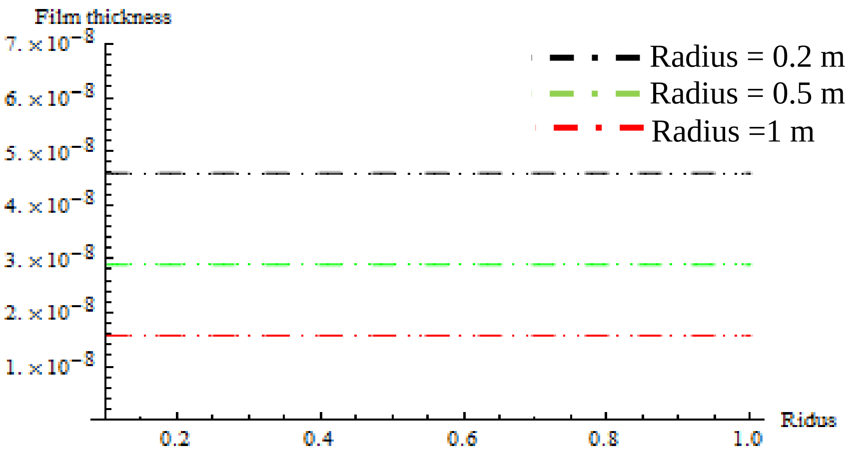
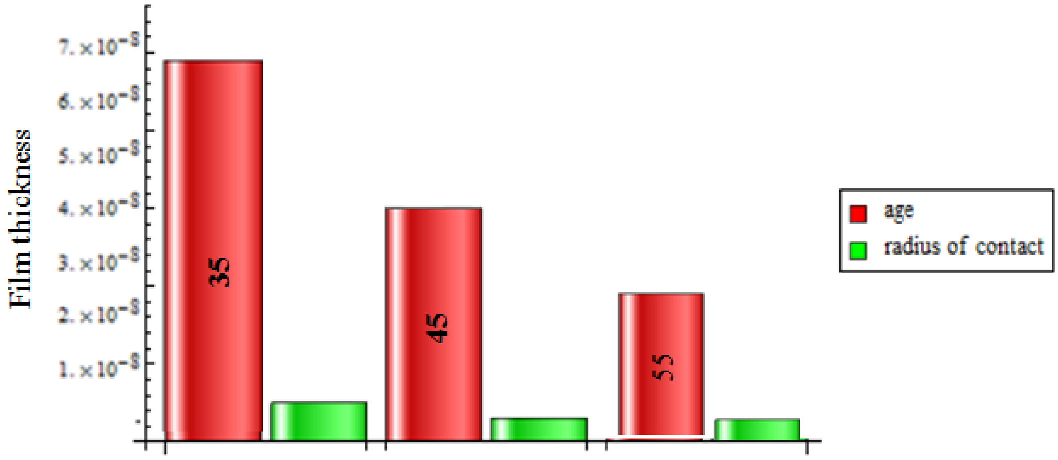


Fig (2.13): That effective curvature of the articular cartilage on film thickness in the stance phase.



Fig(2.14): Effect age and radius on film thickness.

2.9 Film Thickness Calculation in Elastohydrodynamic Lubrication

We find that lubricant thickness in elastohydrodynamic lubrication is independent of the pressure – viscosity characteristics but much more strongly dependent on three main factors:

1. Concentration of Hyaluronic acid in the synovial human joint.
2. Non-dimensional load on the synovial knee joint.
3. Velocity.

To calculate thin of the film in elastohydrodynamic lubrication during mid-stance in the gait cycle, we have applied the following law:

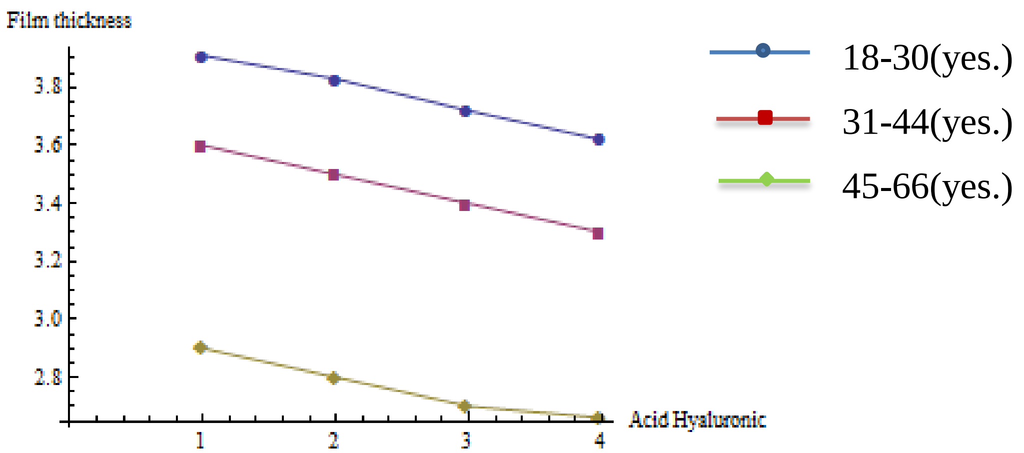
$$h = HA * \frac{U}{W} \qquad (2.4)$$

HA the Hyaluronic Acid, U is speed and W weight when $W \neq 0$.

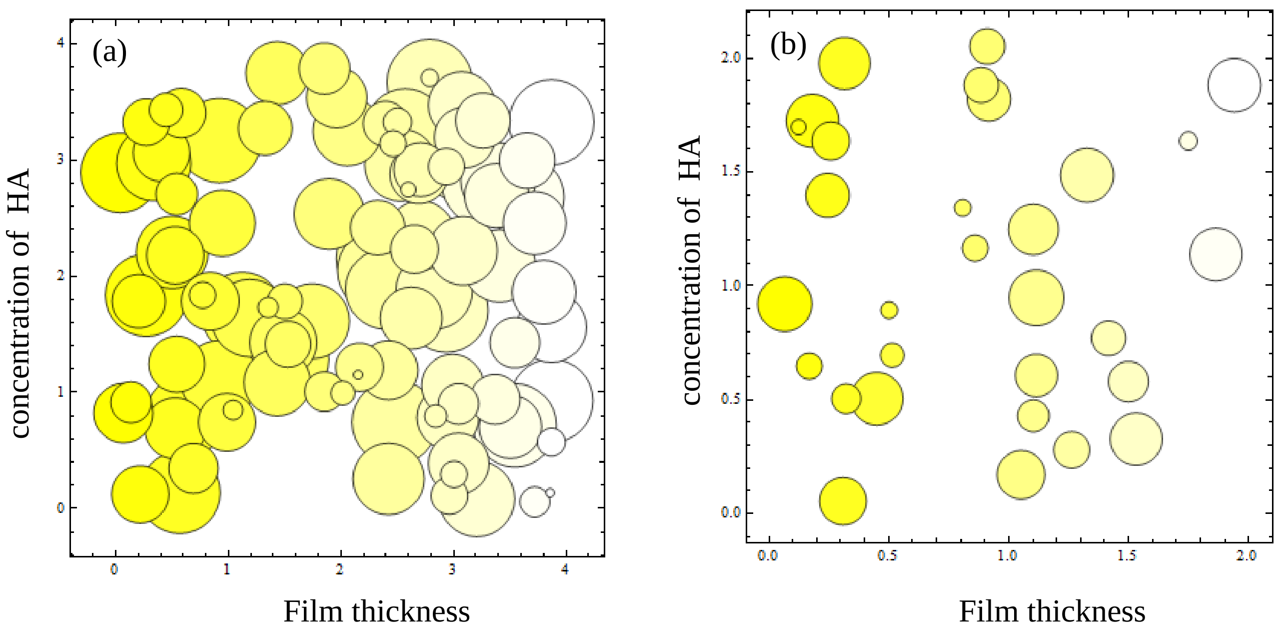
2.9.1 Hyaluronic Acid

Hyaluronic acid primary function is to maintain the vicious of the synovial fluid and reduces the friction between the bones of the joint. Different concentration of Hyaluronic acid in synovial human joint depends on two factors (age and health) where high concentration of hyaluronic acid is 3.9 in age (18-30) year and low in age (45-66) year to reach 2.1, see fig (2.15). A film thickness of elastic- hydrodynamic.

Lubrication is increased when the concentration of Hyaluronic acid is high and decreasing when the concentration of Hyaluronic acid low. The proportion of this substance decreases in case of disease (rheumatoid - arthritis), see fig (2.16); thus, reduces the thickness of film 0.8 µm.



Fig(2.15): Effect hyaluronic acid on film thickness in EHL.



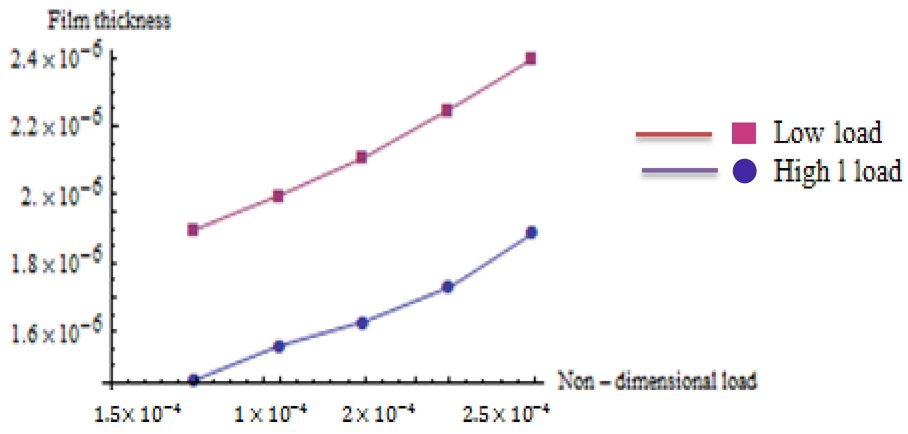
Fig(2.16): Particle lubrication of hyaluronic acid (a) normal joint (b) disease.

2.9.2 Non-Dimensional Load on Synovial Knee Joint

External forces affecting the knee joint (ground reaction force, and acceleration forces of foot and shank). In mid-stance where one or two foot on ground, there are two main factors affecting (ground reaction force, masses) where near-surface of articular cartilage of each other so increases radius of curvature, thin of film effect with different load wherein less load in motion standing up/sitting down then the thickness film ranges between [3.6-1.8] while high load in motion one-legged stance then thickness film ranges between load. [1.7-0.5], see fig (2.17).

Table (2.7): Effect Non-dimensional load on Film thickness articular cartilage.

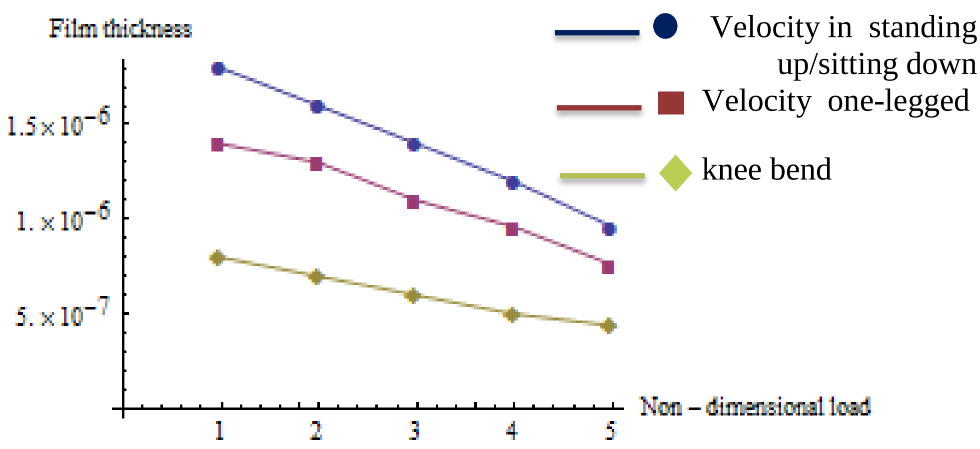
Non-dimensional load	Activity	Film thickness (μm)
1×10^{-4}	standing up/sitting down	3.6
1.5×10^{-4}	standing up/sitting down	2.4
1.6×10^{-4}	standing up/sitting down	2.15
1.7×10^{-4}	standing up/sitting down	2.11
1.8×10^{-4}	standing up/sitting down	2
1.9×10^{-4}	Knee bend	1.89
2×10^{-4}	Knee bend	1.8
2.1×10^{-4}	Knee bend	1.71
2.2×10^{-4}	Knee bend	1.63
2.3×10^{-4}	one legged stance	1.56
2.4×10^{-4}	one legged stance	1.5
2.5×10^{-4}	one legged stance	1.44



Fig(2.17): Film thickness with a different non-dimensional load.

2.9.3 <u>Non-Dimensional Velocity</u>

Classification of motion in mid- stance is classified to (standing up/sitting down- one-legged stance and knee bend), see fig (2.18). Therefore the different of velocity flow synovial lubricant and particle lubrication of porosity articular cartilage,is the high thin film in the standing up/sitting down. The decreasing the films thickness in on legged stance becomes low thin film. This is because of increasing the pressure on the knee joint.



Fig(2.18): The variation of film thickness with Hyaluronic acid for different dimensionless velocity in (EHL).

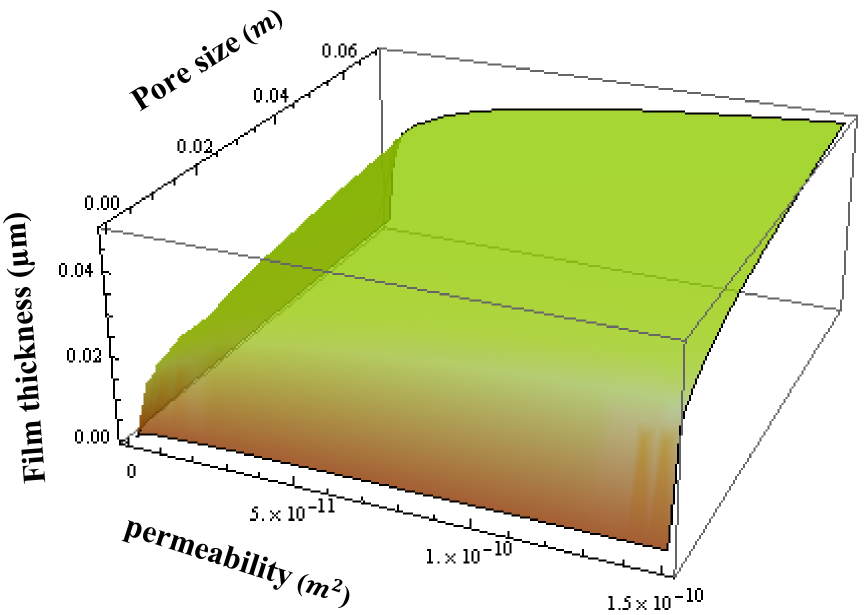
2.10 Film Thickness in Weeping Lubrication

The "weeping" mechanism is due to leakage through the cartilage. The equations for the time of approach and film thickness are stated as follows:

$$t = 0.743(\frac{R}{E})^{3/4}.\frac{(\eta W)^{1/3}}{(\sigma l)^{2/3}} \quad (2.5)$$

$$h = 3.30(\sigma l)^{1/6} \quad (2.6)$$

The permeability of cartilage $\sigma = 1.5 \times 10^{-19}$ m^2 and the pore size of cartilage thickness 0.005 $\boldsymbol{m}$ the assumed values for the parameter gives h_m=0.024 µm; this is much smaller than the value of surface roughness of the articular cartilage. It is well known that during the various events performed by human reflected negatively on the thickness of the film. Therefore, as a joint is loaded, most of the fluid is non-Newtonian crossing the articular surface comes from the cartilage close to the joint surface. Whenever the permeability of the articular cartilage is larging the non- Newtonian fluid flow that supports synovial fluid larger. As a result, it increases the film thickness to a limit in which it prevents the contact of surfaces, see fig (2.19).



Fig(2.19): The variation of film thickness with a pore size of articular cartilage for different permeability.

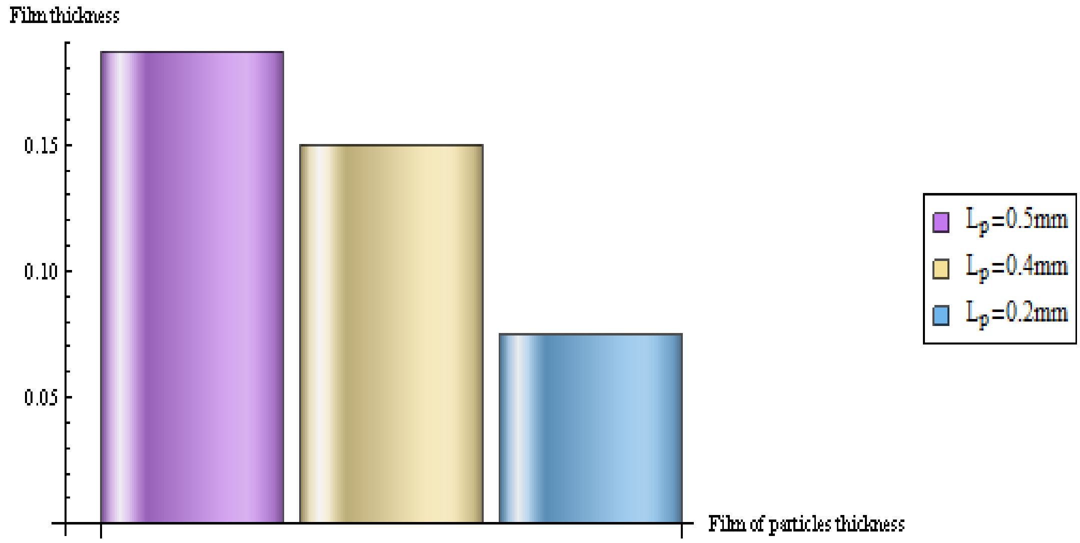
2.11 Film Thickness Calculation Boundary Lubrication

In boundary lubrication, there three main factors (roughness – length of particles – thickness of particles) to calculate film thickness the following law is appled:

$$h = \frac{L_p}{R_a} . h_p \tag{2.7}$$

2.11.1 Length of Particles

It is known that the particles of synovial fluid are from the polymer chains, the length of particles plays an important role in the production of synovial fluid where increasing the length of particles is accompanied by an increase in viscosity.



Fig(2.20): shows the variation of film thickness with a length of particles.

2.12 Friction Force

The friction force is a force combating relative motion of solid surface, fluid layers and material elements sliding against each other.

2.12.1 Classification of Friction Force

1. **Solid Friction:** Two bodies in direct contact with each other experience dry or solid friction.
2. **Fluid Friction:** When they are separated by a solid, liquid or gaseous medium, they experience fluid friction.
3. **Mixed Friction:** Between solid and fluid friction, the situation is known as mixed friction in which some parts of the two bodies are in direct contact while the others are separated by a fluid film.
4. **Internal Friction:** The friction may even involve a single body in which case it is related to the dissipation of the internal energy within the body.

To calculate the friction force of each other we apply the following formula F= μ_k.W (W is the normal force) and (μ_k is the coefficient of kinetic friction). The varying values of the coefficients of friction of the joint as will be in Appendix (B).

2.12.2 Main Factors Effect on Friction Force

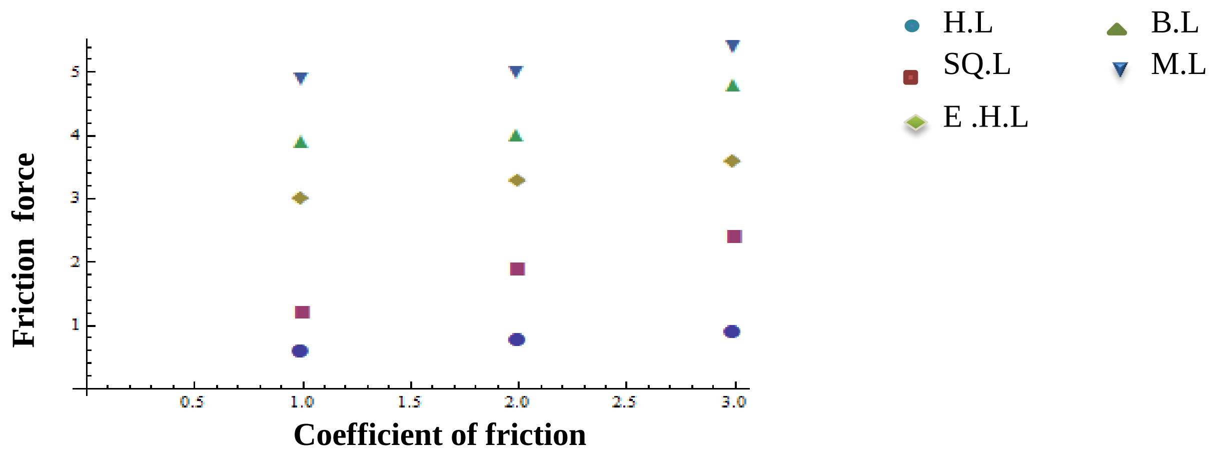
1. **Lubricant:** lubricant plays an important role in determining the friction force between articular cartilage. In fluid film, lubrication is classified to friction force as fluid friction where hydrodynamic lubrication is alot a particular lubrication. This is due two reasons (load- Hyaluronic acid), where in swing –phase(initial swing (acceleration), mid-swing and terminal swing (deceleration)) load is little; this leads to increasing production particles of lubrication from cells of synovial and cells of articular, while squeeze action starts the stance phase (initial contact) in this phase that occurs compering since (mass of person and ground reaction). With different daily active transfers to the second phase (loading response (foot –flat)), in these phases friction force increasing between articular cartilage of knee

joint. In elastic hydrodynamic the knee joint performs three important movements (standing up/sitting down- one-legged stance and knee bend), through mid – stance fluid friction vary deepened on (healthy of knee joint – flexibility of muscle). When staying a long time in seating mode rise to liquid viscosity, therefore, transform fluid friction to internal friction. Load capacity in Boundary lubrication is determined by the particular properties of lubricants and roughness of surfaces, but the viscosity of the fluid does not play an important role that leads to high friction force after many of cycle time, friction force remains classification friction fluid since exist ratio of particles lubrication. Mixed lubrication depended on (asperity contact - pressurized fluid), consequently friction force classified mixed friction after more than 200- cycle time reaches friction force in a normal joint (5.4 N), see fig (2.21).

Table (2.8): Effect Non-dimensional load on Film thickness articular cartilage.

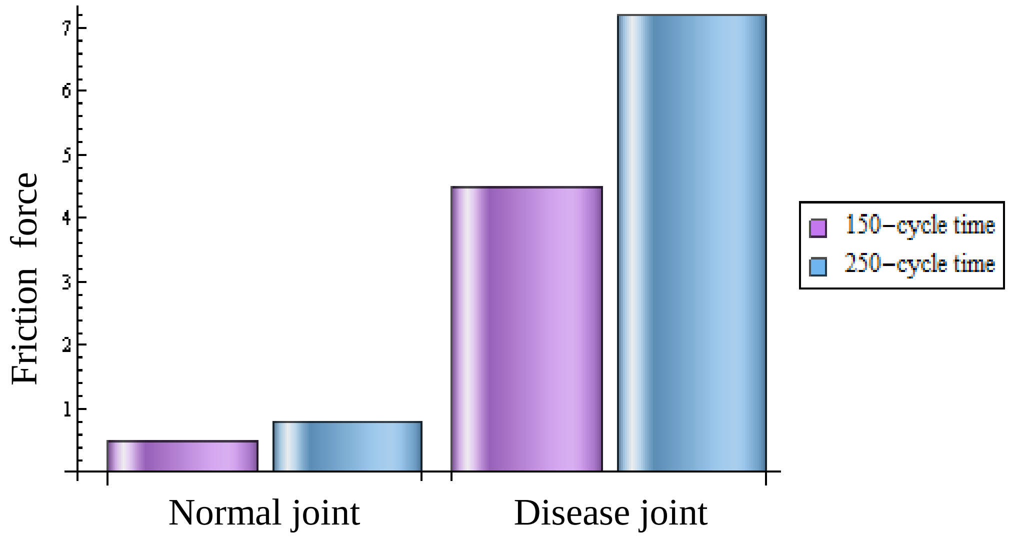
Friction force	Factors effective	Phase	Type of lubrication
swing –phase			
hydrodynamic lubrication	initial swing (acceleration)	Load Hyaluronic acid	0.6
	mid-swing (deceleration)		0.78
	terminal swing		0.9
stance phase			
Squeeze lubrication	initial contact	mass of person ground reaction	1.2
	loading Response(foot –flat)		2.4

<table>
<tr><th colspan="4">mid-stance</th></tr>
<tr><td rowspan="3">elastic hydrodynamic lubrication</td><td>standing up/sitting down</td><td rowspan="3">healthy of knee joint
flexibility of muscle</td><td>3</td></tr>
<tr><td>one legged stance</td><td>3.3</td></tr>
<tr><td>knee bend</td><td>3.6</td></tr>
<tr><td>boundary lubrication</td><td rowspan="2">more cycle time</td><td>particular properties of lubricants roughness of surfaces</td><td>4.8</td></tr>
<tr><td>mixed Lubrication</td><td>asperity contact pressurized fluid</td><td>5.4</td></tr>
</table>



Fig(2.21): The variation of friction force with coefficient of friction for phase of the gait cycle.

2. **Healthy:** In young phase ratio of friction force, (fluid friction-mixed friction- internal friction) range between [0.5 – 3.5] for daily active. Strongly muscle makes a decrease in friction force as well as elastic of articular cartilage. Also particles of lubrication leads to the same function. By the time the movement becomes less and the muscles become weak that conncat carry the weight of person. Therefore the friction force is increased and classified as solid friction, see fig (2.22).



Fig(2.22): The relationship force of knee joint with (gait cycle and lubrication.

2.13 Conclusions

1- Hydrodynamic lubrication analysis in swing phase fluid friction between articular cartilage is less and joint is protected.

2- The viscosity of the synovial fluid in hydrodynamic lubrication the most effect on the thickness of film than speed walk.

3- Gender is also responsible for determining the thickness of the film depended on the walking speed which is higher in male than female.

4- In Squeeze lubrication radius of curvature articular cartilage is increased with cycle time and reduces film thickness.

5- Cycle time is contralled by squeeze films and viscosity through initial contact.

6- The mid-stance sliding motion and load vary by single limb support and double limb support.

7- The thickness of the film varies by distribution non-dimensional load in (standing up/sitting down- Knee bend- one-legged stance).

8- Elastohydrodynamic films having greater thickness relative to the roughness of articular cartilage are operative during normal body movements.

9- Elastohydrodynamic films which little or no sliding takes place, and the synovial fluid will then be required to act as a boundary lubricant to prevent surface damage and to provide low starting friction.

10- Synovial joint failures due to aging or injury and failure of the prosthesis therefore reduce thickness and increased stress resulting from a reduction of conformity and compliance of cartilage surfaces.

11- Friction force of articular cartlige effect of hyaluronic acid produce synovial and cartilaginous cells where they increase production, reduce friction, and represent joint protection.

Chapter Three

Effect of Flexibility on Squeeze Film Characteristcs between Porous Rectangular Plates

Introduction

Flexibility refers to the extensibility of joint tissues to allow normal or physiological motion. Flexibility plays a prominent role in the functional ability of a joint to move through its full range of motion with in stance phase and swing phase without incurring pain or a limit to performance. Flexibility is Classification to many types: Static flexibility that ranges the motion about a joint with no emphasis on speed. Ballistic flexibility is usually associated with bobbing or bouncing motion. Dynamic (functional) flexibility is the ability to use range of motion in the performance of a physical activity. The dynamics and deformations of immersed flexiblility are at the heart of important industrial and biological processes, induce peculiar mechanical and transport properties in the fluids that contain them, and are the basis for novel methods of flow control in porous medium **[37]**.

The study considers a mathematical model for the influence of varying film thickness of articular cartilage and peclet number on squeeze film characteristics for non-Newtonian fluid with variable flexibility through porous medium. The study uses Mathematica 8 to solve the problem. The results of the physical parameter problem are discussed by using the graphs.

3.1 Assumptions of Hydrodynamic Lubrication [15]

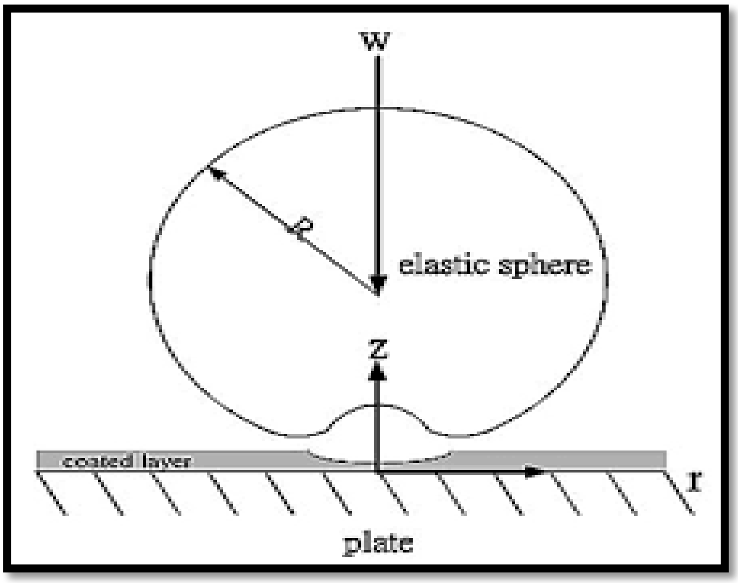
1. Newtonian fluid
2. Los-viscous fluid
3. Incompressible fluid
4. Body force and a inertia force in the equation of motion are negligible.
5. The velocity component across the film is negligible compared with the other velocity component .
6. The pressure gradient across the film could be neglected with respect to the pressure gradient along film .

3.2 Basic Equation

The main equations that describe the flexibility of the synovial human knee joint during different movement in the case stand phase and swing phase are introduced. Pore size of surface of articular cartilage with variable load and peak load will be considered and discussed taking into account the effect of roughness and plecte number on the dynamic joint performance.

3.3 Mathematical Formulation and Solution of The Flexibility Problem

The squeeze film mechanism represents a rigid sphere of radius (R) approaching an infinite plate with a velocity $v = \frac{\partial h}{\partial t}$ with varying flexibility (f) synovial human knee joint. The lubricant is taken to be a stockes couple stress fluid. The geometry and coordinates of the flow domain in the present problem are shown in fig (3.1). Using momentum equations and continuity equation expresses the synovial fluid flow.



Fig(3.1): Squeeze film action between a sphere and a flat plate.

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=X-\mu\frac{\partial^2 u}{\partial z^2}-\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{R_a}\right]-\gamma\frac{\partial^4 u}{\partial z^4} \qquad (3.1)$$

$$\frac{\partial p}{\partial z}=0 \qquad (3.2)$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial(r\rho u)}{\partial r}+\frac{\partial(\rho w)}{\partial z}=0 \qquad (3.3)$$

Where ρ density, X body force, (u,w) are the velocity components of the lubricant in r and z directions respectively, p is pressure, μ dynamic viscosity, γ is a material constant accounting for couple stresses due to polar additives in the lubricant, R_a surface roughness β pore size, and P_e plecte number. Under the assumptions hydrodynamic lubrication theory the fluid film is thin, the fluid inertia is small and body forces are absent, then the momentum equations and continuity equation governing the flow of lubricant in polar coordinates reduce to the form:

$$\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{R_a}\right]=\mu\frac{\partial^2 u}{\partial z^2}-\gamma\frac{\partial^4 u}{\partial z^4} \qquad (3.4)$$

$$\frac{\partial p}{\partial z}=0 \qquad (3.5)$$

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \qquad (3.6)$$

The Boundary conditions for the velocity component at the surfaces of the plate and spheres are:

$$u(r,0)=\frac{\partial^2 u(r,0)}{\partial z^2}=0 \qquad ,\ \mathrm{w(r,0)}=0 \qquad (3.7)$$

$$u(r,h)=\frac{\partial^2 u(r,h)}{\partial z^2}=0 \qquad ,\ \mathrm{w(r,h)}=\frac{\partial h}{\partial t} \qquad (3.8)$$

If the ratio of hyaluronic acid lion is high in synovial and cartilaginous cells between the spherical and plane, h_m is minimum film thickness and R is a curvature radius ,then the film thickness h can be written as follows :

$$h=h_m+\frac{r^2}{2R} \qquad (3.9)$$

3.4 Governing Equation

The velocity can be acquired by integrating equation (3.4) subject to the boundary conditions (3.7) and (3.8), Dividing the equation (3.4) by (μ) to get:-

$$\frac{\partial p}{\partial r}\frac{1}{\mu}\left[\beta-\frac{\beta.P_e}{Ra}\right]=\frac{\partial^2 u}{\partial z^2}-\frac{\gamma}{\mu}\frac{\partial^4 u}{\partial z^4} \tag{3.10}$$

The ratio ($\frac{\gamma}{\mu}$) is about of dimensional square length and; hence,it characterizes the chain length of the polymer additives.

$$l=\sqrt{\frac{\gamma}{\mu}} \tag{3.11}$$

For homogenous part the equation (3.10) becomes :-

$$\frac{\partial^4 u}{\partial z^4}-\frac{1}{l^2}\frac{\partial^2 u}{\partial z^2}=0 \tag{3.12}$$

It integrated the equation (3.12) twice with respect to z and thus it obtained the general solution .

$$u(r,z)=c_1\cosh\left(\frac{z}{l}\right)+c_2\sinh\left(\frac{z}{l}\right) \tag{3.13}$$

To obtain a particular solution integrate equation (3.10) twice with respect to z :-

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}-l^2\frac{\partial^3 u}{\partial z^3}\right)=\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right] \tag{3.14}$$

$$\frac{\partial u}{\partial z}-l^2\frac{\partial^3 u}{\partial z^3}=\int\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]d\,z=\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]z+C_3 \tag{3.15}$$

$$u-l^2\frac{\partial^2 u}{\partial z^2}=\int\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]z+C_3\,dz \tag{3.16}$$

$$u-l^2\frac{\partial^2 u}{\partial z^2}=\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\frac{z^2}{2}+C_3 z+C_4 \tag{3.17}$$

Where C_3, C_4 are integration constant, assume the non-homogenous part particular solution takes the form:-

$$Az^2+Bz+C=u(r,z) \tag{3.18}$$

Now using the equations (3.13) and (3.17), it the formula has been obtained, following velocity component of the synovial fluid:

$$u(r,z) = C_1\, cosh\frac{z}{l} + C_2\, sinh\frac{z}{l} + \frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] z^2 + C_3 z + C_4 + \frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] \quad (3.19)$$

Where c_1, c_2 it is calculated through the derivation twice by the equation(3.19) with respect to u :

$$\frac{\partial u}{\partial z} = \frac{C_1}{l}sinh\frac{z}{l} + \frac{C_2}{l}cosh\frac{z}{l} + \frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] z + C_3 \quad (3.20)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{C_1}{l^2}cosh\frac{z}{l} + \frac{C_2}{l^2}sinh\frac{z}{l} + \frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] \quad (3.21)$$

Applying the boundary conditions $u(r,0) = \frac{\partial^2 u(r,0)}{\partial r^2} = 0$ on equations(3.19) and (3.21), it is obtained the integration constant $C_1 = -\frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right]$ and $C_4 = 0$ Substituting for c_1 and c_4 into the equation (4.19), to get general form:

$$u(r,z) = -\frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] cosh\frac{z}{l} + C_2\, sinh\frac{z}{l} + \frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] z^2 + C_3 z + \frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] \quad (3.22)$$

After derivative of the equation (3.22) twice with respect to z and applying the boundary conditions $u(r,h) = \frac{\partial^2 u(r,h)}{\partial r^2} = 0$ hence it was obtained the integration constant $C_2 = \left[\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] cosh\frac{h}{l} - \frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right]\right]\frac{l^2}{sinh\frac{h}{l}}$ and $C_3 = \frac{-h}{2\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right]$ **.** Substituting for C_2 and C_3 into equation (3.22) it is obtained velocity component of the synovial fluid in the general form:

$$u(r\,,z) = \frac{-l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] cosh\frac{z}{l} + [\left[\frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right]\right] cosh\frac{h}{l} - \frac{1}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right]]\frac{l^2}{sinh\frac{h}{l}} .sinh\frac{z}{l} + \frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] Z^2 - \frac{h}{2\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] Z + \frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right] \quad (3.23)$$

$$u(r,z)=\frac{-l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]cosh\frac{z}{l}+\left[\frac{\left[\frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]cosh\left(\frac{h}{l}\right)-\frac{l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\right]}{sinh\left(\frac{h}{l}\right)}\right]sinh\left(\frac{z}{l}\right)+$$

$$\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[z^2-hz+2\,l^2\right] \quad (3.24)$$

It depends on hyperbolic sine and cosine, simplifying the equation (3.24) and it is obtained the final form of velocity in polar coordinates:-

$$u(r\,,z)=\frac{-l^2}{\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[cosh\left(\frac{z}{l}\right)+\frac{\left(cosh\left(\frac{h}{l}\right)-1\right)\,sinh\frac{z}{l}}{sinh\frac{h}{l}}\right]+$$

$$\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[z^2-hz+2l^2\right] \quad (3.25)$$

$$u(r,z)=\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[z^2-hz+2l^2-2l^2\left(\frac{\frac{cosh\ (2z-h)}{2l}}{cosh\left(\frac{h}{2l}\right)}\right)\right] (3.26)$$

To determine velocity squeeze (w) of synovial fluid during daily activities integrate the continuity equation (3.6) with respect to z.

$$\frac{\partial w}{\partial z}=-\frac{1}{r}\frac{\partial}{\partial r}\,r\,u(r,z) \quad (3.27)$$

$$w=\frac{-1}{r}\frac{\partial}{\partial r}\int_0^h u(r,z)r.dz \quad (3.28)$$

Now, it will integrate sliding motion (u) that represent the equation (3.26)

$$\int u(r,z)dz=\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[\frac{z^3}{3}-h\frac{z^2}{2}+2zl^2-\frac{2l^3}{cosh\left(\frac{h}{2l}\right)}\left(sinh\frac{(2z-h)}{2l}\right)\right]+$$

$$A \quad (3.29)$$

Where A is the integration constant. Substitute in equation (3.29) in equation (3.28) then we get the following result.

$$w=-\frac{\partial}{\partial r}\cdot\frac{1}{2\mu}\cdot\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[\frac{z^3}{3}-h\frac{z^2}{2}+2zl^2\ \frac{2l^3}{cosh\left(\frac{h}{2l}\right)}\left(sinh\frac{(2z-h)}{2l}\right)\right]$$

$$+A \quad (3.30)$$

Using the boundary condition $w(r,0)=0$ on the solution (3.30), hence we obtain the integration constant $A=-\frac{\partial}{\partial r}\cdot\frac{1}{2\mu}\cdot\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]2l^3tanh\left(\frac{h}{2l}\right)$

and substitute the value A in equation (3.30), We get the following expression:

$$w = -\frac{\partial}{\partial r}\cdot\frac{1}{2\mu}\cdot\frac{\partial p}{\partial r}\left[\beta-\frac{\beta.P_e}{Ra}\right]\left[\frac{z^3}{3}-h\frac{z^2}{2}+2zl^2-\frac{2l^3}{cosh\left(\frac{h}{2l}\right)}\left(sinh\frac{(2z-h)}{2l}\right)-2l^3\tanh\left(\frac{h}{2l}\right)\right] \quad (3.31)$$

The flow of hyaluronic acid fluid in a porous matrix (articular cartilage) is specific by the modified Darcy law that show up the simple proportional relationship between the immediate discharge rate through a porous medium. The viscosity of the fluid and the direction pressure in the gap between articular cartilage with different type of lubrication expression of law is given by:

$$Q = -\frac{f}{\mu}\nabla p \quad (3.32)$$

Q is the total discharge, and $Q=(u^*,w^*)$ where u^*,w^* represent modified Darcy velocity components in r,z directions, respectively, u^*,w^* have expression following :

$$u^* = -\frac{f}{\mu}\frac{\partial p^*}{\partial r} \qquad w^* = -\frac{f}{\mu}\frac{\partial p^*}{\partial z} \quad ,\mu \neq 0 \quad (3.33)$$

Where p^* is porous region, f is the flexibility of synovial human knee joint that represents the ratio of the stride length and viscosity to time and weight. If flexibility of knee joint is high and velocity of synovial fluid so then the microstructure additives present in the lubricant block the pores in the porous layer and thus reduce the Darcy flow through the porous matrix. When the microstructure size of the particle is very small compared to the pore size, it will be noted as a negative sign because fluid flows from high pressure to low pressure. p^* in the porous region, due to continuity, satisfies the Laplace equation:

$$\nabla^2 p^* = \frac{\partial^2 p^*}{\partial r^2}+\frac{\partial^2 p^*}{\partial z^2}=0 \quad (3.34)$$

Integrating equation (3.34) with respect to (z) and using the boundary conditions of solid bearing ($z = -H_{\circ}\ at\ z = 0$) where $H_{\circ}$ is the porous layer thickness so noted negative sign because fluid flows less, therefore film thickness decreased.

$$\frac{\partial p^*}{\partial z} = -\int_{-H_0}^{0} \left(\frac{\partial^2 p^*}{\partial r^2}\right) dz \tag{3.35}$$

Assuming that the porous layer thickness $H_{\circ}$ is very small using the pressure continuity condition pressure generated in film region (p) equal to the pressure generated in the porous region at the interface $z = 0$ of porous matrix and fluid film in healthy synovial knee joint , integral equation (3.35) becomes:

$$\frac{\partial p^*}{\partial z} = -H_0 \frac{\partial^2 p^*}{\partial r^2} \tag{3.36}$$

We will focus our attention on the squeeze action w^* that changes with the movement where the person's weight presses downward and the reaction of the ground upwards with different type of flexibility, That will ignore the speed slide u^* and make equation (3.32) form:

$$w^* = \frac{f}{\mu} H_{\circ} \frac{\partial^2 p}{\partial r^2} \tag{3.37}$$

The relevant boundary conditions for the velocity components are:

(i) at the static flexibility $z = 0$

$$u(r,0) = \frac{\partial^2 u(r,0)}{\partial z^2} = 0 \quad , \qquad \mathrm{w(r,0) = 0} \tag{3.38}$$

(ii) at the dynamic (functional) flexibility $z = h$

$$u(r,h) = \frac{\partial^2 u(r,h)}{\partial z^2} = 0 \quad , \qquad \mathrm{w(r,h)} = \frac{-\partial \mathrm{h}}{\partial \mathrm{t}} - \mathrm{w}^* \tag{3.39}$$

The squeeze action is obtained by integrating the continuity equation (3.6) with respect to z and applying boundary condition squeeze action on the upper and lower surface of the synovial human knee joint. From equation(3.31), we obtain:

$$w(r,h) = \frac{\partial}{\partial r} \cdot \frac{1}{12\mu} \cdot \frac{\partial p}{\partial r}\left[\beta - \frac{\beta.P_e}{Ra}\right][N(h,l)] \tag{3.40}$$

Where:

$$N(h,l) = h^3 - 12l^2h + 24l^3 \tanh[\frac{h}{2l}] \tag{3.41}$$

Then the modified Reynolds equation governing the film pressure can be written as:

$$-12\mu \frac{\partial h}{\partial t} = \frac{\partial^2 p}{\partial r^2}\left[12 fH_\circ + \left[\beta - \frac{\beta.P_e}{Ra}\right] N(h,l)\right] \tag{3.42}$$

3.5 Squeeze Film Pressure

Introducing the non-dimensional parameters in the governing equations for the pressure is of importance for both theoretical and computational purposes. It is also of importance to present the various parameters in the lubrication system, in non-dimensional form.

$$p^* = -\frac{ph_\circ^2}{\mu R \frac{\partial h}{\partial t}} \qquad l^* = \frac{l}{h_\circ} \qquad h^* = \frac{\mathrm{h}}{h_\circ}$$
$$r^* = \frac{r}{R} \qquad \beta = \frac{R}{h_\circ} \qquad f = \frac{S^2\,\mu}{t\,W} \tag{3.43}$$

Where S,W are the stride length and body weight of human. Apply equation (3.43) into equation (3.42), the final form is obtained of dimensionless modified Reynolds equation as:-

$$\frac{12\,R^2}{\beta} = \frac{\partial p^*}{\partial r^*}\left[12fH^* + \left[\beta - \frac{\beta.P_e}{Ra}\right] N(h^*,l^*)\right]\frac{\partial}{\partial r^*} \tag{3.44}$$

Boundary condition for the fluid film pressure and radial of chain polymer is as follows:

$$\left.\begin{aligned} p^* &= 0 \;\; at \quad r^* = 2 \\ \frac{\partial p^*}{\partial r^*} &= 0 \; at \quad r^* = 0 \end{aligned}\right\} \tag{3.45}$$

To find the solution to the modified Reynolds equation, we integrate equation (3.44) one with respect to r^*, and thus we obtain from following:

$$\frac{\partial p^*}{\partial r^*} = \frac{12.R^2\, .r^*}{\beta\left[12.f.H^* + \left[\beta - \frac{\beta.P_e}{Ra}\right].h^{*3} - 12l^{*2}h^* + 24l^{*3}\tanh\left[\frac{h^*}{2l^*}\right]\right]} + A \quad (3.46)$$

Where A is the integration constant, it is applied the boundary condition, hence, it is obtained the integration constant $A = 0$, then the squeeze film pressure is given by .

$$p^* = \frac{6.R^2\left(4 - r^{*2}\right)}{\beta\left[12.f.H^* + \left[\beta - \frac{\beta.P_e}{Ra}\right].h^{*3} - 12l^{*2}h^* + 24l^{*3}\tanh\left[\frac{h^*}{2l^*}\right]\right]} \quad (3.47)$$

With the film pressure known, the squeeze film characteristics can now be calculated.

3.6 Load Carrying Capacity for Synovial Knee Joint

The load carrying capacity during with different activities performed by human and varying flexibility level.The load carrying capacity of the porous flat plate (W) can be determined:

$$w = 2\pi \int pr\, dr \quad (3.48)$$

Introduce the dimensionless load carrying capacity in consideration to flexibility knee joint.

$$w^* = -\frac{Wh_\circ^2}{\mu R^3 \frac{\partial h}{\partial t}} \quad (3.49)$$

Substituted quantity (3.49) into equation (3.48) yield

$$w^* = 2\pi \int_0^2 p^* r^*\, dr^* \quad (3.50)$$

We integrate dimensionless pressure with respect to the dimensionless radial and thus we obtain the general form:

$$w^* = \frac{48\pi R^2}{\beta\left[12.f.H^* + \left[\beta - \frac{\beta.P_e}{Ra}\right].h^{*3} - 12l^{*2}h^* + 24l^{*3}\tanh\left[\frac{h^*}{2l^*}\right]\right]} = g\left(h^*, \beta, P_e, l^*\right) \quad (3.51)$$

3.7 Squeeze Time-Film for Flexibility Knee Joint [37]

Film thickness between two articular cartilages is different with cycle time where squeeze projected on knee joint transfers film thickness to minimal film. Time film thickness is depends on load carrying capacity and flexibility knee joint from the equation (3.51), we obtain the time of approach as follows:

$$\frac{dh_m^*}{dt^*} = \frac{1}{12\pi \int_0^2 g\,(h^*,\beta,P_e,l^*)dr^*} \tag{3.52}$$

where $t^* = \dfrac{W\,h_0^2}{\mu R^2 \phi} t$ the dimensionless time of approach.

$$t^* = 12\pi \int_{h_m^*}^{1} g\,(h^*,\beta,P_e,l^*)dh_m^* \tag{3.53}$$

If the dimensionless time approach tends to zero, then the minimum film thickness will be high. Equation (3.53) is a highly non-linear differential.

$$t^* = \frac{226.06.(R)^4}{\{\beta.12.f.4+(\beta-\frac{\beta.P_e}{R_a})((h^*)^3-12.(l^*)^2+24.(l^*)^3.\mathrm{Tanh}[\frac{h^*}{2.(l^*)}])\}} - \frac{226.06.(R)^4.h^*{}_m}{\{\beta.12.f.4+(\beta-\frac{\beta.P_e}{R_a})((h^*)^3-12.(l^*)^2+24.(l^*)^3.\mathrm{Tanh}[\frac{h^*}{2.(l^*)}])\}} \tag{3.54}$$

3.8 Peak load

Maximum peak load to the lower limbs is the force exerted by the body weight of human and the act of the ground can be calculated maximum permissibility to bear depending on squeeze film characteristics,

$$P = \frac{w^*.S}{\dfrac{6.R^2\,(4-r^{*2})}{\beta\left[12.f.H^*+\left[\beta-\frac{\beta.P_e}{R_a}\right].h^{*3}-12l^{*2}h^*+24l^{*3}tanh[\frac{h^*}{2l^*}]\right]}} \tag{3.55}$$

3.9 Numerical Results

The governing equations lubrication of the human knee joint is very difficult if not impossible to solve or to study the mechanism estimated in vivo. For this reason, the need for a numerical study arises, and the selection of factors values for the best estimation to get good analytics results, see table (3.1). The parameters chosen are based on previously measured selected values used in researches, specializing with problem to reach two parameters that are close to reality. So numerical results are displayed through fig (3-2) - (3-23) to show the effects of various parameters such as surface roughness (R_a), permeability (ϕ),film thickness(h), couple stress length (l). The values of film thickness are obtained in chapter three.

Table (3.1): the estimated values of the parameters involved in the knee

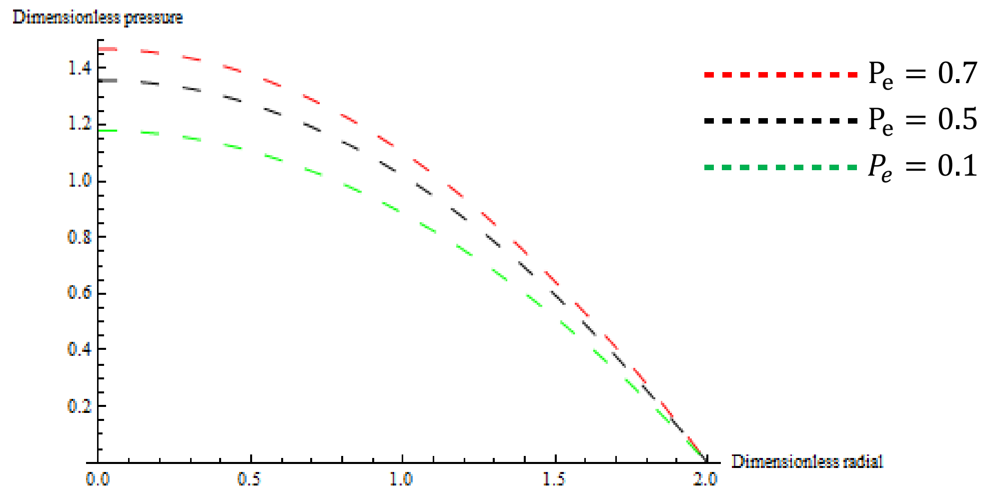
Parameters	Symbols	Numerical values
The effective radius of curvature	R	0.1-1
Flexibility	f	0-5
Couple stress length	l	0.1-0.7
Radial coordinate	r	0.1-2
The ratio of the microstructure size to the pore size	β	0.01-0.05
Surface roughness	R_a	2-4
The film thickness of gab between two articular	h	0.7-20
Peclet number	P_e	0.7-0.1
The Thickness of the porous	H	4-7
Weight	w	40-100
Stride length	S	1.09-1.66
Permeability	ϕ	10^{-18}

3.10 Result and Discussion

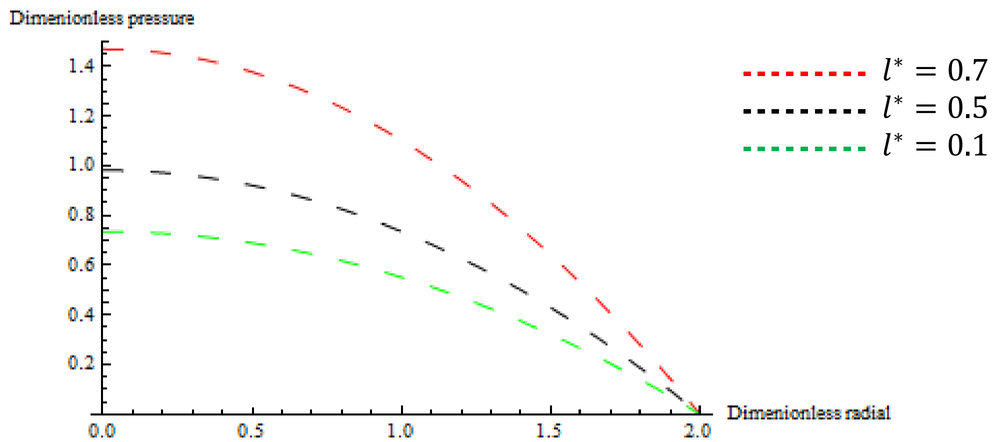
On the basis of momentum equations and continuity equation, this chapter discusses effective of plecte number and flexibility of the articular cartilage on squeeze film characteristics in synovial human knee joint in daily active, and determines the type of flexibility in different lubrications.

3.10.1 Squeeze Film Pressure

The variation of the dimensionless squeeze film pressure (p^*) is shown in fig (3.2) with using equation (3.47), generated by the squeeze film action as a function of dimensionless radial coordinate (r^*) for different values of peclet number parameters (P_e). It is observed that the effect of the peclet number fluid be increase film pressure, especially in the nearby areas of the position radial coordinate $(r^* = 0)$, see table (3.2). This result appears important peclet number relationship with the fluid flow about increased pressure distribution. The effect of couple stress parameter (l^*) on the variation of (p^*) with radial coordinate (r^*) is shown in fig (3.3). It is observed that the pressure film (p^*) increases with increasing values of (l^*). The effect of flexibility parameter (f) on the variation of $(p^*)$with radial coordinate (r^*) is shown in fig (3.4). It is observed that the pressure film (p^*) increases with decreasing values of (f), see table(3.3), when the joint has high flexibility then the pressure distribution is lower. The effect of a film thickness of gab between two articular parameter (h^*)on the variation of $(p^*)$with radial coordinate (r^*) is shown in fig (3.5). It is observed that the pressure film $(p*)$ increases with decreasing values of (h^*) in different type lubrication (hydrodynamic, squeeze and elastohydrodynamic).The effects of the surface roughness parameter(R_a), and effective radius of curvature parameter (R) on the variations of $(p^*)$with radial coordinate (r^*) are shown in fig (3.6) and (3.7). It is observed that the pressure film (p^*) increases with decreasing (R_a) and increasing values of (R).



Fig(3.2): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate (r^*) for different peclet number parameters (P_e) $(\beta = 0.02, f = 1$ and $h^* = 10)$.



Fig(3.3): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate (r^*) for different couple stress length parameter (l^*) $(\beta = 0.02, f = 1$and $h^* = 10)$.

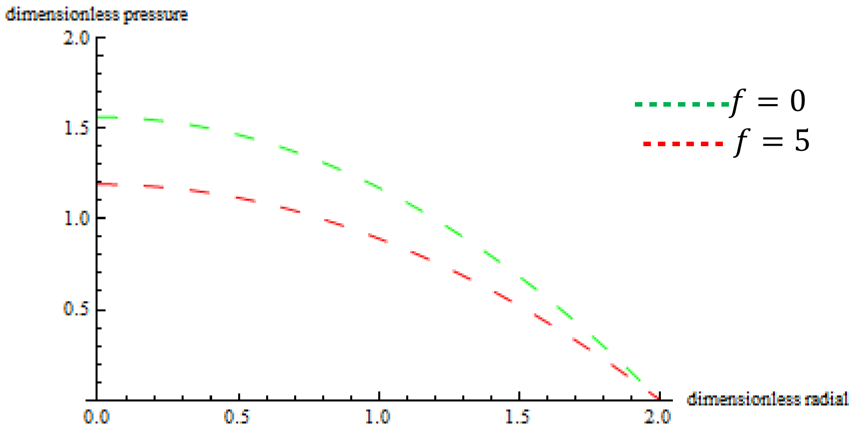


Fig (3.4): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate (r^*) for different flexibility parameter $(f)(\beta = 0.02, \mathrm{P_e} = 0.7$and $h^* = 16)$.

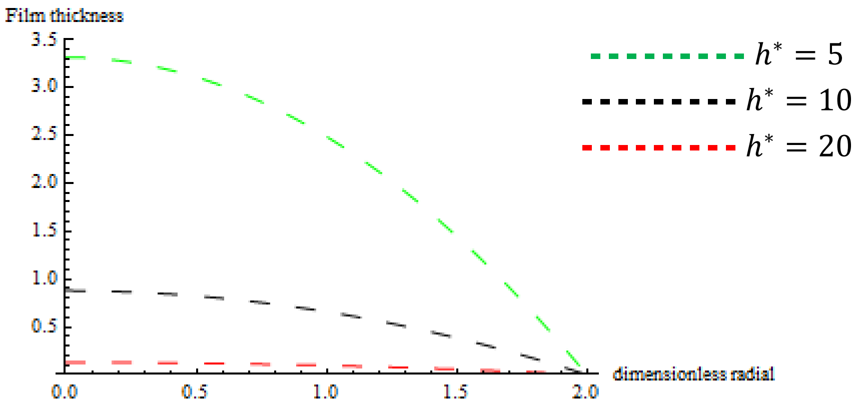
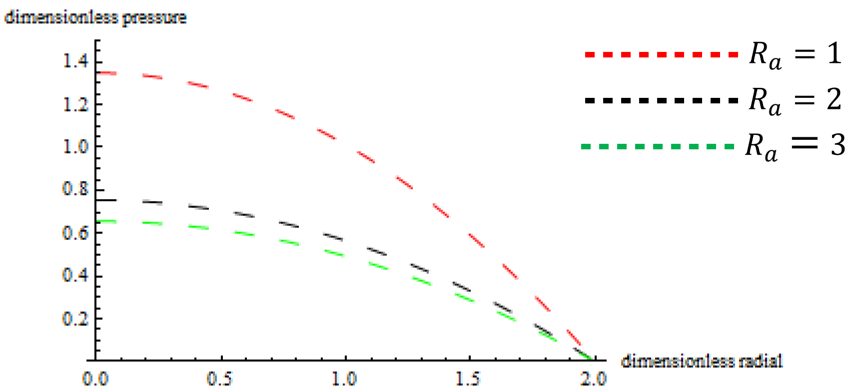
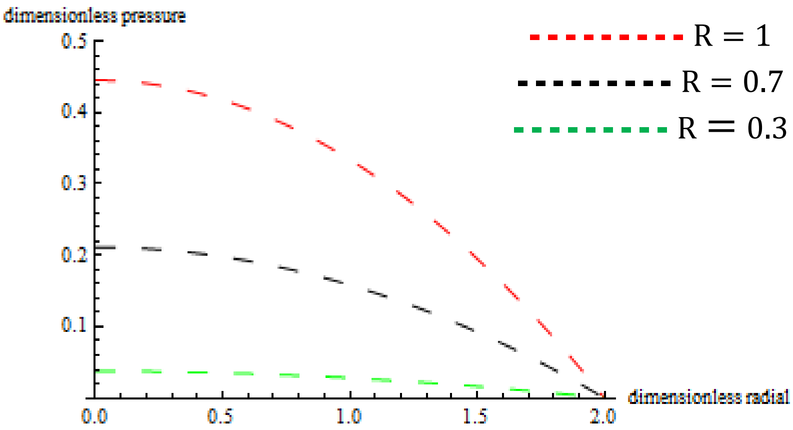


Fig (3.5): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate $(r^*)$for the different film thickness of gab between two articular parameters (h^*) $(\beta = 0.03, \mathrm{P_e} = 0.7$and $f = 3)$.



Fig(3.6): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate (r^*) for different the surface roughness parameter $(\mathrm{R_a})(\beta = 0.04, h^* = 10$and $f = 3)$.



Fig(3.7): The variation of dimensionless pressure (p^*) with dimensionless radial coordinate (r^*) for the different effective radius of curvature parameter (R) $(\beta = 0.04,\ h^* = 11$and $f = 4)$.

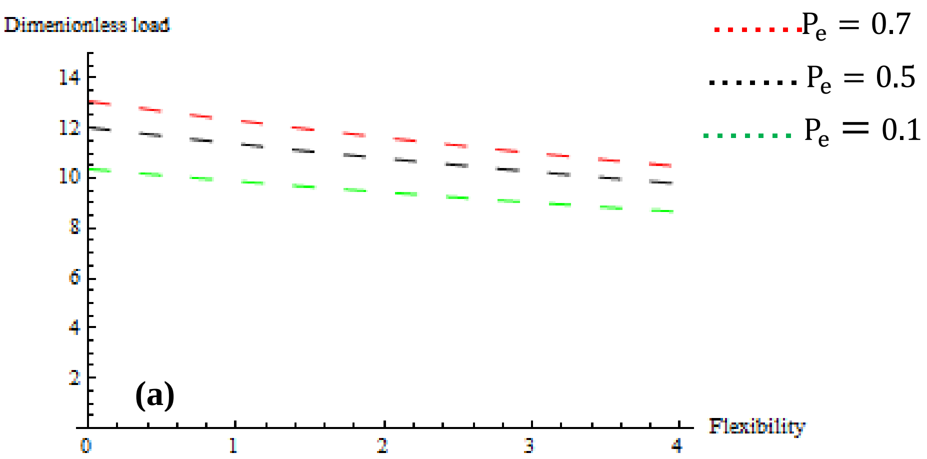
3.10.2 <u>Load Carrying Capacity</u>

The dimensionless load carrying capacity (w^*) as a function of dimensionless flexibility (f) for different values of peclet number parameters (P_e) is shown in fig (3.8). After applying equation (3.51) in the computer program, it is observed that the effect of the peclet number when film thickness $(h^* = 10)$ increases load carrying capacity, especially in the nearby areas of the position $(f\ = 0)$ because plecte number effects give a higher film pressure so in case SQL. where film thickness $(h^* = 7)$. Load carrying capacity different with time cycle appears clearly in EHL, as we have explained in the table(3.4).

The dimensionless load carrying capacity (w^*) as a function of dimensionless flexibility (f) for different dimensionless couple stress is shown in fig (3.9). It is observed that the effect of the couple stress fluid $(l^* \neq 0)$ is to increase load carrying capacity especially in the vicinity of the position $(h* = 4)$ and it has found different a couple stress with type of flexibility.

The dimensionless load carrying capacity (w^*) as a function of the effective radius of curvature (R) for different values of flexibility parameters (f) is shown in fig (3.10). It is observed that it increases the flexibility (f) of knee joint when film thickness $(h^* = 10)$ that leads to increase load carrying capacity, since they expand the tissue and thus increase the flow of fluid responsible for generating pressure. So, EHL. when film thickness $(h^* = 4)$ increased flexibility leads to increased load carrying capacity of joint, as we have explained in the table(3.5).

The effect of a film thickness of gab between two articular parameter (h^*) on the variation of (w^*) with (f) is shown in fig (3.11). It is observed that the load carrying capacity increases with decreasing values of (h^*). The effect of the surface roughness parameter (R_a) on the variations of $(w^*)$with (f) is shown in fig (3.12). It is observed that the load carrying capacity increases with decreasing values of (R_a), since the roughness of the knee indicates a decrease in joint endurance and with age, the joint loses its ability to bear weight. The effect of the effective radius of curvature parameter (R) on the variation of $(w^*)$with (f) is shown in fig (3.13). It is observed that the load carrying capacity increases with increasing values of (R).



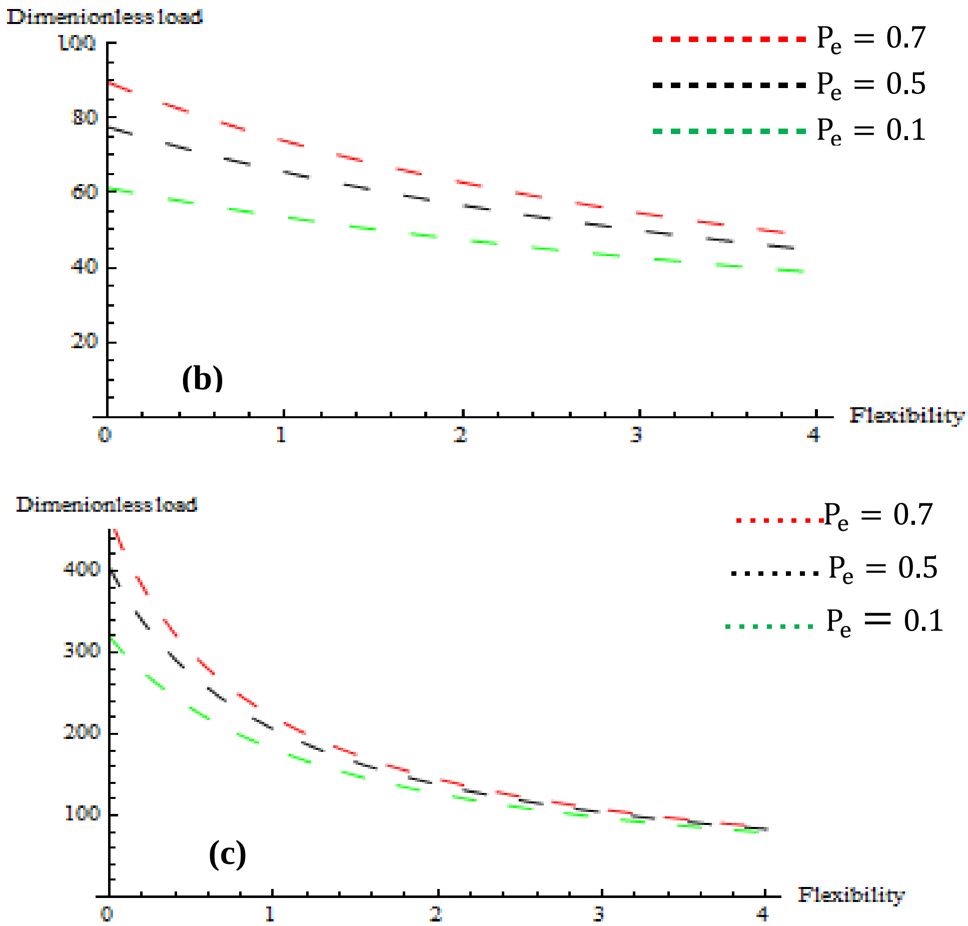


Fig (3.8): The variation of dimensionless load carrying capacity (W^*) of with flexibility (f) for different peclet number parameters (P_e) $(\beta = 0.02, R_a = 3, R = 1)$. (a) HL ,(b) SQL and (c) EHL.

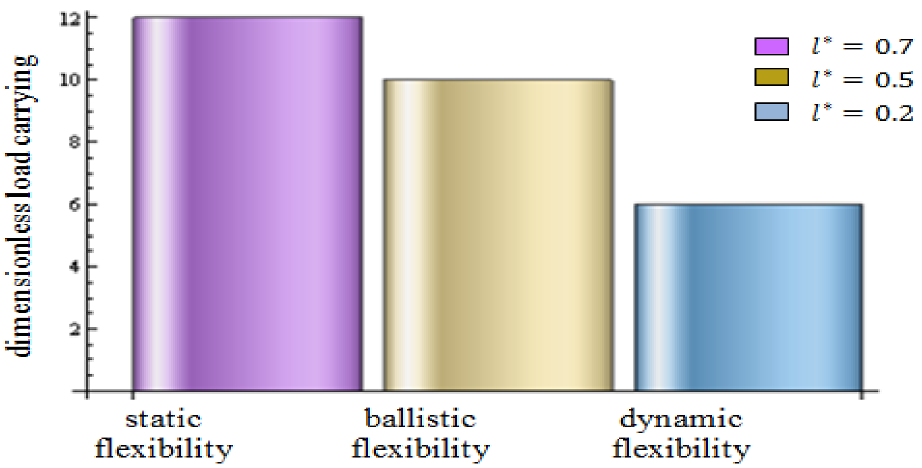


Fig (3.9): The variation of dimensionless load carrying capacity (W^*) with dimensionless flexibility (f) for different couple stress length parameter (l^*).

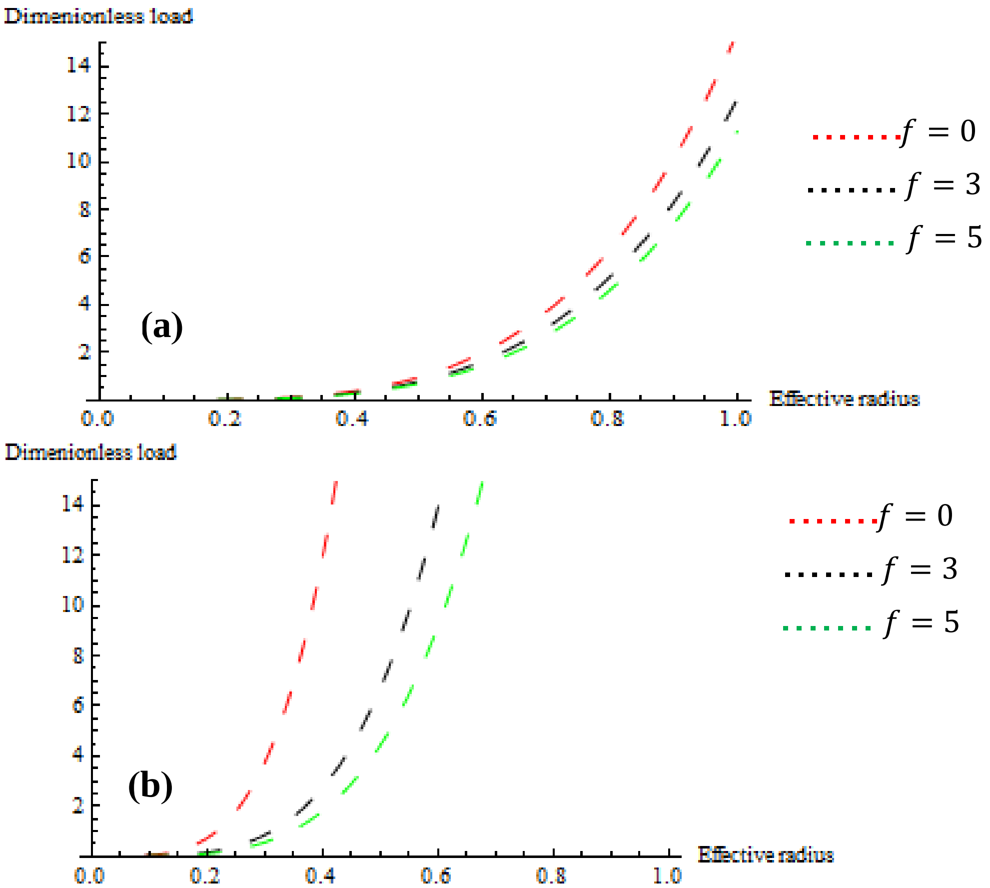


Fig (3.10): The variation of dimensionless load carrying capacity (w^*) of with a dimensionless effective radius of curvature(R) for different flexibility parameters (f) (β= 0.02, Ra=2,Pe=0.7, and h^*=10). (a) HL, (b) SQL.

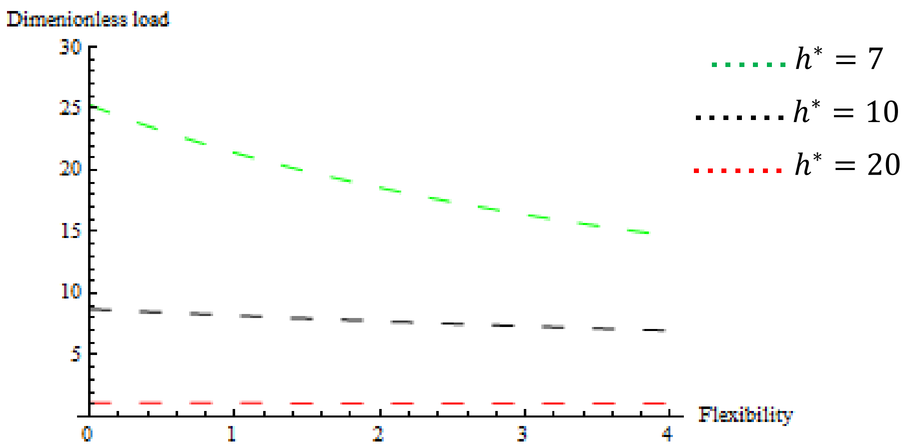


Fig (3.11): The variation of dimensionless load carrying capacity (w^*) with dimensionless flexibility (f) for the different film thickness of gab between two articular parameters (h^*) ($\beta = 0.03, P_e = 0.7, R_a = 3$, and $R = 1$).

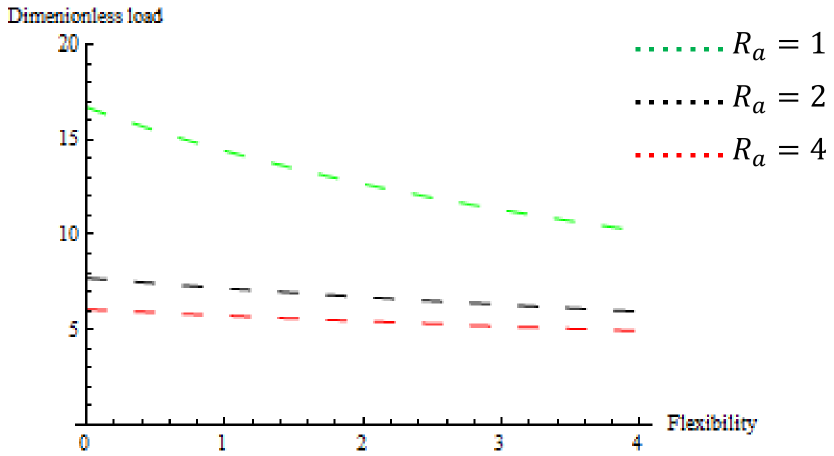


Fig (3.12): The variation of dimensionless load carrying capacity (W^*) with dimensionless flexibility (f) fo different the surface roughness parameter $(\mathrm{R_a})$ $(\beta = 0.04, \mathrm{P_e} = 0.7, h^* = 10$, and $\mathrm{R} = 1)$.

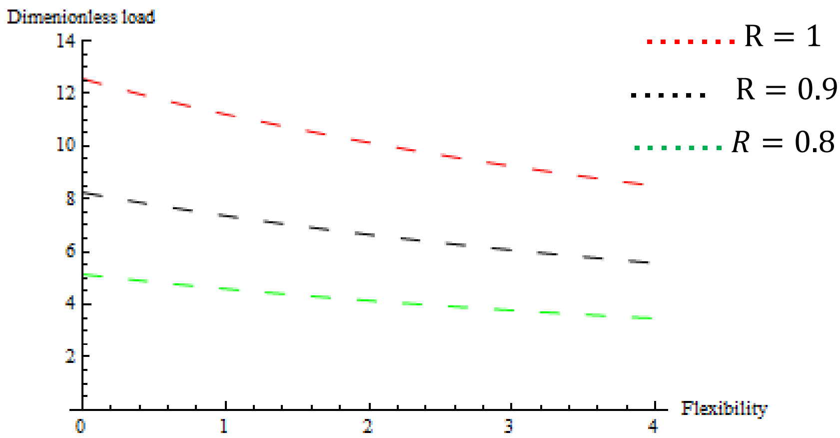


Fig (3.13): The variation of dimensionless load carrying capacity (W^*) with dimensionless flexibility (f) for the different effective radius of curvature parameter (R) $(\beta = 0.04, \mathrm{P_e} = 0.4, h^* = 11$, and $\mathrm{R_a} = 1)$.

3.10.3 Squeeze Time-Film Thickness

The height time of the film thickness is an important factor in describing squeeze film characteristics. This is the time elapsed to reduce the film thickness of different type lubrication to the minimum permissible squeeze film height. The variation of the flexibility (f) with the dimensionless height time (t^*) for the different values of peclet number parameters $(\mathrm{P_e})$ is shown in fig (3.14) by solving equation (3.54) in the computer program. In non- Newtonian lubricant the film thickness turns into minimum film thickness for different peclet number. This process needs longer time compared to the Newtonian lubricant, see table (3.6). The effect of couple stress length parameter (l^*) on the variation of $(t^*)$with (f) is shown in fig (3.15). It is observed that the time increases with increases values of (l^*). The presence of hyaluronic molecules makes the hydrodynamic pressure very high. The effect of flexibility (f) on the variations of (t^*) with (R) is shown in fig (3.16). It is observed that the squeeze time increases with decreasing values of (f) where it appears clearly in EHL more than HL, as we have explained in the table(3.7), since the type of movements performed by human make the time of transformation film thickness to minimum film thickness more. The effect of a film thickness of gab between two articular parameters (h^*) on the variation of (t^*) with (f) is shown in fig (3.17). It is observed that the time (t^*) increases with decreasing values of (h^*) in different stage lubrication (hydrodynamic, squeeze and elastohydrodynamic). The fig (3.18) explains the surface roughness parameter (R_a) on the dimensionless response time (t^*) with a value of (f). The response time of the squeeze film (t^*) decreases with increasing the value of the surface roughness parameter (R_a). The fig (3.19) explains the effective radius of curvature parameter (R) on the dimensionless response time (t^*) with a value of (f). The response time of the squeeze film (t^*) increases with increasing value the effective radius of curvature parameter (R), effect different parameters squeeze film characteristics see appendix (c).

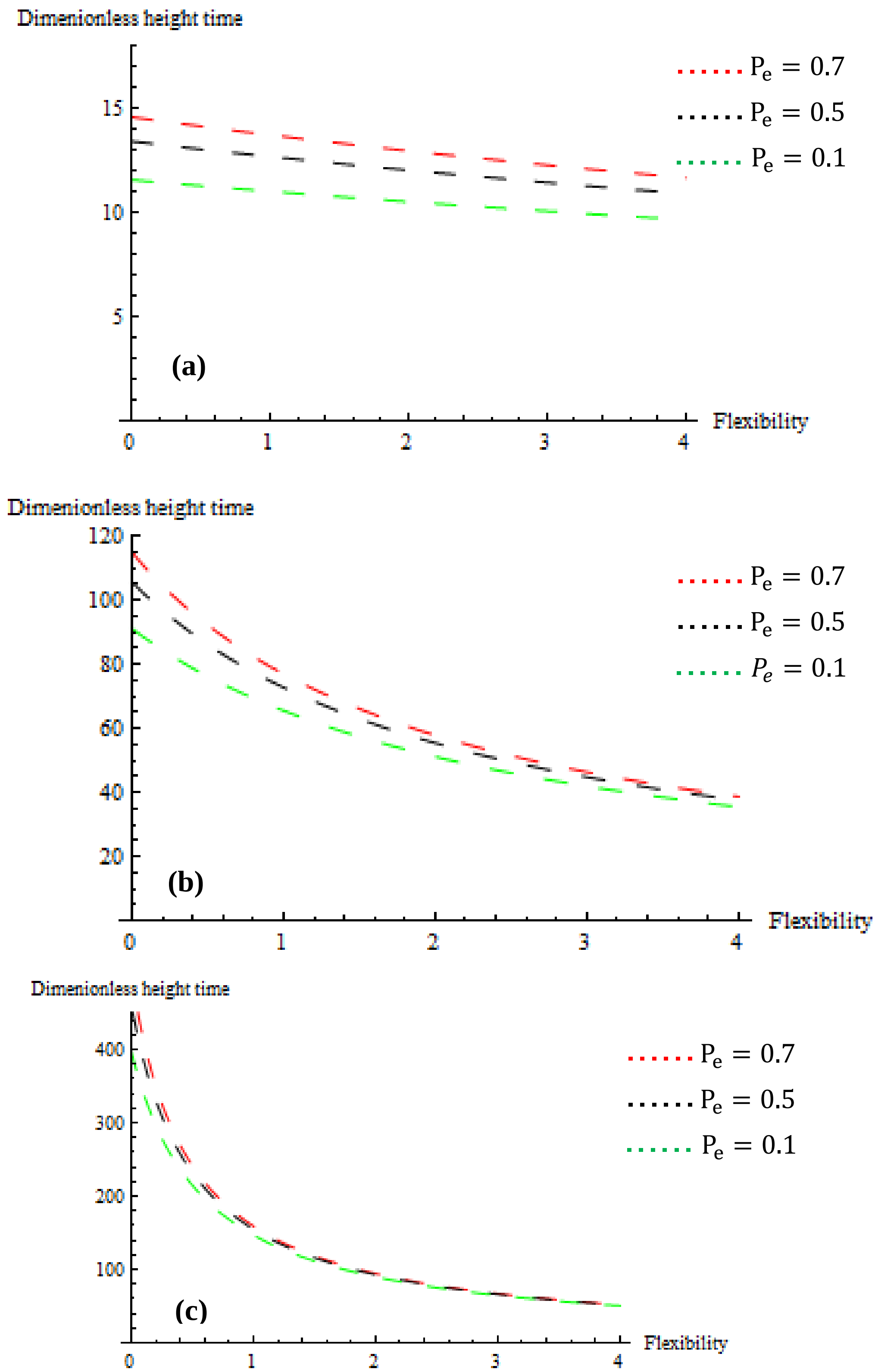


Fig (3.14): The variation of the dimensionless time (t^{*}) of with flexibility (f) for different peclet number parameters, (a)HL (b)SQL and (c) EHL.

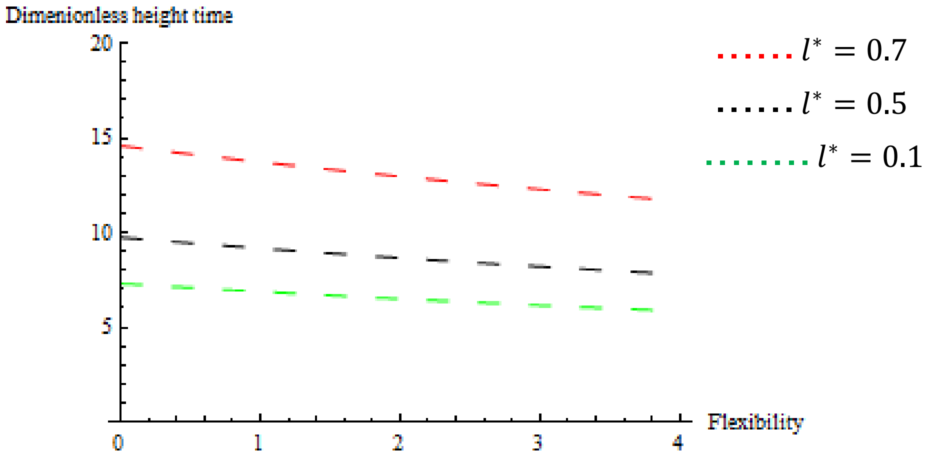
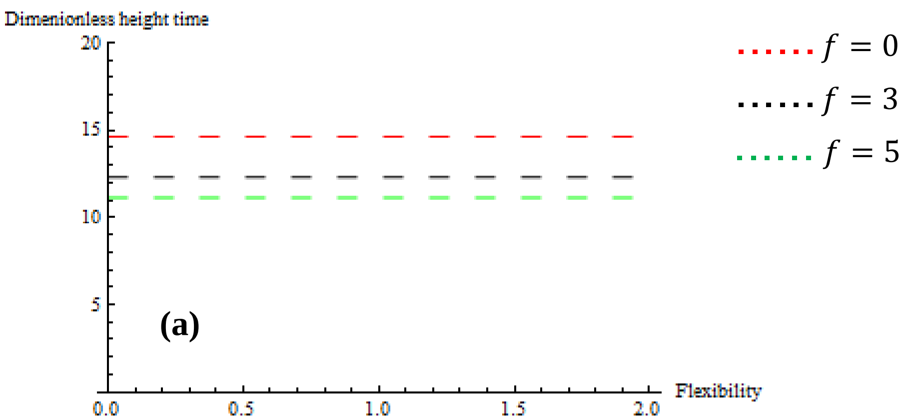


Fig (3.15): The variation of the dimensionless time (t^*) with dimensionless flexibility (f) for different couple stress length parameter (l^*).



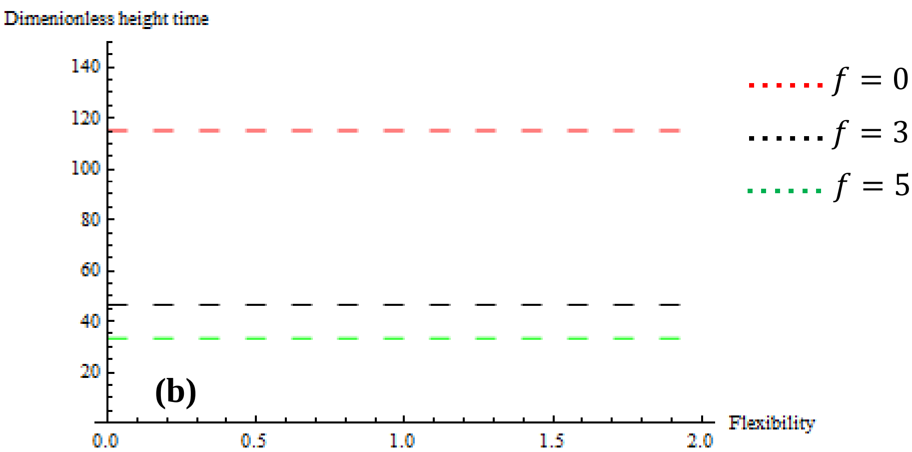
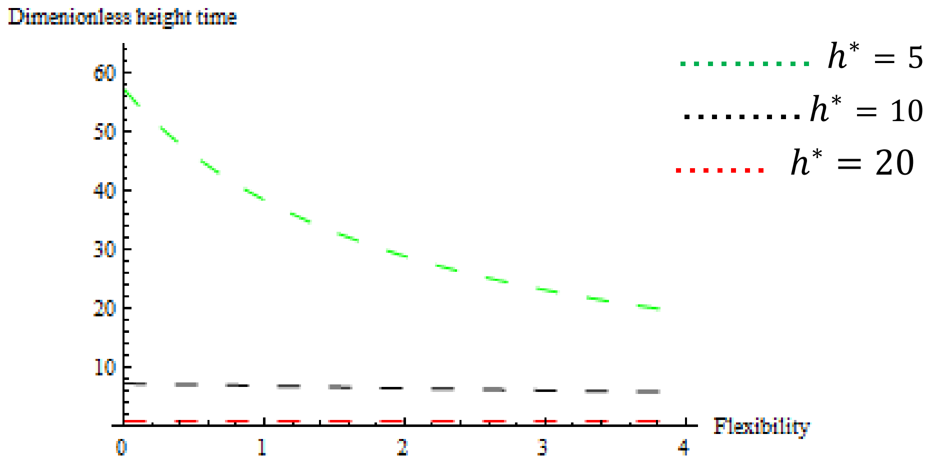


Fig (3.16): The variation of the dimensionless time (t^*) of with a dimensionless effective radius of curvature(R) for different flexibility parameters(f) ($h^*_m = 0.005, \beta = 0.02, P_e = 0.7, h^* = 10, R_a = 3$). (a) HL (b) SQL.



Fig(3.17): The variation of the dimensionless time (t^*) with dimensionless flexibility (f) for the different film thickness of gab between two articular parameters(h^*) ($h^*{}_m = 0.007, \beta = 0.04, P_e = 0.7, R_a = 3, R = 1$).

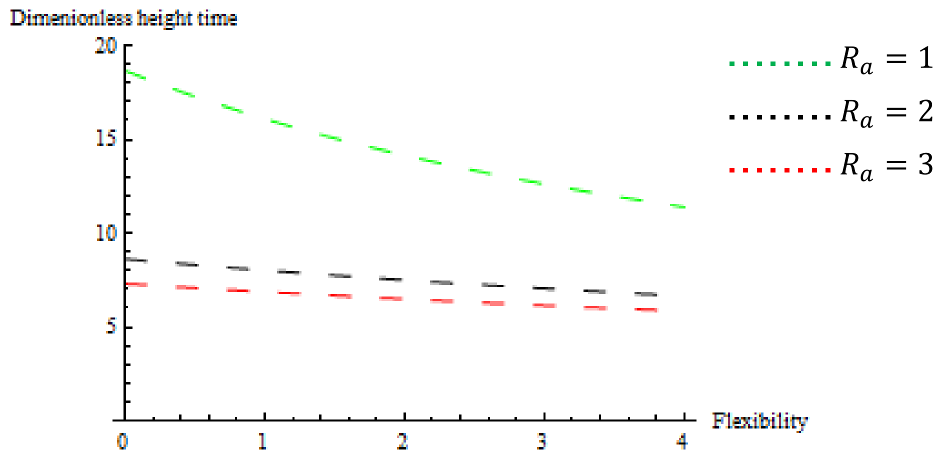


Fig (3.18): The variation of the dimensionless time (t^*) with dimensionless flexibility (f) for different the surface roughness parameter (R_a) $h^*{}_m = 0.007, \beta = 0.04, P_e = 0.7, h^* = 10, R = 1$.

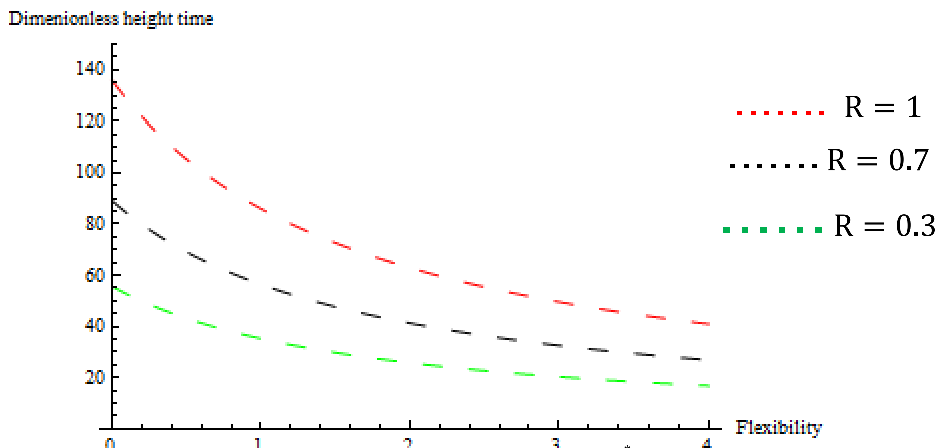


Fig (3.19): The variation of the dimensionless time (t^*) with dimensionless flexibility (f) for the different effective radius of curvature parameter (R) $h^*{}_m = 0.009, \beta = 0.04, P_e = 0.7, h^* = 10, , R_a = 2$.

3.10.4 <u>Peak Load:</u>

Fig (3.20) shows the variation of peak load (P) as a function of radial coordinate (r^*) for various value of stride length parameter (S). After applying equation (3.55), the effect peaks load to increasing with increasing value of stride length, as shown in Table (3.8) in the case of males and Table (3.9) in the case of females. This is because the length of stride makes the pressure increase on the joint. The fig (3.21) explains the effective weight (W) on the peak load (P) with a value of radial coordinate (r^*), the peak load (P) increasing with increasing value body weight (W). The fig (3.22) explains the effective pore size (β) on the response peak load on articular cartilage (P) with a value of radial coordinate (r^*)the peak load (P) decreasing with increasing value of pore size (β) explanation of this increase pore size that leads to increased flow of synovial fluid and change in the thickness of film with different types of lubrication . The fig (3.23) explains the effective flexibility (f) on the peak load (P) with a value of radial coordinate (r^*)the peak load (P) increasing with increasing value of flexibility (f).

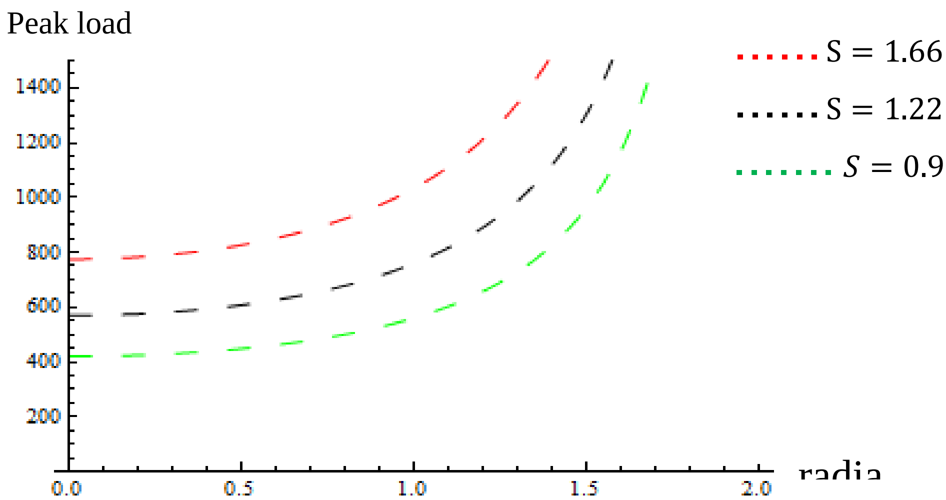


Fig (3.20): The variation of the dimensionless peak load (P) with dimensionless radial coordinate (r^*) for the different stridec length parameter (S) $\beta \to 0.02, f \to 1, P_e \to 0.7, h^* \to 10, l^* \to 0.7$).

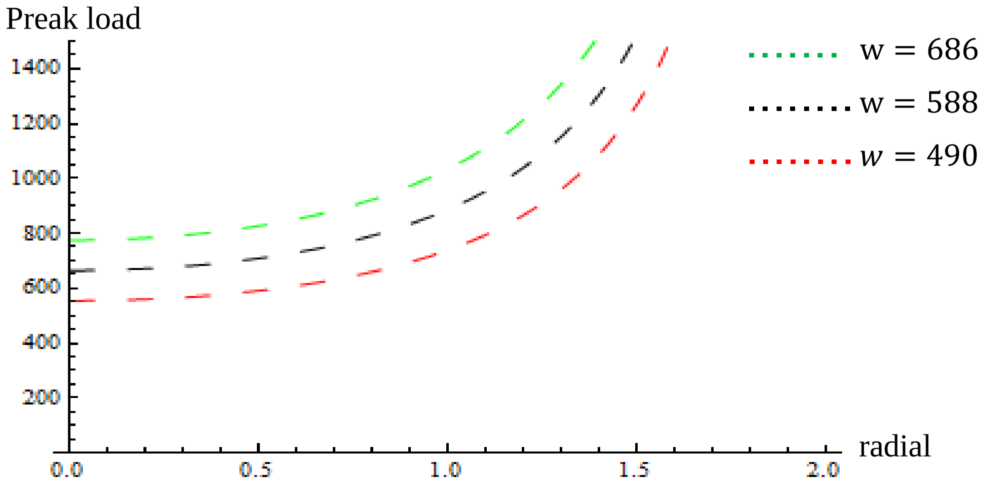


Fig (3.21): The variation of the dimensionless peak load (P) with dimensionless radial coordinate (r^*) for the different weight parameter (w) $\beta \to 0.02, f \to 1, P_e \to 0.7, h^* \to 10, l^* \to 0.7$.

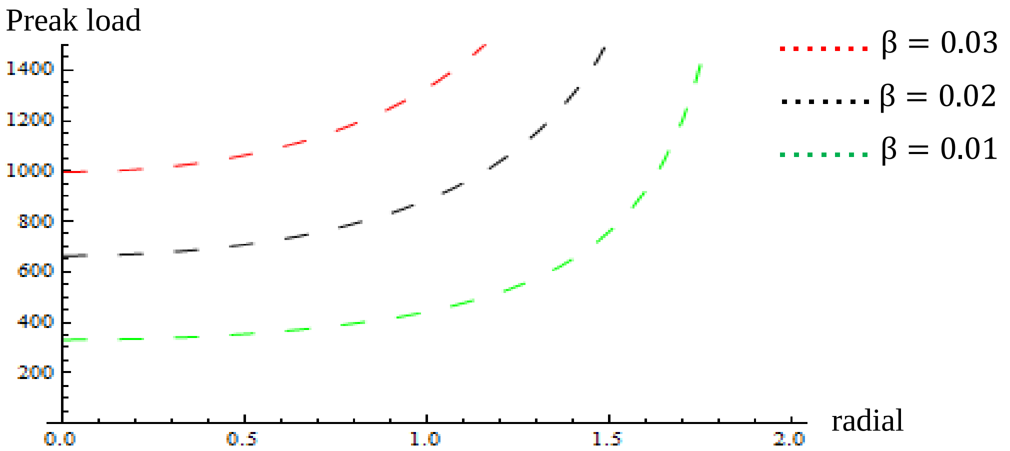


Fig (3.22): The variation of the dimensionless peak load (P) with dimensionless radial coordinate (r^*) for the different pore size parameter (β) $f \to 1, P_e \to 0.7, h^* \to 10, l^* \to 0.7, w \to 588$.

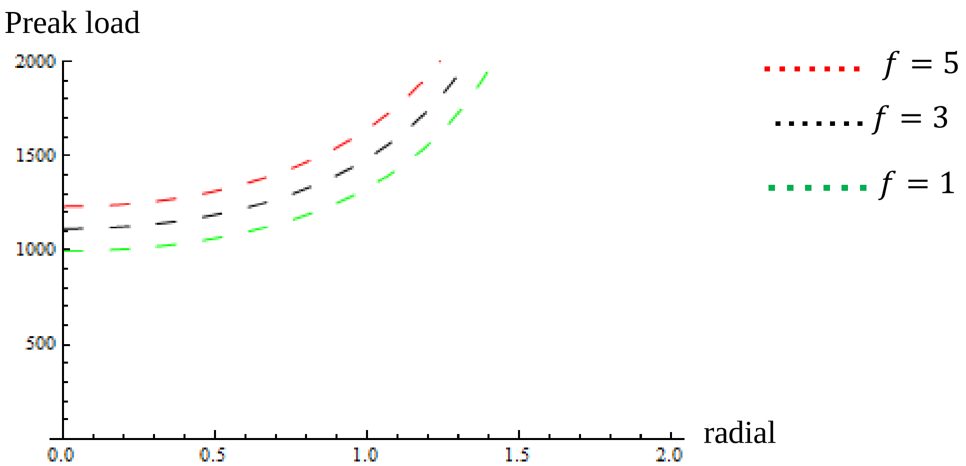


Fig (3.23): The variation of the dimensionless peak load (P) with dimensionless radial coordinate (r^*) for the different fexibility parameter (f) $\beta \to 0.03, f \to 5, P_e \to 0.7, h^* \to 10, l^* \to 0.7, w \to 588$.

3.11 Comparison

This section discusses the comparison of pressure and load carry capacity and time and peak load change in each type of lubrication. All results are obtained and discussed under various forms of relevant parameters from the tables in this section.

Table (3-2) shows the effect of changing the value of Peclet number (P_e) on the value of pressure in each type of lubrication. It is noted that decreasing in the pressure value decreases the value of Peclet number. Also, the note in every type of lubrication that is decreasing film thickness of gab between two articular (h^*) increases the pressure in each type of lubrication.

The table (3-3) displays the effect of flexibility at constant values of (P_e,β,h^*,l^* ,R_a). It is noticed that the pressure increases with decreases of flexibility. Also, the note that the pressure increases when the value of film thickness of gab between two articular (h^*) decreases of each type of lubrication.

The table (3-4) displays the effect of Peclet number (P_e) at constant values of (β,f , h^*,l^*,R_a). It is noticed that the load carry capacity (w^*) decreases with decreases of Peclet number. Also, it note is that the load carry capacity increases when the value of film thickness of gab between two articular (h^*) decreases of each type of lubrication.

The table (3-5) displays the effect of flexibility at constant values of (P_e,β, h^*,l^*,R_a). It is noticed that the load carry capacity (w^*)increase with decreases of flexibility.

The table (3-6) displays the effect of Peclet number (P_e) at constant values of (β,,f, h^*,l^*R_a). It is noticed that the time (t^*) decreases with decreases of Peclet number.

The table (3-7) displays the effect of flexibility at constant values of (P_e,β, h^*,l^*,R_a). It is noticed that the time (t^*) increase with decreases of flexibility. Also, it note is that the time increases as the value of film thickness of gab between two articular (h^*) decreases of each type of lubrication.

The table (3-8) displays the effect of Stride length (S) of male at constant values of (P_e,β, h^*,l^*,f). It is noticed that the peak load (P) decreases with decreases of Stride length.

The table (3-9) displays the effect of Stride length (S) of female at constant values of (P_e,β, h^*,l^*,f). It is noticed that the peak load (P) decreases with decreases of Stride length.

The table (3-10) displays the effect of weight (w) at constant values of (P_e,β, h^*,l^*,f). It is noticed that the peak load (P) increases with increases of weight.

The table (3-11) (appendix (C)) shows that effect different parameters on (pressure – load carrying capacity –time approach – peak load) on squeeze film characteristics. It is found that plecte number has more effect in pressure while in load it appears a film thickness of articular cartilage in time approach in surface roughness more effect. Finlay, flexibility is very important on peak load.

Table (3.2): Peclet number (P_e) relationship of pressure (p^*) at $\beta = 0.02, f = 2, h^* = 10, l^* = 0.7$ hydrodynamic , $\beta = 0.02, f = 2, h^* = 5, l^* = 0.7$ squeeze film, $\beta = 0.02, f = 2, h^* = 2, l^* = 0.7$ elasto hydrodynamic.

Peclet number	Pressure		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
P_e	p^*	p^*	p^*
0.7	1.33261	5.9496	11.1573
0.6	1.28302	5.82197	11.1233
0.5	1.23699	5.6997	11.0896
0.4	1.19415	5.58246	11.056
0.3	1.15417	5.46995	11.0227
0.2	1.116679	5.36189	10.9895
0.1	1.08175	5.25801	10.9566

Table(3.3): Flexibility (f) relationship of pressure (p^*) at $\beta = 0.03, P_e = 0.7, h^* = 16, l^* = 0.7$ hydrodynamic, $\beta = 0.03, P_e = 0.7, h^* = 10, l^* = 0.7$ squeeze film, $\beta = 0.03, P_e = 0.7, h^* = 5, l^* = 0.7$ elasto hydrodynamic

Flexibility	Pressure		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
f	p^*	p^*	p^*
5	0.22708	0.76155	2.2747
4	0.230349	0.799609	2.65169
3	0.233714	0.841672	3.17846
2	0.237178	0.888406	3.9664
1	0.240747	0.940635	5.27376
0	0.244425	0.999389	7.86671

Table (3.4): Peclet number (P_e) relationship of load carry capacity (w^*) at $\beta = 0.02, h^* = 10, l^* = 0.7, R_a = 3, R = 1$ hydrodynamic, $\beta = 0.01, h^* = 7, l^* = 0.7, R_a = 2, R = 1$ squeeze film, $\beta = 0.01$, $h^* = 4, l^* = 0.7, R_a = 2, R = 1$elasto hydrodynamic.

Peclet number	load carry capacity (w^*)		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
P_e	w^*	w^*	w^*
0.7	12.3066	73.7527	220.608
0.6	11.8228	69.3573	212.882
0.5	11.3756	65.4564	205.642
0.4	10.961	61.9709	198.895
0.3	10.5755	58.8379	192.577
0.2	10.2163	56.0064	186.648
0.1	9.88062	53.4349	181.073

Table (3.5): Peclet number (P_e) relationship of load carry capacity (w^*) at $\beta = 0.02, P_e = 0.7, h^* = 10, l^* = 0.7, R_a = 3$ hydrodynamic , $\beta = 0.01, P_e = 0.7, h^* = 7, l^* = 0.7, R_a = 2$ squeeze film, $\beta = 0.01$, $P_e = 0.7, h^* = 4, l^* = 0.7, R_a = 2$ elasto hydrodynamic.

Flexibility	load carry capacity (w^*)		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
f	w^*	w^*	w^*
5	11.2705	43.2657	70.9871
4	11.9119	48.2522	85.4808
3	12.6306	54.5378	107.411
2	13.4416	62.7063	144.478
1	14.364	73.7527	220.608
0	15.4222	89.5231	466.334

Table (3.6): Peclet number (P_e) relationship of time (t^*) at $\beta = 0.02, h^* = 10,$ $l^* = 0.7, R_a = 3, R = 1, h^*{}_m = 0.009$ hydrodynamic, $\beta = 0.02,$ $h^* = 5, l^* = 0.7, R_a = 3, R = 1, h^*{}_m = 0.009$ squeeze film, $\beta = 0.02,$ $h^* = 3, l^* = 0.7, R_a = 3, R = 1, h^*{}_m = 0.009$ elasto hydrodynamic.

Peclet number	Time (t^*)		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
P_e	t^*	t^*	t^*
0.7	13.7192	76.9178	159.263
0.6	13.1798	74.7393	157.094
0.5	12.6813	72.6809	154.984
0.4	12.2191	70.7328	152.93
0.3	11.7894	68.8864	150.929
0.2	11.3889	67.1339	148.98
0.1	11.0147	65.4684	147.08

Table(3.7): Flexibility (f) relationship of time (t^*) at $\beta = 0.02, P_e = 0.7,$ $h^* = 10, l^* = 0.7, R_a = 3, h^*{}_m = 0.005$ hydrodynamic, $\beta = 0.01,$ $P_e = 0.7, h^* = 5, l^* = 0.7, R_a = 2, h^*{}_m = 0.005$ squeeze film, $\beta = 0.01,$ $P_e = 0.7, h^* = 3, l^* = 0.7, R_a = 2, h^*{}_m = 0.005$ elasto hydrodynamic.

Flexibility	Time (t^*)		
	Swing phace	Stance phace	
	hydrodynamic (1)	squeeze film (2)	elasto hydrodynamic (3)
f	t^*	t^*	t^*
5	11.152	69.6872	86.8677
4	11.7094	81.861	106.635
3	12.3253	99.1883	138.05
2	13.0097	125.82	195.704
1	13.7745	172.004	336.049
0	14.6349	271.752	1188

Table(3.8): Stride length (S) of male relationship of peak load (P) at ($\beta = 0.02, f = 2, P_e = 0.7, h^* = 10, l^* = 0.7$).

Stride length(s) (Male)	Peak load(P)
1.66	822.409
1.56	722.866
1.51	748.095
1.47	728.278
1.38	668.827

Table(3.9): Stride length (S) of female relationship of peak load (P) at ($\beta = 0.02, f = 2, P_e = 0.7, h^* = 10, l^* = 0.7$).

Stride length (Female)	Peak load(P)
1.61	797.638
1.51	748.095
1.47	728.278
1.42	703.507
1.35	668.827

Table (3.10): weight (w) relationship of peak load (P) at ($\beta = 0.02, f = 2, P_e = 0.7, h^* = 10, l^* = 0.7$).

Weight(w)	Peak load(P)
490	587.435
588	704.922
686	822.409
784	939.896
882	1057.38

3.12 Conclusions

The effects of couple stresses on the pressure film are presented in the synovial knee joint on the basis of momentum equations and continuity equation. The modified Reynolds equation, which governs the squeeze film, numerically solved using "(Wolfram Mathematic 8)". We obtained the following results:

1. The effect of couple stress is to increase when film pressure increase, load carrying capacity increase, time in one side increase and decrease friction coefficient on the other side significantly as compared to Newtonian case.
2. The effect of film thickness parameters is to increases when film pressure decrease, load carrying capacity increase and time decrease.
3. The effect decrease flexibility parameters is to increases in film pressure, time, and increases of flexibility parameters is to increases load carrying capacity.

Chapter Four

Non-Newtonian Fluid with Elastic Deformation on Synovial Effect Knee Joint

Introduction

In this chapter, the theoretical analysis of the problem is presented through a mathematical module that has been depended on a Navier-Stokes equations, module studied effective elastic deformation of articular cartilage surface on squeeze film characteristic. A mathematical module has been developed to estimate the pressure distribution, load carry capacity, friction force and coefficient of friction of synovial human knee joint. The governing equation for elastic deformation is solved, which results in obtaining a mathematical equation that measures the pressure distribution. Basing on friction force and load carrying capacity to being completed the calculated values coefficient of friction. The wear in layers of articular cartilage has been calculated, parameters effective on wear and elastic deformation has been studied in normal and disease knee joint relationship friction force and wear of layers articular cartilage under condition lubrication is the discussion. The results obtained of the elastic deformation reduce wear of layers so effects on the performance of synovial human knee joint from the side medical and dynamics very important have been compared with non- elastic (Osteoarthritic).

4.1. <u>Elastic Deformation</u> [27]

A sufficient load is applied to a material or articular cartilage, it will cause the material to change shape. This change in shape is called "deformation". A tentative shape change that is self- reverberate after the force is removed, so that the object returns to its original shape, is called "elastic deformation". In other words, elastic deformation is a change in the shape of a material at low stress that is recoverable after the stress is removed.

Elastic deformations can be approximated applied Hertzian theory for a static dry contact and it is defined as :

$$a = \sqrt[3]{\frac{3\,W\,R}{4\,E\,h^3}} \tag{4.1}$$

Where $W, R,$ h and E are the load applied on metal or articular cartilage, radius film thickness and Young's modulus respectively when $E, h \neq 0$.

<u>Properties of Elastic deformation</u> [13]

1. Reversible.
2. Depends on the initial and final states of stress and strain.
3. No strain hardening effects

4.2. <u>Governing Equation</u>

It is suitable and allowable to represent synovial fluid flow in the gap between two bone surfaces (femur- tibia), by two planes configuration for the purpose of analysis; the plane solid is assumed to be rigid and the opposing component consists of a rigid core (representing the bone) covered by a layer of porous, elastic material (representing the articular cartilage), synovial fluid serves as a lubricant (preventing bone-to-bone friction), see Fig (4.1). The journal of eccentricity ratio ϵ with sliding motion w on the basis of the preceding assumption for the quasi-static squeeze thin film the continuity equation and Navier- Stokes equations is reduced to:

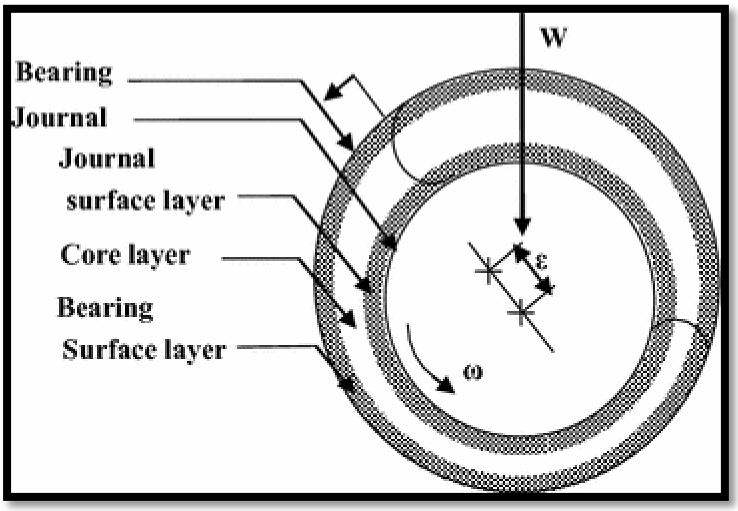


Fig (4.1): The physical configuration of partial journal bearing.

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial r}\varepsilon - av \tag{4.2}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial r} \tag{4.3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{4.4}$$

Where ε permeability, a elastic deformation, (u, w) are the velocity components of the lubricant in r and z directions respectively, p is pressure, μ dynamic viscosity. The Boundary conditions for the velocity component at the surfaces of the plate and spheres are:

$$v = 0 \qquad at \qquad z = 0 \tag{4.5}$$

$$v = 0 \qquad at \qquad z = h \tag{4.6}$$

If a lubricating exists between the spherical and plane, h_m is minimum film thickness and R is a curvature radius, then the film thickness h can be written as follows:

$$h = h_m + \frac{r^2}{2R} \tag{4.7}$$

4.3. Derivation of Velocity

Integrating is Navier- Stokes equations (4.3) twice with respect to z.

$$\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) = \frac{1}{\mu}\ \frac{\partial p}{\partial r} \tag{4.8}$$

$$\frac{\partial v}{\partial z} = \frac{1}{\mu}\ \frac{\partial p}{\partial r} \int dz \tag{4.9}$$

$$\frac{\partial v}{\partial z} = \frac{1}{\mu}\ \frac{\partial P}{\partial r}\ z + c_1 \tag{4.10}$$

$$v = \frac{1}{\mu}\ \frac{\partial p}{\partial r}\ \frac{z^2}{2} + c_1 z + c_2 \tag{4.11}$$

And using the boundary conditions the tangential component of the fluid velocity in the film region satisfies:

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial r}(z^2 - hz) \tag{4.12}$$

Substituting the tangential component of the fluid velocity in the film region equation (4.12) into the Navier- Stokes equations (4.2) gives

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{2\mu}\frac{\partial p}{\partial r}\left(2\varepsilon - \frac{a}{2\mu}\frac{\partial p}{\partial r}z^2 + \frac{a}{2\mu}\frac{\partial p}{\partial r}hz\right) \tag{4.13}$$

To obtain sliding motion u integrating equations (4.13) twice with respect to z.

$$\frac{\partial u}{\partial z} = \frac{1}{2\mu}\frac{\partial p}{\partial r}\ \int(2\varepsilon - az^2 + ahz)\, dz \tag{4.14}$$

$$\frac{\partial u}{\partial z} = \frac{1}{2\mu}\frac{\partial p}{\partial r}\ \left(2\varepsilon z - a\frac{z^3}{3} + ah\frac{z^2}{2}\right) + c_1 \tag{4.15}$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r}\ \int 2\varepsilon z - a\frac{z^3}{3} + ah\frac{z^2}{2} + c_1\ \ dz \tag{4.16}$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r}\ \ [\ \varepsilon z^2 - a\frac{z^4}{12} + ah\frac{z^3}{6} + c_1 z +\ c_2\ ] \tag{4.17}$$

Applying the no- slip condition on both surfaces radially:

$$u = 0 \qquad at \qquad z = 0 \tag{4.18}$$

$$u = 0 \qquad at \qquad z = h \tag{4.19}$$

Integrating (4.17) with the above condition the component of the fluid velocity in the region becomes:

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\varepsilon z^2 - a\frac{z^4}{12} + a\frac{hz^3}{6} - \varepsilon hz - a\frac{h^3}{12}z\right] \tag{4.20}$$

The boundary conditions for velocity component $w(r,z)$ at the surfaces of the plate and sphere are:

$$w(r,0) = 0 \qquad and \qquad w(r,h) = \frac{\partial h}{\partial t} \tag{4.21}$$

To determine squeeze action (w) of lubricant inside the gap between articular cartilage during swing phase and stance phase integrates the continuity equation (4.4) with respect to z.

$$\frac{\partial w}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r}\, u(r,z)\, r \tag{4.22}$$

$$w = \frac{-1}{r}\,\frac{\partial}{\partial r} r \int u(r,z)\, dz \tag{4.23}$$

$$w = \frac{-1}{r}\,\frac{\partial}{\partial r}\; r\,\frac{1}{2\mu}\,\frac{\partial p}{\partial r}\left[\frac{\varepsilon z^3}{3} - \frac{a\,z^5}{60} + \frac{ah\,z^4}{24} - \frac{\varepsilon\, hz^2}{2} - \frac{ah^3z^2}{24}\right] + A \tag{4.24}$$

Where A is the integration constant using the boundary condition $w(r,0) = 0$ on the solution (4.24); hence, we obtain the integration constant= 0. Now substitution boundary condition $w(r,h) = \frac{\partial h}{\partial t}$. Then the modified Reynolds equation governing the film pressure:

$$\frac{\partial h}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} \cdot r\frac{1}{2\mu}\frac{\partial p}{\partial r}\,[g(\varepsilon, h)] \tag{4.25}$$

Where:

$$g(\varepsilon, h) = \frac{\varepsilon h^3}{3} - \ \ a\frac{h^5}{60} - \frac{\varepsilon h^3}{2} \tag{4.26}$$

4.4. A Module Elastic Deformation Analysis in a Knee Joint

Introducing a mathematical module is to take into account elastic deformation (α) and wear layers of articular cartilage during swing phase and stance phase. The conjunction between the femur and the tibia in the knee joint can be represented by an equivalent spherical elastic porosity near a plane, the film thickness of types lubrication is taken from the tables in chapter two and discussed its influence this module on pressure distribution, load carry capacity, friction force.

4.4.1. Squeeze Film pressure of The Knee Joint [23]

Introducing the non-dimensional parameters in the governing equations for the pressure is of importance for both theoretical and computational purposes. It is also of importance to present the various parameters in the lubrication system, in non-dimensional form.

$$\bar{p} = -\frac{ph^2}{\mu R\frac{\partial h}{\partial t}} \qquad \bar{\varepsilon} = \frac{\varepsilon}{R^2} \qquad \bar{\mathrm{h}} = \frac{\mathrm{h}}{\mathrm{R}}$$
$$\bar{r} = \frac{r}{R} \qquad \beta = \frac{R}{h_0} \qquad \epsilon = \frac{e}{c} \tag{4.27}$$

Where e = module elastic. c = radial clearance inches

ϵ = eccentricity ratio β = pore size

r = radial h_m= minimal film thickness

Apply equation (4.27) into equation (4.25) it obtained the final form of dimensionless modified Reynolds equation as:

$$\frac{2}{\beta^2 R^2} = \frac{\partial}{\partial \bar{r}}\left[g(\varepsilon, \bar{h})\frac{\partial \bar{p}}{\partial \bar{r}}\right] \tag{4.28}$$

Where:

$$g(\varepsilon, \bar{h}) = R^5[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5] \tag{4.29}$$

The boundary conditions for the pressures in the film and bearing are:

$$\begin{aligned} &\bar{p} = 0 \quad && at \quad && \bar{r} = 1 \\ &\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \quad && at \quad && \bar{r} = 0 \end{aligned} \tag{4.30}$$

4.4.2. <u>Pressure Distribution</u>

To obtain the solution to the modified Reynolds equation. We integrate the equation (4.28):

$$\int \frac{2}{R^7\beta^2[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]}\ d\bar{r} = \frac{\partial \bar{p}}{\partial \bar{r}} \tag{4.31}$$

$$\frac{2\bar{r}}{\beta^2 R^7[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]} + A = \frac{\partial \bar{p}}{\partial \bar{r}} \tag{4.32}$$

Where A is the integration constant, by using the boundary condition, it obtained the integration constant $A = 0$, then the squeeze film pressure is given by:

$$\frac{1}{\beta^2 R^7[-\frac{\bar{\varepsilon}}{6}\bar{h}^3-\frac{a}{60}\bar{h}^5]}\int_{\bar{r}}^{1} 2\bar{r} \quad d\bar{r}=\bar{p} \tag{4.33}$$

$\beta \neq 0$ with the film pressure known, the squeeze film characteristics can be calculated now.

4.4.3. <u>Load Carrying Capacity</u>

Considering the pressure distribution on the journal bearing during phases of gait (swing phase – stance phase) for normal and disease human knee joint, we can compute the load - carrying capacity for joint by integrating pressure distribution.

$$w = 2\pi\int_0^1 a\,pR\,dr \tag{4.34}$$

Introducing the dimensionless quantity:

$$\bar{W} = -\frac{w\varepsilon}{\mu u c R^2} \tag{4.35}$$

The dimensionless load carrying capacity is given by

$$\bar{W} = 2\pi\int_0^1 \bar{p}\,a\,d\bar{r} \tag{4.36}$$

Although the values of the dimensionless pressure distribution ($\bar{p}$) and the dimensionless load- carrying capacity ($\overline{W}$) in equations (4.33) and (4.36) cannot be calculated by direct integration, they could be numerically evaluated by the methods of (power series, Gaussian Quadrature and Simpson method). In this study, the power series is used and both terms have been substituted with the well-fitting approximate function which has been substituted with power series, then integrating the output result from power series to get the dimensionless load. All mathematical analyses and output resulting curves are carried out by “wolfram Mathematica (8)”. This is a computational software program used in scientific, engineering, and mathematical fields and other areas of technical computing. The equation (4.36) becomes:-

$$\overline{W} = \frac{240\,\pi\, a}{3\beta^2 \bar{h}^2 R^2(-\bar{\varepsilon}+a\bar{h}^2\ )} \tag{4.37}$$

4.4.4. Friction Force

Shear tress τ on the surface for a Newtonian fluid is $\mu(\frac{\partial u}{\partial z})$ hence total friction F is:

$$F = \int \tau\, dr = L\,\mu \int \frac{\partial u}{\partial z}\, dr \tag{4.38}$$

$$\frac{\partial u}{\partial z} = \frac{\partial p}{\partial r}\left(\frac{z}{\mu} - \frac{L}{2\mu}\right) + \frac{u}{e} \tag{4.39}$$

Where L is the length of articular cartilage to knee Joint. The frictional force needed on two articular cartilage surface $z = h\ \ and\ \ z = 0$

$$\frac{F}{L} = \int_0^2 \left(-\frac{\partial p}{\partial r}\frac{L}{2} + \frac{u}{e}\mu\right) dr \tag{4.40}$$

$$\frac{F}{L} = \int_0^2 - \frac{\partial p}{\partial \bar{r}}\frac{L}{2}\, d\bar{r} + \frac{u\mu\, R}{e}\, d\bar{r} \tag{4.41}$$

$$\frac{F}{Lu\mu\, R} = \int_0^2 \frac{\partial}{\partial \bar{r}}\left(-\frac{p}{u\mu\, R}\right)\frac{L}{2} d\bar{r} + \frac{1}{e}\, d\bar{r} \tag{4.42}$$

The dimensionless friction force is given by:

$$\bar{F} = \frac{Fc}{\mu u R L} \tag{4.43}$$

$$\bar{F} = \int_0^2 \frac{\partial}{\partial \bar{r}}\left(\frac{phc}{\mu u R}\ \frac{\bar{L}}{2}\right) d\bar{r} + \frac{1}{\epsilon}\, d\bar{r} \tag{4.44}$$

$$\bar{F} = \int_0^2 \frac{(1-\bar{r}^2)\bar{L}}{2\beta^2 R^7[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]} d\bar{r} + \frac{1}{\epsilon} d\bar{r} \tag{4.45}$$

$$\bar{F} = \frac{2\,\bar{L}\epsilon}{6\,\beta^2 R^7 \epsilon[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]} + \frac{2\,\beta^2 R^7[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]}{6\,\beta^2 R^7 \epsilon[-\frac{\bar{\varepsilon}}{6}\bar{h}^3 - \frac{a}{60}\bar{h}^5]} \tag{4.46}$$

4.4.5. Coefficient of Friction

Coefficient of friction is a value that shows the relationship between the force of friction between two particles of articular cartilages and the normal reaction between the particles of articular cartilages during (standing up/sitting down- Knee bend- one legged stance). Coefficient of friction is given by:

$$C_f = \frac{\bar{F}}{\bar{W}} \tag{4.47}$$

Substituting for ($\bar{W}$) and ($\bar{F}$) from equation (4.37) and equation (4.46) respectively in equation (4.47) to get the final expression of the non-dimensional coefficient of friction:

$$C_f = \frac{6\,\bar{L}\epsilon\beta^2\bar{h}^2R^2(-\bar{\varepsilon}+a\bar{h}^2\,)}{6\,\beta^2R^7\epsilon[-\frac{\bar{\varepsilon}}{6}\bar{h}^3-\frac{a}{60}\bar{h}^5]240\,\pi\,a} + \frac{6\,\beta^2R^7[-\frac{\bar{\varepsilon}}{6}\bar{h}^3-\frac{a}{60}\,\bar{h}^5]\beta^2\bar{h}^2R^2(-\varepsilon+a\bar{h}^2\,)}{6\,\beta^2R^7\epsilon[-\frac{\bar{\varepsilon}}{6}\bar{h}^3-\frac{a}{60}\bar{h}^5]240\,\pi\,a} \tag{4.48}$$

4.4.6. Wear [13]

Wear of cartilage layers is the result of loss of articular cartilage elastic deformation and flexibility, synovial human knee joint is most joint exposed to wear due to the constant pressure on the joint. Wear layers (superficial zone - middle zone -deep zone) that contain particles generated from articulations are progressive into the synovial fluid and may be implicated in the biological activities in the knee, articular cartilage has a limited repair strength. If the rejuvenation cannot hold pace with the degradation of the matrix due to reasons such as aging, osteoarthritis (OA) will occur general formula to determine wear of layers be:

$$\omega = \frac{W\ h}{F\ C} \tag{4.49}$$

4.5. Results and Discussion

On the basis of quasi-static squeeze film equations, this paper discusses effectiveness of elastic deformation of the articular cartilage on squeeze film characteristics in synovial human knee joint in case (normal -disease) joint, and determine wear layers of articular cartilage (superficial zone – middle zone – deep zone) in young and elderly people.

4.5.1. Squeeze Film Pressure

Fig (4.2) illustrates dimensionless the pressure distribution on layers of articular cartilage generated by squeeze film action during the normal walk with different values of film thickness for (hydrodynamic – squeeze- elasto hydrodynamic) lubrication. It is found that dimensionless pressure distribution ($\bar{p}$) increases and becomes more with decreasing value of film thickness parameters ($h_0^* = 3$) inversely, increaseing film thickness ($h_0^* = 5$) that leads to decreasing in dimensionless pressure distribution. The percentage rate of increase in dimensionless pressure in swing phase is approximate 5% at (a = 0.044,$\varepsilon = 1.5 \times 10^{-18}$), while it has been found that the percentage rate of increase in dimensionless pressure ($\bar{p}$) instance phase is approximately 80% at (a = 0.044). The different dimensionless pressure distribution as a function of the radius of the sphere for different value elastic deformation is seen in fig (4.3).It is observed in which elastic deformation is small, it means that the hydrodynamic pressure between the articular cartilage is very high; this is protected for layers of articular cartilage (superficial zone – middle zone – deep zone), when low hydrodynamic pressure inside gap leading to increasing elastic deformation of superficial zone in health knee joint. Table (4.1) shows the relationship between pressure distribution and elastic deformation . In fig (4.4), normal articular cartilage of knee joint is a higher water content in articular cartilage, this provides an increased stiffness and lower hydraulic permeability to knee cartilage, The result is a higher pressure distribution that could protect knee cartilage from continuous different movements that determine the percentage rate of increase in dimensionless pressure that is approximately (80% at $\varepsilon = 1.5\times 10^{-18}$) in knee joint synovial fluid of patients suffering from osteoarthritis (OA) that are determined and apparent synovial permeability (SP) to each protein calculated. The results show that the normal synovial are significantly more permeable (ε = 1.5×10^{-18}) than the osteoarthritic synovial Relation between size pore of articular cartilage and pressure was shown in fig (4.5). When pore size is small then the pressure on at the cartilage edge is high; therefore, synovial fluid in cell synovial flows free, it determines the percentage rate of increase in dimensionless pressure that is approximately (80% at $\beta = 0.01$) of stance phase while swing phase observed decrease dimensionless pressure that is approximately (16% at $\beta = 0.05$).

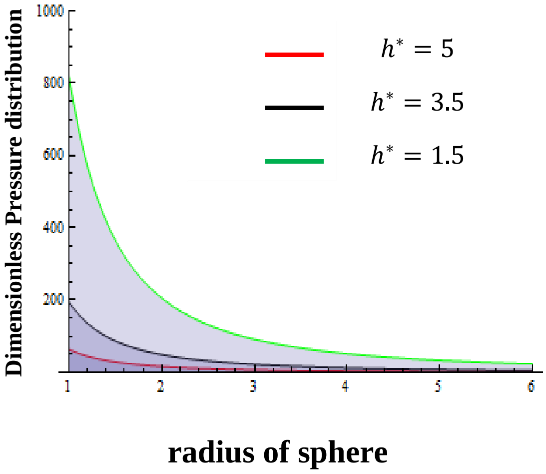


Fig (4.2): Variation of dimensionless pressure distribution with a radius of a sphere for different values of film thickness.

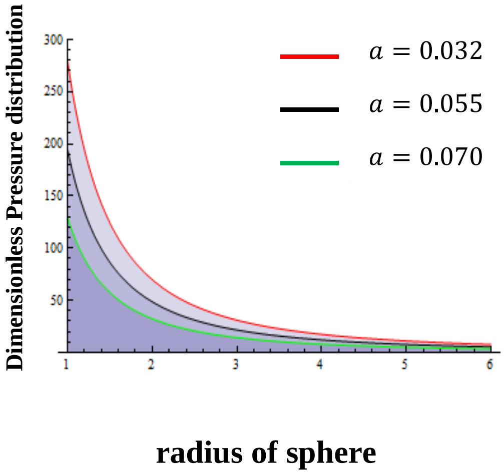


Fig (4.3): Variation of dimensionless pressure distribution with a radius of a sphere for different values of elastic deformation.

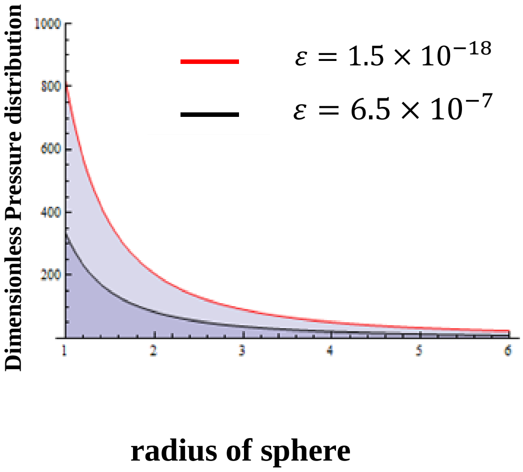


Fig (4.4): Variation of dimensionless pressure distribution with a radius of a sphere for different values of permeability.

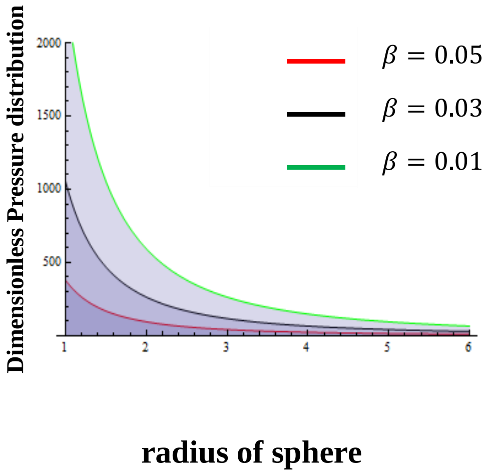


Fig (4.5): Variation of dimensionless pressure distribution with a radius of a sphere for different values of pore size.

4.5.2. <u>Load Carry Capacity</u>

Fig (4.6) appears the effects of film thickness on the load carrying capacity for a radius of the sphere after applying equation (4.37). The effect of film thickness parameter h^* signifies a growing in load carrying capacity especially for small values of $(h^* = 1.5, t = 1.4s)$ compared with that of the film thickness of hydrodynamic where $(h^* = 1.5, t = 1.4s)$. The effect of pore size on ability articular cartilage to bear different weights, is shown in Fig (4.7). Pore size of articular cartilage in swing phase is large since load on joint is few. Therefore load carry capacity is approximately (5% at a = 0.055) when foot touches the earth start stance phase this when increaseing load on lower joint that leads to pore diameters becomes smaller when it reaches to ($\beta = 0.01$ in toe off) and load carrying capacity is approximately (60% at a = 0.055). The relationship elastic deformation and load through cycle time we have seen in fig (4.8). loads are originated from the main body weight which doubles during movement (normal walk, run) and thus leads to elastic of layer articular cartilage that increases in stance phase where strain is high while decreasing in swing phase where strain (10^{-3}), meaning relation load and elastic be positive. The variation in dimensionless load capacity with dimensionless permeability for different walking patterns is depicted in fig (4.9), cartilage is compressed, its permeability decreases. Therefore, as a joint is loaded, most of the fluid that pass the articular surface comes from the cartilage closest to the joint surface under increasing load carry capacity fluid flux that will decrease because of the decrease in permeability that accompanies compression.

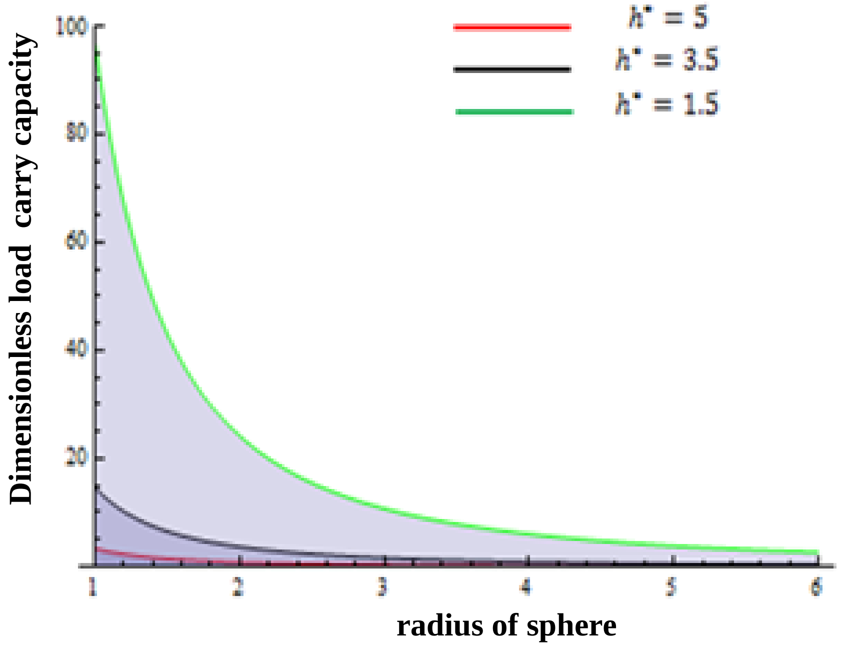


Fig (4.6): Variation of dimensionless load carrying capacity with a radius of the.

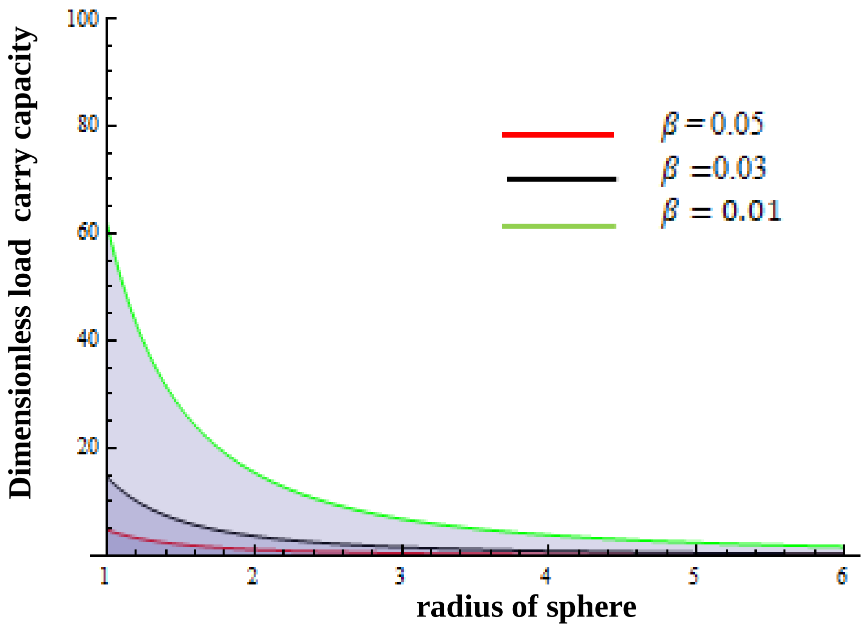


Fig (4.7): Variation of dimensionless load carrying capacity with a radius of the sphere for different values of pore size.

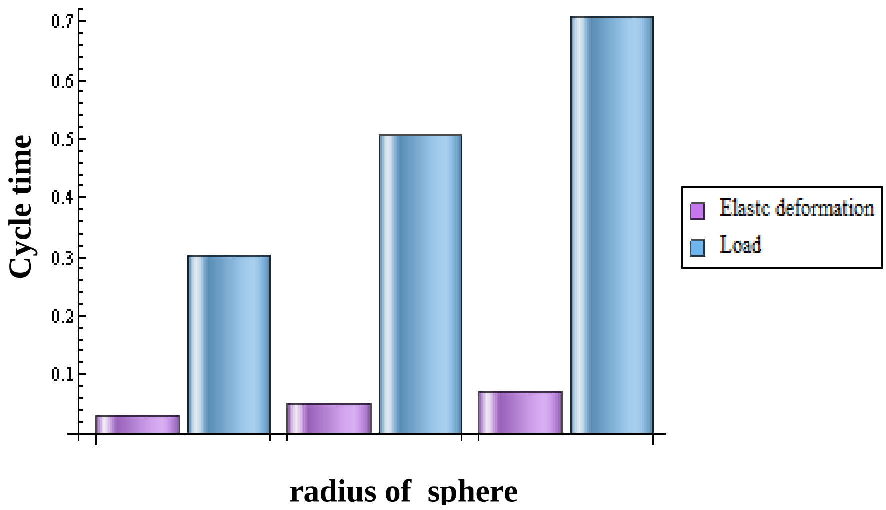


Fig (4.8): Effective of cycle time on elastic deformation and load.

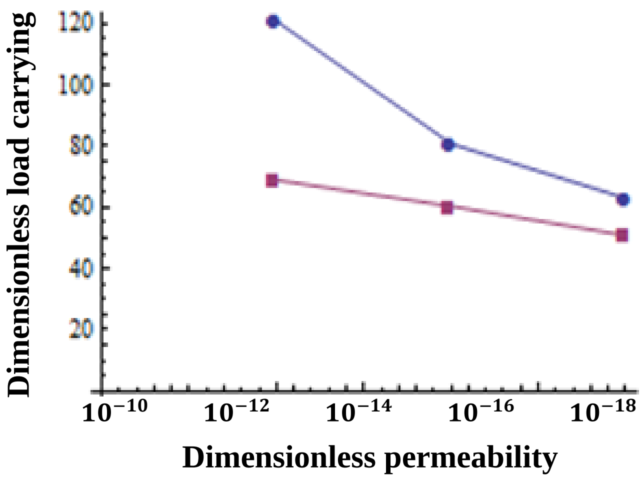


Fig (4.9): Variation of dimensionless load carrying capacity with dimensionless permeability for different walking patterns.

4.5.3. <u>Frication Force</u>

Friction force and film thickness of (hydrodynamic–squeeze- elasto hydrodynamic) are seen in Fig (4.10), after applying equation (4.46). when film thickness between articular cartilage becomes sufficiently thick i.e $h > 2$ friction tends towards friction produced after 3 cycle time i.e 4.2 s, after 5 cycle time is observed film thickness is low; this leads to high friction force where it reaches (50.55%) when h < 1.1.

The effective of lubrication and elasto deformation on friction force is showen in fig (4.11). Elasto deformation is different with respect to lubrication knee joint. In hydrodynamic lubrication that shown elasto deformation small thus the value of friction force is low. By continuing activities of life, lubrication transforms to elasto-hydrodynamic where velocity flow synovial fluid and load are played main in lubrication. Therefore, value of elasto deformation becomes large in comparison to its previous values thus friction force becomes large. See table (4.2).

Dimensionless film thickness

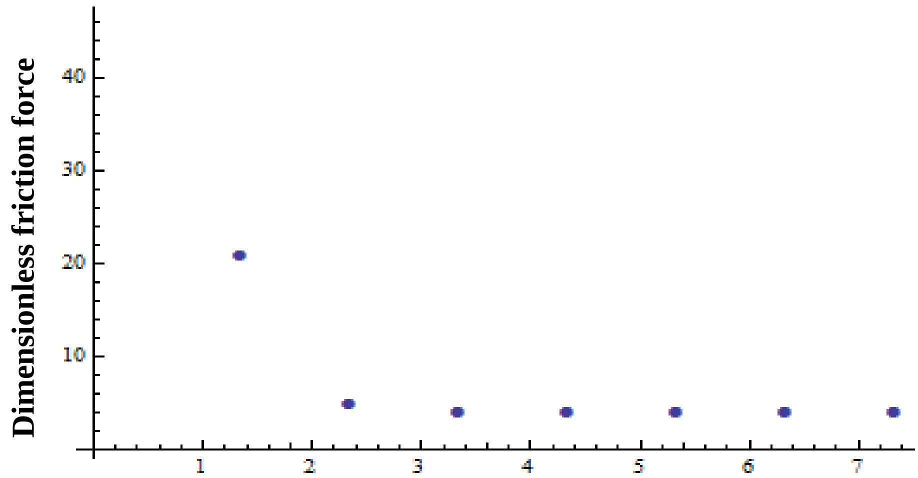


Fig (4.10): Relationship between dimensionless friction force and film thickness of type lubrication.

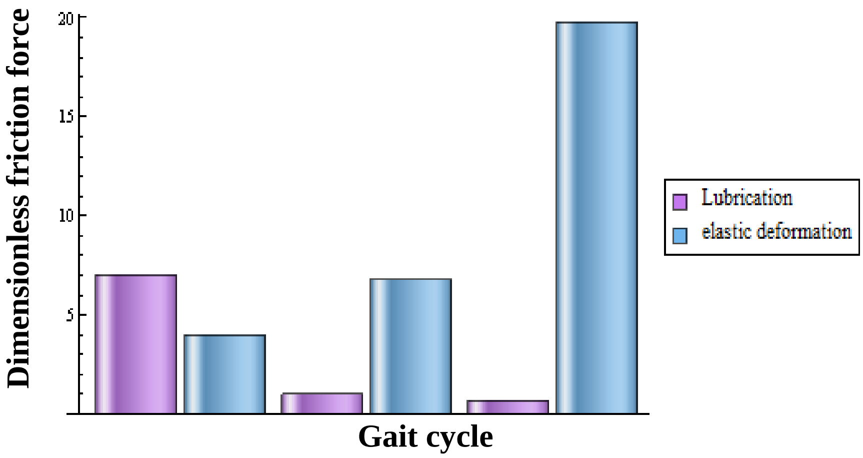


Fig (4.11): Effective lubrication e & elastic deformation on dimensionless friction force.

Influential pore size and elasto deformation on friction force are shown in fig (4.12). Pore size of articular cartilage is one of the most important adsorbent parameters that determin the ability of the molecules fluid on flow through tissue. When molecules fluid are smalles than the pore size then pressure and load carrying capacity become high, while friction force and elasto deformation is low, with changing patterns of movement and aging become pore size big than molecules fluid. Resulte that friction force was appered high comparison with elastic.

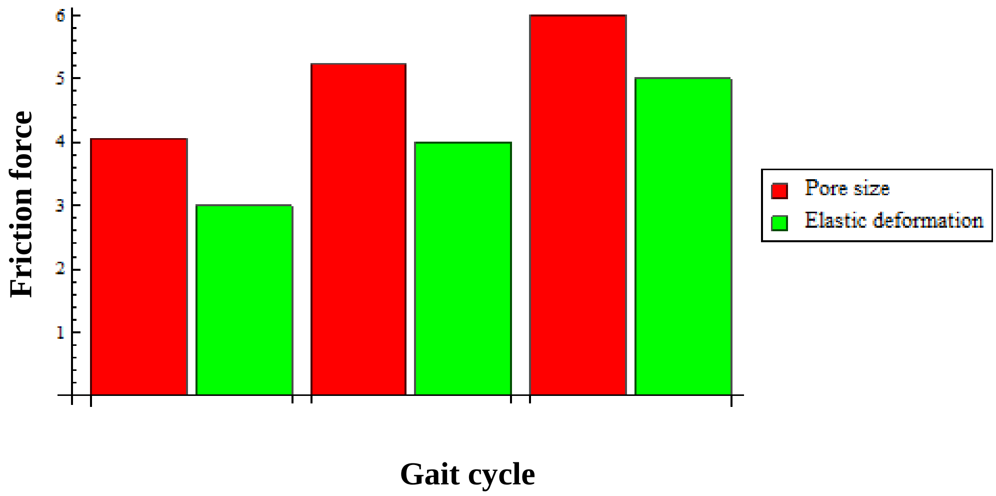


Fig (4.12): Effective pore size & elastic deformation on dimensionless friction force.

4.5.4. <u>Coefficient of Friction</u>

The relationship between weight of human and coefficient of friction with different age is shown in Fig (4.13). In children phase where range weight between [10-33] with different coefficient of friction respect to lubricant, we find the rang increasing coefficient of friction [28%-42%], friction coefficients are affected by increased weight in young, but this effect is simple on the joint because of the presence of lubricants that reduce friction. This situation changes with progresses age where weight increases and friction factors have a significant impact on cartilage.

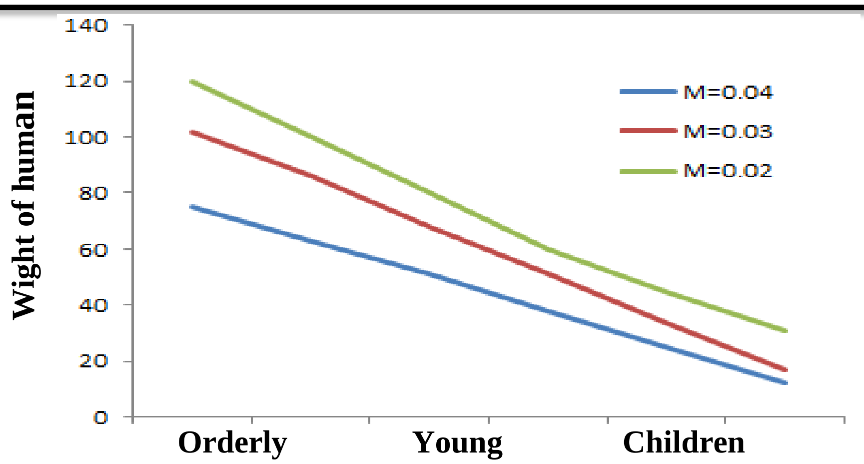


Fig (4.13): Variation of Friction force with age of human for different values of coefficient of friction.

4.5.5. Elastic Deformation

Parameters are effective on elastic deformation during motion (walk, run, jump) with different load as shown in fig (4.14) after applying formula (4.1). One parameter radius that change in stages of human life is accompanied this change increases in elastic deformation. This increasing begins little in (children – young) stages, when a person reaches to aging, the curvature increases and deformation its highest level. Elastic modules are two parameters effective on deformation when elastic modules ($E = 10^7$) then the percentage of deformation (35%) while when elastic modules ($E = 10^9$) then the percentage of deformation (72%). Relationship between film thickness and elastic deformation is shown in fig (4.15), since film thickness is higher in hydrodynamic lubrication then elastic deformation that infects cartilage surfaces that are very few in stance phase deffrent film thickness through cycle time (1.4 s). It is found that elastic deformation in squeeze lubrication is higher as compared to hydrodynamic, in case of body wieght increase and weak bone inceasesing elastic deformation significantly with time loss joint my qualities that are elastic and flexibilty. This is consistent with both types of lubrication (mixed –boundray).

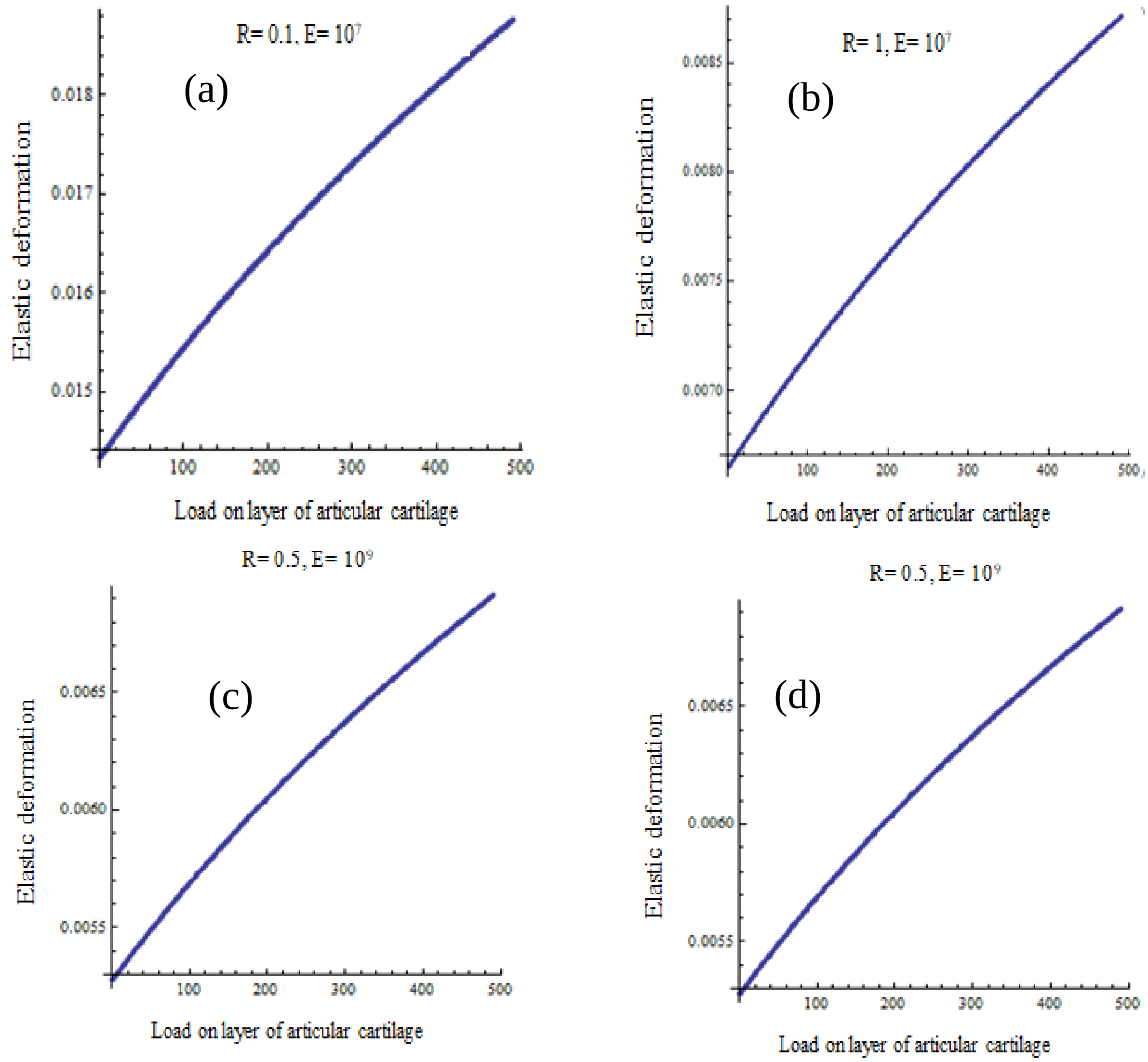
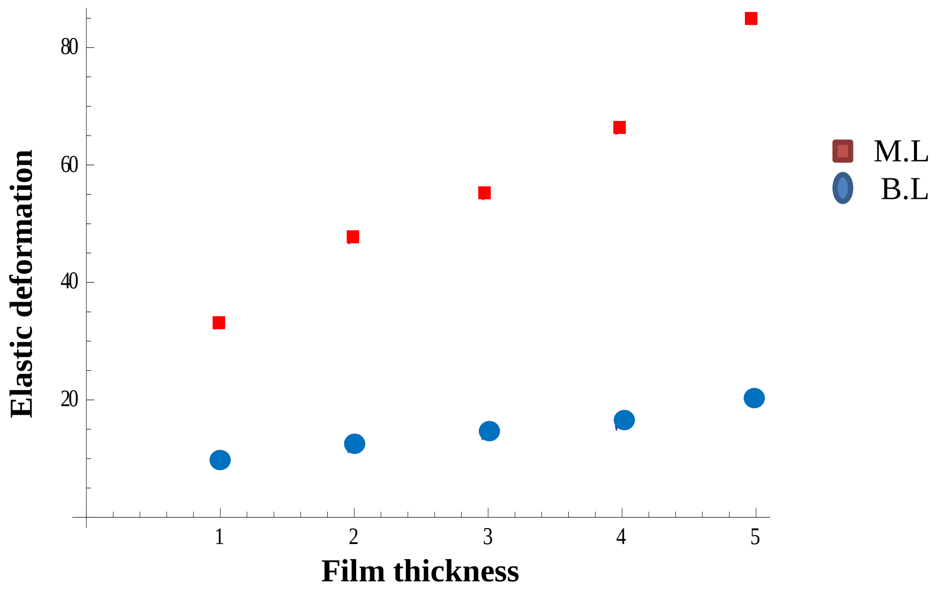


Fig (4.14): parameters effective on elastic deformation with different load (a) radius of articular cartilage in elasto hydrodynamic lubrication, (b) radius of articular cartilage in hydrodynamic, (c) elastic module in stance phase (d) elastic module in swing phase.



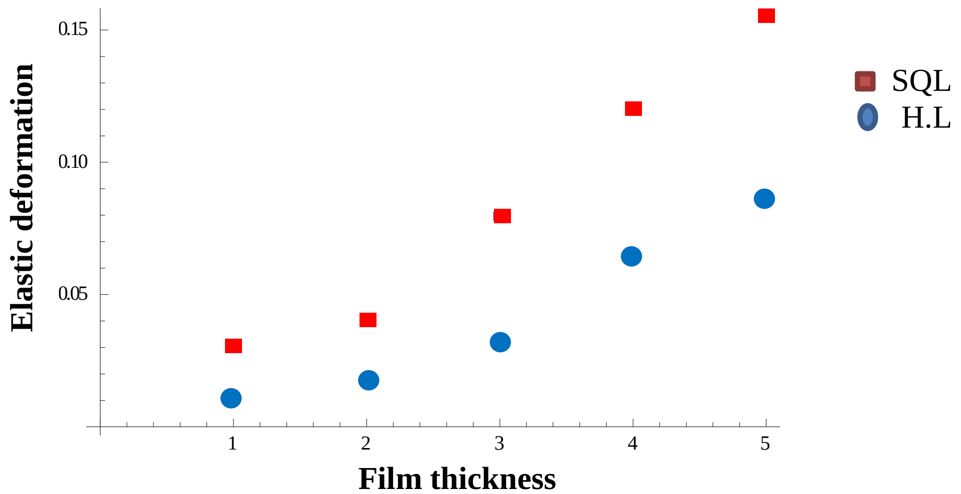


Fig (4.15): Relationship between film thickness and elastic deformation (a) in (hydrodynamic and squeeze) lubrication (b) in (mixed and boundary) lubrication.

4.5.6. <u>Wear Analysis of Layer Articular Cartilage For Knee Joint</u> [42]

Wear layers of articular cartilage, as shown in fig (4.16), set up in human knee joints are by-products of cartilage in the wear process. Wear layers hold privileged information on knee joint conditions and wear mechanism of a human knee. Classification joint that it is healthy or injured (Osteoarthritic) dependes on the rate of wear of layers as shown in fig (4.17), parameters that are effective on wear load carrying capacity and friction force where high load carrying capacity reduce Wear of layer articular cartilage (superficial zone, subsurface zone, middle zone) reach wear to middle and in case Osteoarthritic also plays elastic deformation and friction force an important role in increasing wear articular cartilage for both sexes.

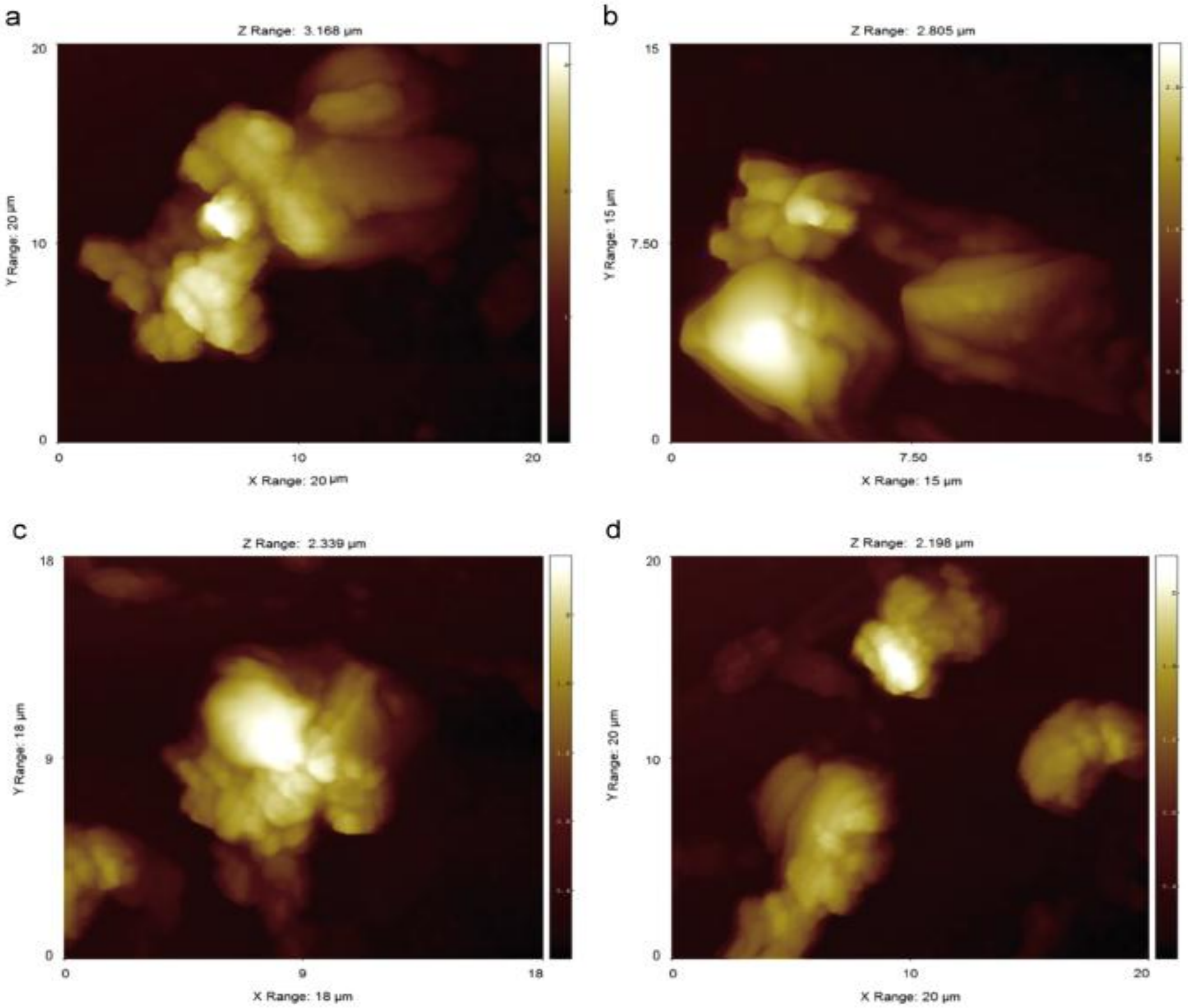


Fig (4.16): AFM images of wear particles found in knee synovial fluid samples collected from severe osteoarthritic patients (OA grades 3 and 4). (a) An irregular shaped wear particle; (b) a square wear particle with a smooth surface; (c) an irregular and rough particle; and (d) rough particles **[35]**.

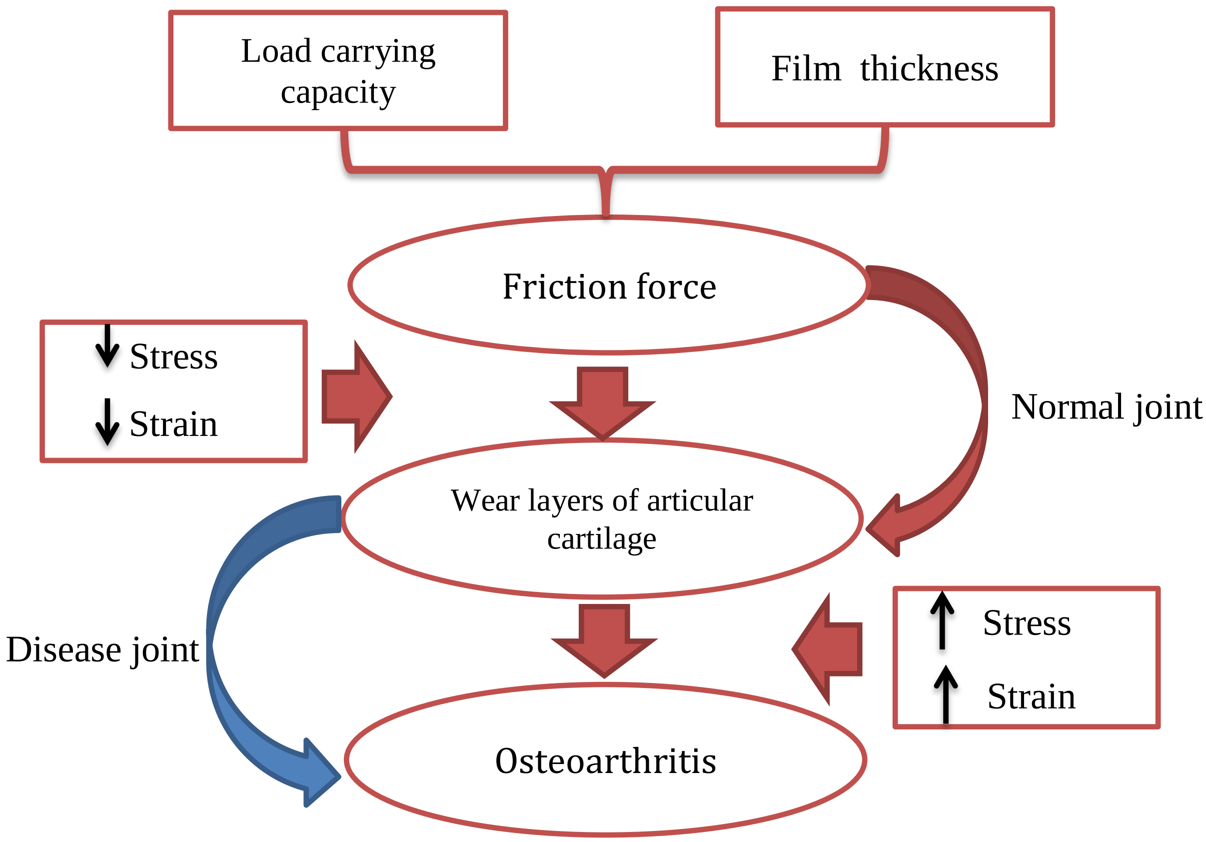


Fig (4.17): Phase transformation healthy joint to disease joint.

Fig (4.18) displays the relationship between wear of articular cartilage and film thickness, since the friction force gives a effectiveness lower in film thickness between articular cartilage and with different system of lubrication and friction force in hydrodynamic lubrication confined corrosion in fires layer and continues for 35 years. Lack of sport and increase weight with progress age make wear in to the second layer and continues for 60 years, with friction reaches its highest point, wear reaches the last layer.

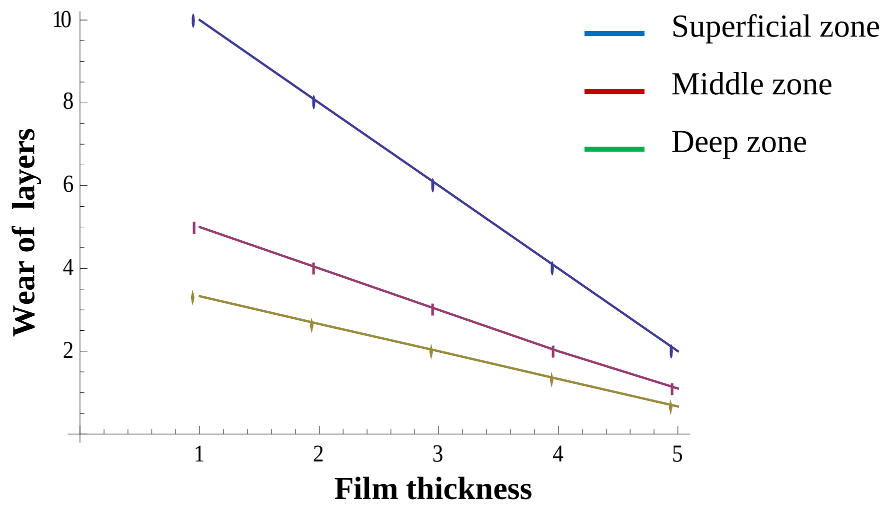


Fig (4.18): Wear of layers articular cartilage withdifferent film thickness.

4.6. Comparison

This section discusses the comparison of variable effects system lubrication on the elastic deformation and wear. All solutions are obtained and discussed under different forms of the relevant parameters of the tables in this section. The relevance is also incorporated from simplification tables for various physical activities parameters.

The table (4-1) displays the effect of elastic deformation on pressure distribution with in (stance-swing) phase. It is noticed that the increases elastic deformation leads to decreases in pressure distribution. Similarly, there is a significant decrease in squeeze of pressure distribution during various movements performed by knee joint.

The table (4-2) states the effect of film thickness on friction force with different value of elastic deformation in each type lubrication. It has found the relationship between film thickness of articular cartilage and friction force inversely and the friction force appears obvious in mixed lubrication. It should be noted that the increase friction force between two articular cartilage leads to cartilage lose recipe elastic.

The table (4-3) indicates the wear of layer articular cartilage with different load carrying capacity of knee joint. It has been found with reduced load carrying capacity and film thickness increases layers of cartilage, wear simple in normal joint and become more pronounced when the disease.

Table (4.1): Relationship between elastic deformation and pressure distribution in normal walk

Hydrodynamic lubrication			Squeeze Lubrication		
Elastic	Pressure	phase	Elastic	Pressure	phase
0.032	1.5	acceleration	0.032	285.5	Initial contact
0.046	1.04	acceleration	0.046	198.6	Initial contact
0.055	0.87	Mid-swing	0.055	166.1	foot –flat
0.058	0.82	Mid-swing	0.058	157.5	foot –flat
0.061	0.78	Mid-swing	0.061	149.8	foot –flat
0.69	0.064	deceleration	0.69	0.064	heel-off
0.71	0.067	deceleration	0.71	0.067	heel-off
0.75	0.069	deceleration	0.75	0.069	toe-off.

Table (4.2): Variation in friction force with different value film thickness and elastic deformation.

Film thickness	Friction force	Elastic deformation
Hydrodynamic lubrication		
8	4.00012	0.070
7	4.00027	0.060
6	4.00066	0.050
Squeeze lubrication		
5	4.00849	0.055
4	4.0644	0.040
Mixed lubrication		
1	6.0625	0.044
0.7	16.2717	0.030

Table (4.3): wear layer of articular cartilage.

Hydrodynamic lubrication ,F=10 N, a =[0.070-0.050],superficial zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	10	Normal	Very low
80	8	Normal	Very low
60	6	Normal	Low
40	4	Normal	Low
20	2	Osteoarthritic	High
10	1	Osteoarthritic	Very high
Squeeze lubrication, F=20 N, a =[0.050-0.040], middle zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	5	Normal	Very low
80	4	Normal	Very low
60	3	Normal	Low
40	2	Normal	Low
20	1	Osteoarthritic	High
10	0.5	Osteoarthritic	Very high

Elasto-Hydrodynamic lubrication, F=30 N, α=[0.044-0.030], deep zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	3.33	Normal	Very low
80	2.66	Normal	Low
60	2	Osteoarthritic	Low
40	1.33	Osteoarthritic	Low
20	0.666	Osteoarthritic	Very high
10	0.333	Osteoarthritic	Very high

4.7.<u>Conclusions</u>

1. When film thickness is sufficiently thick, pressure generally between articular cartilages will reduce asperity contact, the effect of a film thickness of lubrication improves the performance of the joint when permeably value is small.
2. Under the condition studied, elastic deformation of surface in the normal knee joint becomes low; this is related to high pressure generally and small pore size that synovial fluid flows through it.
3. .Load carrying capacity is divided on stance phase swing phase, it has been found in stance phase film thickness and pore size becomes very little to protect the joint from damage then load carrying increasing marked increases reversed swing phase.
4. It is found that the radius of shape and module of elastic has a material influence on the performance of knee joint. It has been found that elastic deformation with high radius and lessening lubricant becomes considerable.

5. Friction force looks more important and effects on the safety of articular when reducing thin film thickness which in turn raises elastic.

6. With progress age, increasing body weight occur specialling female than male since (sickness, less move) associate with increasing coefficient of friction.

7. The wear of the layer of the articular cartilage varies by the type of lubrication and load carrying capacity where hydrodynamic lubrication wear of superficial zone below in normal knee joint in squeeze lubrication reaches wear to middle zone in late OA the damage reaches to deep zone. It height elastic deformation forms protect the cartilage layers and reduce damage and early injury OA.

Recommendations for Future Work

In the light of the study and results obtained in this thesis, the following future work suggestions are given:

1. The study elastic deformation for other synovial joints such as hip and ankle joint by using couple stress fluid as the lubricant.
2. The study elastic deformation and flexibility on artifcial joint.
3. Determining wear of articular cartilage in artifcial knee joint.

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Appendix (A)

The variation of the parameters of gait cycle between male and female.

Age	Sex	Stride length (m)	time	Speed(m/s)	Time cycle
20-29	M	1.02	15.20	1.31	0.38
30-39	M	1.28	18.87	0.872	0.58
40-49	M	1.06	20.85	1.059	0.49
50-59	M	1.35	21.1	0.947	0.570
60-70	M	1.26	22.91	0.959	0.6515
Age	Sex	Stride length (m)	Time	Speed(m/s)	Time cycle
20-29	F	1.16	16.99	0.88	0.52
30-39	F	1.24	20.16	0.99	0.50
40-49	F	1.3	22.76	1.17	0.47
50-59	F	1.38	24.35	0.89	0.62
60-70	F	1.26	25.19	0.79	0.029

Appendix (B)

The coeffient of friction in human synovial joint.

Joint tested	Coefficient of friction
Human knee	(0.010-0.03)
Healthy human articular cartilage	0.01
Pathological human articular cartilage	0.09
Healthy joint	0.0071
Friction of human joint between bones joints	0.003

Appendix(C)

Table (3.11): shows that effect different parameters squeeze film characteristics.

Pressure (p^*)		
Peclet number	P_e	p^*
	0.7	1.5%
	0.5	1.35%
	0.1	1.2%
Couple stress length	l^*	p^*
	0.7	1.45%
	0.5	0.95%
	0.1	0.75%
Flexibility	f	p^*
	5	1.2%
	0	1.6%
The film thickness of gab between two articular	h^*	p^*
	20	0.1%
	10	0.9%
	5	3.3%
The surface roughness	R_a	p^*
	3	1.35%
	2	0.75%
	1	0.65%
The effective radius of curvature	R	p^*
	1	0.45%
	0.7	0.22%
	0.3	0.004%
Load carrying capacity (w^*)		
Peclet number	P_e	w^*
	0.7	13%
	0.5	12%
	0.1	10.5%
Couple stress length	l^*	w^*
	0.7	12%
	0.5	10%
	0.1	6%
Flexibility	f	w^*
	0	0.1%

	3	0.2%
	5	0.3%
The film thickness of gab between two articular	h^*	w^*
	20	1%
	10	9%
	7	25%
The surface roughness	R_a	w^*
	4	6%
	2	8%
	1	17%
The effective radius of curvature	R	w^*
	1	4.5%
	0.8	1.90%
	0.5	0.4%
time (t^*)		
Peclet number	P_e	t^*
	0.7	14.5%
	0.5	13.5%
	0.1	11.5%
Couple stress length	l^*	t^*
	0.7	14.5
	0.5	9.5%
	0.1	7%
Flexibility	f	t^*
	0	0.38%
	3	0.25%
	5	0.20%
The film thickness of gab between two articular	h^*	t^*
	20	1%
	10	8%
	5	58%
The surface roughness	R_a	t^*
	3	7%
	2	8.5%
	1	18.5%
The effective radius of curvature	R	t^*
	1	8.5%
	0.7	2%
	0.3	0.8%

Peak load (P)		
Stride length	S	P
	1.66	790%
	1.22	580%
	0.9	410%
weight	W	P
	490	570%
	588	620%
	686	780%
Pore size	B	P
	0.03	990%
	0.02	620%
	0.01	280%
Flexibility	f	P
	5	1300%
	3	1100%
	1	990%

Appendix(D)

Published Papers	
Paper title	**Journal**
1. Influence of Gait Cycle on Lubrication of Synovial Human Knee Joint	International journal of enineering and information (IJ EAIS), Vol.3 Issue7, July-2019 Pages:88-99
2. A Study of Wear Articular Cartilage of Synovial Human Knee Joint Using Non-Newtonian Elastic Mathematical Model	Iraq journal of Science and information (IJS), Vol.(61) Issue(6), June-2020
3. Design a non-Newtonaian porosity mathematical model for the effect of flexibility on knee joint performance	(قيد النشر)

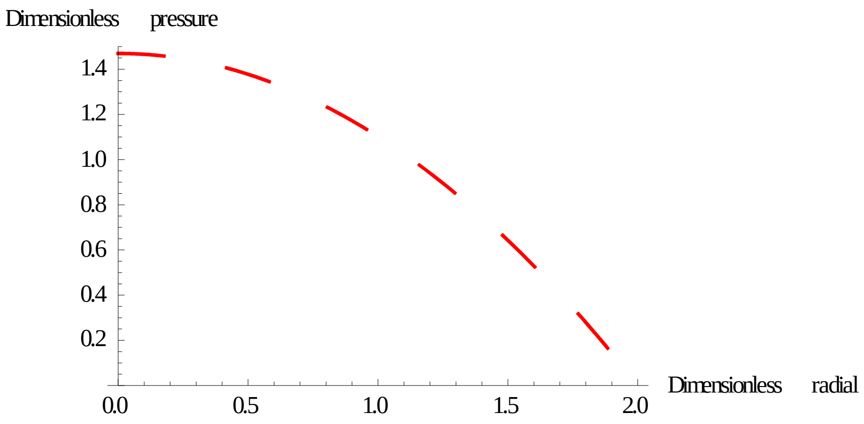
Appendix(E)

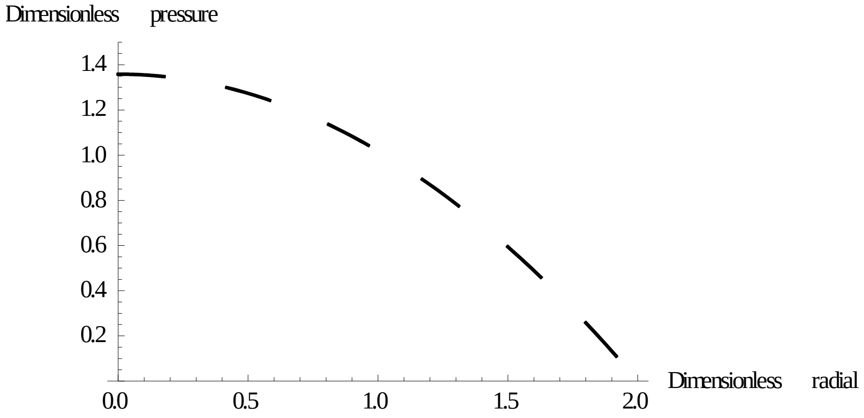
1. HYDRODYNMICLUBRICTION(P_e)

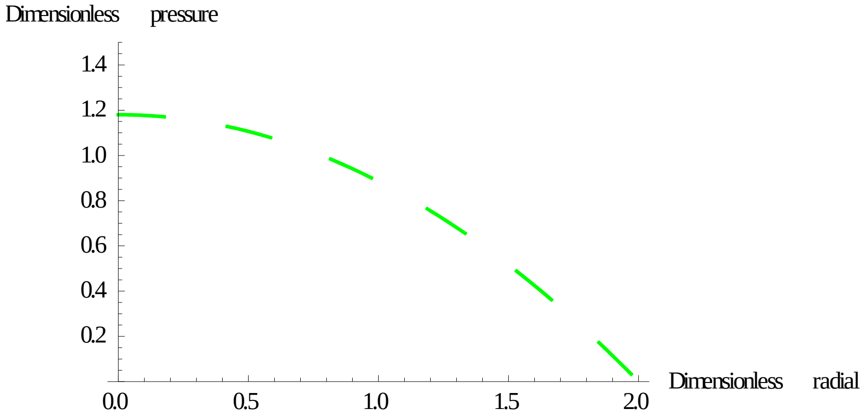
$$\text{P1} = \frac{6*(1)^2*(4-r^2)}{\{\beta*12*F*4+(\beta-\frac{\beta*P_e}{3})((h)^3-12*(l)^2+24*(l)^3*\text{Tanh}[\frac{h}{2*(l)}])\}}$$

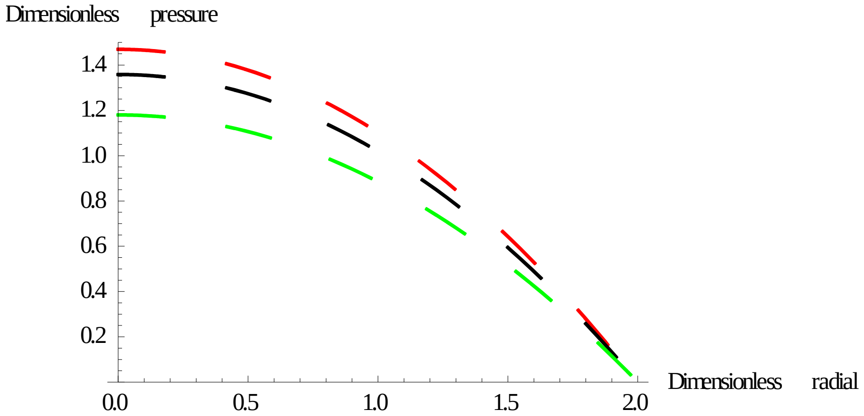
```
b1=P1/.{β→0.02,F→1,Pₑ→0.7,h→10,l→0.7};
b2=P1/.{β→0.02,F→1,Pₑ→0.5,h→10,l→0.7};
b3=P1/.{β→0.02,F→1,Pₑ→0.1,h→10,l→0.7};
g1=Plot[b1,{r,0,2},PlotStyle→{Red,Thickness
[0.001],Dashing[Large]},PlotRange→{0,1.5},A
xesLabel→{Dimensionless
radial,Dimensionless pressure}]
g2=Plot[b2,{r,0,2},PlotStyle→{Black,Thickne
ss[0.001],Dashing[Large]},PlotRange→{0,1.5}
,AxesLabel→{Dimensionless
radial,Dimensionless pressure}]
g3=Plot[b3,{r,0,2},PlotStyle→{Green,Thickne
ss[0.001],Dashing[Large]},PlotRange→{0,1.5}
,AxesLabel→{Dimensionless
radial,Dimensionless pressure}]
Show[g1,g2,g3]
```

$$\{\frac{6(4-r^2)}{48F\beta+(\beta-\frac{\beta P_e}{3})(h^3-12l^2+24l^3\text{Tanh}[\frac{h}{2l}])}\}$$


Dimensionless pressure
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
Dimensionless radial


Dimensionless pressure
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
Dimensionless radial


Dimensionless pressure
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
Dimensionless radial


Dimensionless pressure
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
Dimensionless radial

الخلاصة :

الهدف الرئيسي من هذه الدراسة هو تحديد خصائص انضغاط الغشاء باستخدام المعلمات الرياضية للحصول على مظهر تحليلي للضغط الهيدروديناميكي للغشاء، وقدرة التحمل لمفصل الركبة الزليلي البشري أثناء النشاط اليومي (المشي- الركض)، قوة الاحتكاك، الحد الأدنى لسماكة طبقة الغشاء مع وقت اقتراب عظم الفخذ إلى الساق وتحديد قمم الإجهاد على مفصل الركبة مع مختلف أنواع التزييت ، الموديل الرياضي ركز على مرونة مفصل الركبة (معلمة خارجية) والتشوه المرن للغضاريف المفصلية (معلمة داخلية)، وتحديد تأثيرها في تحسين أداء المفصل: البحث سلط الضوء على تأكل طبقات الغضاريف ومعدل التآكل في كل طبقة وفقًا لقوة الاحتكاك بين المفصل ومعدل التشوه المرن.

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة الكوفة
كلية التربية للبنات
قسم الرياضيات

دراسة التأثيرات المركبة للتزييت والتشويه المرن على مفصل الركبة باستخدام النموذج الرياضي

رسالة تقدمت بها الطالبة **هاله خضير عباس الزغيبي** إلى مجلس كلية التربية للبنات – جامعة الكوفة وهي جزء من متطلبات شهادة الماجستير في الرياضيات

بأشراف

أ.م.د . ايناس يحيى عبدالله

جماد الثاني/ 1441هـ **كانون الثاني/2020م**