On Neutrosophic Crisp Relations

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Abstract

The aim of this paper is to introduce a new types of neutrosophic crisp relations as a generalization to intuitionistic relations due to Indira et al.[9], and study some of its properties. Finally, the concepts of the star and retract neutrosophic relations are introduces and studied and some properties of these concepts will be investigated.
Keywords: Crisp sets relations; Neutrosophic crisp set; Star neutrosophic crisp set; Neutrosophic crisp relation; Star neutrosophic crisp relation.

1.Introduction

Established by Florentin Smarandache, neutrosophy [15] was presented as the study of origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The main idea was to consider an entity,"A" in relation to its opposite"Non-A", and to that which is neither"A" nor "Non-A",denoted by "Neut-A". And from then on, neutrosophy became the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics. According to this theory every idea "A" tends to be neutralized and balanced by "neutA" and "nonA" ideas - as a state of equilibrium. In a classical way"A","neutA", and"antiA" are disjoint two by two. Nevertheless, since in many cases the borders between notions are vague and imprecise, it is possible that "A", "neutA", and "antiA" have common parts two by two, or even all three of them as well. In [18, 19, 20], Smarandache introduced the fundamental concepts of neutrosophic set, that had led Salama et al. in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] to provide a mathematical treatment for the neutrosophic phenomena which already existed in our real world. Moreover, the work of Salama et al. formed a starting point to construct new branches of neutrosophic mathematics. Hence, neutrosophic set theory turned out to be a generalization of both the classical and fuzzy counterparts [2, 3, 6, 21]. This paper is devoted for introducing a new type of neutrosophic crisp relation called the retract neutrosophic crisp set, and studying some of its properties. On the application of neutrosophic theory, the readers can referes [22-24].

2. Preliminaries:

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In this section, we recall some definitions for essential concepts of neutrosophic crisp relations on neutrosophic crisp sets and study their properties, which were introduced by Salama and Smarandache in [11,14].

2.1 Definition[11]

Consider any two neutrosophic crisp sets, A on X and B on Y; where $A = \langle A_1 , A_2 , A_3 \rangle$ and $B = \langle B_1 , B_2 , B_3 \rangle$

The Cartesian product of A and B is defined as the triple structure: $A \times B = \langle A_1 \times B_1 , A_2 \times B_2 , A_3 \times B_3 \rangle$.

where each component is a subset of the Cartesian product $X \times Y$; $A_i \times B_i = \{(a_i, b_i): a_i \in A_i , b_i \in B_i\}, \forall\, i = 1,2,3$

2.1 Corollary

In general if $A \neq B$, then $A \times B \neq B \times A$

2.2 Definition[11]

A neutrosophic crisp relation R from a neutrosophic crisp set A to B, namely $R : A \rightarrow B$, is defined as a triple structure of the form $R = \langle R_1, R_2, R_3 \rangle$, where$R_i \subseteq A_i \times B_i, \forall\, i = 1,2,3$ that is

2.3 Definition

A neutrosophic crisp inverse relation R^{-1} is a neutrosophic crisp relation from a neutrosophic crisp set B to A, $R^{-1}: B \rightarrow A$, and to be defined as a triple structure of the form: $R^{-1} = \langle R_1^{-1}, R_2^{-1}, R_3^{-1} \rangle$, where $R_i^{-1} \subseteq B_i \times A_i, \forall\, i = 1,2,3$ that is: $R_i^{-1} = \{(a_i, b_i) : (a_i, b_i) \in R_i\} R_i = \{(a_i, b_i): a_i \in A_i , b_i \in B_i\}$

2.1 Example

Let $X = \{1, 2, 3,4 \}$, $A = \langle \{1,2\}, \{3\}, \{4\} \rangle$ and $B = \langle \{1\}, \{3\}, \{4,2\} \rangle$, if R is relation from A to B be defined as $R = \{(a, b): a \in A, b \in B : a \geq b\}$, then the products of two neutrosophic crisp sets are given by:

$$A \times B = \langle \{(1,1), (2,1)\}, \{(3,3)\}, \{(4,4), (4,2)\} \rangle,$$
$$B \times A = \langle \{(1,1), (1,2)\}, \{(3,3)\}, \{(4,4), (2,4)\} \rangle,$$
$$R_1 = \langle \{(1,1)\}, \{(3,3)\}, \{(4,4)\} \rangle, R_1 \subseteq A \times B ,$$
$$R_2 = \langle \{(1,2)\}, \{(3,3)\}, \{(4,4), (2,4)\} \rangle, R_2 \subseteq B \times A ,$$
$$R_1^{-1} = \langle \{(1,1), (1,2)\}, \{(3,3)\}, \{(4,4), (2,4)\} \rangle,$$
$$R_2^{-1} = \langle \{(2,1)\}, \{(3,3)\}, \{(4,4), (4,2)\} \rangle.$$

3. Domain and Range of Neutrosophic Crisp Relations

For any neutrosophic crisp relation $R : A \rightarrow B$, we define the following:

– The domain of R, is defined as: $Dom(R) = \langle dom(R_1), dom(R_2), dom(R_3) \rangle$

– The range of R, is defined as: $Rng(R) = \langle rng(R_1), rng(R_2), rng(R_3) \rangle$

– The Domain of R, is defined as: $Dom(R) = dom(R_1) \cup dom(R_2) \cup dom(R_3)$

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– The Range of R, is defined as: $Rng(R) = rng(R_1) \cup rng(R_2) \cup rng(R_3)$

3. 1 Corollary

From the definitions given in 2.5, one may notice that for any ultra neutrosophic crisp relation $R : A \to B$, we have:

– The domain of R is a crisp subset of X, namely, $Dom(R) \subseteq X$.

–The range of R is a crisp subset of Y, namely, $Rng(R) \subseteq Y$.

3.2 Corollary

For any neutrosophic crisp relation $R : A \to B$, we have that:

$$Dom(R^{-1}) = Rng(R)$$
$$Rng(R^{-1}) = Dom(R)$$

4 Neutrosophic Crisp Relations' Operations

In the following definitions we consider R and S are two neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, neutrosophic crisp sets A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ in X, $B = \langle B_1, B_2, B_3 \rangle$ on Y.

4.1 Definition [14]

The neutrosophic crisp relation R is a neutrosophic crisp subset of the neutrosophic crisp set S($R \subseteq S$), may be defined as one of the following two types:

Type 1: $R \subseteq S \Leftrightarrow A_{R1} \subseteq B_{S1}$, $A_{R2} \subseteq B_{S2}$ and$A_{R3} \supseteq B_{S3}$.

Type 2:$R \subseteq S \Leftrightarrow A_{R1} \subseteq B_{S1}$, $A_{R2} \supseteq B_{S2}$ and $A_{R3} \supseteq B_{S3}$.

4.2 Definition [14]

The neutrosophic intersection and neutrosophic union of any two neutrosophic sets A and B, may be defined as follows:

1. The neutrosophic intersection, $A \cap B$, may be defined as one of the following two types:

   Type 1:$R \cap S \Leftrightarrow A_{R1} \cap B_{S1}$, $A_{R2} \cap B_{S2}$ and $A_{R3} \cup B_{S3}$.

   Type 2: $R \cap S \Leftrightarrow A_{R1} \cap B_{S1}$, $A_{R2} \cup B_{S2}$ and $A_{R3} \cup B_{S3}$.

2. The neutrosophic intersection,$A \cup B$, may be defined as one of the following two types:

   Type 1:$R \cup S \Leftrightarrow A_{R1} \cup B_{S1}$, $A_{R2} \cup B_{S2}$ and $A_{R3} \cap B_{S3}$.

   Type 2:$R \cup S \Leftrightarrow A_{R1} \cup B_{S1}$, $A_{R2} \cap B_{S2}$ and $A_{R3} \cap B_{S3}$.

4.1 Theorem

Let R, S and Q be three neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, then:

a. $R \subseteq S \to R^{-1} \subseteq S^{-1}$.
b. $(R \cup S)^{-1} \to R^{-1} \cup S^{-1}$, $(R \cap S)^{-1} \to R^{-1} \cap S^{-1}$.
c. $(R^{-1})^{-1} = R$.
d. $R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q), R \cup (S \cup Q) = (R \cup S) \cap (R \cup Q)$.
e. If $S \subseteq R, Q \subseteq R$, then $S \cup Q \subseteq R$.

4.3 Definition

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The neutrosophic crisp relationsR $\in \mathrm{NCR}(\mathrm{X}, \mathrm{X})$are called:

1) Neutrosophic Reflexive Relation, if for every $x \in X$, there is

$$(x, x) \in R_i \quad \forall\, i = 1,2,3$$

2) Neutrosophic Symmetric Relation, if $R = R^{-1}$, that is for every $(x, y) \in X \times Y$ such that

$$\forall\, (x, y) \in R_i \Rightarrow (y, x) \in R_i \quad \forall\, i = 1,2,3$$

3) Neutrosophic Transitive Relation, if there is$(x, y), (y, z) \in X \times Y$ such that

$$\forall\, (x, y), (y, z) \in R_i \Rightarrow (x, z) \in R_i \quad \forall\, i = 1,2,3$$

4) Neutrosophic Equivalence Relation, if R is reflexive, symmetric and transitive relations

5. Composition of Neutrosophic Crisp Relations

Consider the three neutrosophic crisp sets: A of$X$, B of Y and C of$Z$; and the two neutrosophic crisp relations:$R : A \to B$ and $S : B \to C$; where $R = \langle R_1, R_2, R_3 \rangle$, and $S = \langle S_1, S_2, S_3 \rangle$. The composition of R and S, is denoted and defined as:

$R \odot \mathrm{S} = \langle R_1 \circ S_1, R_2 \circ S_2, R_3 \circ S_3 \rangle$, such that:

$R_i \circ S_i : A_i \to C_i$, where $R_i \circ S_i = \{(a_i, b_i) : \exists \mathrm{b}_i \in B_i, (a_i, b_i) \in R_i \text{and} (b_i, c_i) \in S_i\}$.

5.1 Corollary

For any two neutrosophic crisp relations:$R : A \to B$ and $S : B \to C$;

$$Dom(R \odot \mathrm{S}) \subseteq Dom(R)$$

$$Rng(R \odot \mathrm{S}) \subseteq Rng(S)$$

5.2 Corollary

Consider the three neutrosophic crisp relations: $R : A \to B$ and $S : B \to C$, and

$$K : C \to D;$$

$$R \odot (S \odot \mathrm{K}) = (R \odot \mathrm{S}) \odot \mathrm{K}$$

6 Star Neutrosophic Crisp Relations

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In this section, we consider R^* and S^* are two star neutrosophic crisp relations between X and Y for every$(x,y) \in X \times Y$, star neutrosophic crisp sets A^* and B^*in the form $A^* = \langle {A_1}^*, {A_2}^*, {A_3}^* \rangle$ in X, $B^* = \langle {B_1}^*, {B_2}^*, {B_3}^* \rangle$ on Y.

6.1 Definition

Consider any two neutrosophic crisp sets, A on X and B on Y; where $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$, two star neutrosophic crisp sets A^*, B^* is the structure $A^* = \langle {A_1}^*, {A_2}^*, {A_3}^* \rangle, B^* = \langle {B_1}^*, {B_2}^*, {B_3}^* \rangle$where ${A_1}^* = A_1 \cap co(A_2 \cup A_3)$, ${A_2}^* = A_2 \cap co(A_1 \cup A_3)$, ${A_3}^* = A_3 \cap co(A_1 \cup A_2)$, ${B_1}^* = B_1 \cap co(B_2 \cup B_3)$, ${B_2}^* = B_2 \cap co(B_1 \cup B_3)$ and ${B_3}^* = B_3 \cap co(B_1 \cup B_2)$. Then:

The Cartesian product of A^* and B^* is defined as the triple structure:

$$A^* \times B^* = \langle A_1^* \times B_1^*, A_2^* \times B_2^*, A_3^* \times B_3^* \rangle.$$

where each component is a subset of the Cartesian product $X \times Y$;

$${A_i}^* \times {B_i}^* = \{(a_i, b_i): a_i \in {A_i}^*, b_i \in {B_i}^*\}, \qquad \forall\, i = 1,2,3$$

6.2 Definition

A star neutrosophic crisp relation R^* from a star neutrosophic crisp set A^* to B^*, namely$R^* : A^* \to B^*$, is defined as a triple structure of the form $R^* = \langle {R_1}^*, {R_2}^*, {R_3}^* \rangle$, where $R_i^* \subseteq A_i^* \times B_i^*, \forall\, i = 1,2,3$, that is

$${R_i}^* = \{(a_i, b_i): a_i \in A_i^*, b_i \in B_i^*\}$$

6.3 Definition

A star neutrosophic crisp inverse relation $R^{*^{-1}}$is a star neutrosophic crisp relation from a neutrosophic crisp set B^* to A^*,$R^{*^{-1}}: B^* \to A^*$, and to be defined as a triple structure of the form: $R^{*^{-1}} = \langle R_1^{*^{-1}}, R_2^{*^{-1}}, R_3^{*^{-1}} \rangle$, where $R_i^{*^{-1}} \subseteq B_i^* \times A_i^*, \forall\, i = 1,2,3$ that is: $R_i^{*^{-1}} = \{(a_i, b_i) : (a_i, b_i) \in R_i^{*^{-1}}\}$

6.1 Example

Let$X = \{a, b, c, d, e, f\}$, $A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$ and $B = \langle \{a, b, c\}, \{d\}, \{e\} \rangle$, are neutrosophic crisp sets. Then $A^* = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$, $B^* = \langle \{a, b, c\}, \{d\}, \{e\} \rangle$,then the products of two starneutrosophic crisp sets are given by:

$$A^* \times B^* = \langle \{(a,a), (a,b), (a,c)(b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,a), (d,b), (d,c)\}, \{(e,d)\}, \{(f,e)\} \rangle,$$

$$B^* \times A^* = \langle \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d)\}, \{(d,e)\}, \{(e,f)\} \rangle,$$

$$R_1^* = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle, R_1 \subseteq A \times B,$$

$$R_2^* = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d), (b,d)\} \rangle, R_2 \subseteq B \times A,$$

$$R_1^{*-1} = \langle \{(a,a), (a,b)\}, \{(c,c)\}, \{(d,d), (b,d)\} \rangle,$$

$$R_2^{*-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d), (d,b)\} \rangle.$$

7 .Domain and Range of Star Neutrosophic Crisp Relations

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For any neutrosophic crisp relation$R^* : A^* \to B^*$, we define the following:

– The domain of R^*, is defined as: $Dom(R^*) = \langle dom({R_1}^*), dom({R_2}^*), dom({R_3}^*) \rangle$

– The range of R^*, is defined as: $Rng(\,R^*) = \langle Rng({R_1}^{\tau}), Rng({R_2}^{\tau}), Rng({R_3}^{\tau}) \rangle$

– The Domain of R^*, is defined as: $Dom(R^*) = dom({R_1}^*) \cup dom({R_2}^*) \cup\ dom({R_3}^*)$

– The Range of R^*, is defined as: $Rng(R^*) = rng({R_1}^*) \cup\ rng({R_2}^*) \cup\ rng({R_3}^*)$

7.1 Corollary

From the definitions given in 3.5, one may notice that for any star neutrosophic crisp relation $R^* : A^* \to B^*$, we have:

– The domain of R^* is a crisp subset of X, namely, $\mathrm{Dom}(R^*) \subseteq X$.

– The range of R^* is a crisp subset of Y, namely, $Rng(R^*) \subseteq Y$.

7.2 Corollary

For any neutrosophic crisp relation $R^* : A^* \to B^*$, we have that: $Dom\left({R^*}^{-1}\right) = Rng(R^*),\ Rng\left({R^*}^{-1}\right) = Dom(R^*)$

8 Star Neutrosophic Crisp Relations' Operations

In this section, we call some definitions relations on star neutrosophic crisp sets and its properties.

8.1 Definition

The complement of a star neutrosophic crisp relation R^*(co R^*, for short) may be defined as:

$$co\ R^* = \langle\ R_3^*, R_2^*,\ R_1^* \rangle.$$

8.2 Definition

Consider R^* and S^* are two retract neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, retract neutrosophic crisp sets A^* and B^* in the form $A^* = \langle {A_1}^*,\ {A_2}^*,\ {A_3}^* \rangle$ in X, $B^* = \langle {B_1}^*,\ {B_2}^*,\ {B_3}^* \rangle$on Y.

1. The retract neutrosophic crisp relation R^{τ} is a neutrosophic crisp subset of the retract neutrosophic crisp set S^* ($R^* \subseteq S^*$), may be defined as one of the following two types:

$$R^* \subseteq S^* \iff {A^*}_{R1} \subseteq {B^*}_{S1}\ , {A^*}_{R2} \subseteq {B^*}_{S2} \text{ and } {A^*}_{R3} \supseteq {B^*}_{S3}.$$

$$R^* \subseteq S^* \iff {A^*}_{R1} \subseteq {B^*}_{S1}\ , {A^*}_{R2} \supseteq {B^*}_{S2} \text{ and } {A^*}_{R3} \supseteq {B^*}_{S3}.$$

2. The neutrosophic intersection, $R^* \cap S^*$, of any two neutrosophic sets retract A^{τ} andB^{τ}, may be defined as follows:

$$R^* \cap S^* \iff {A^*}_{R1} \cap {B^*}_{S1}\ , {A^*}_{R2} \cap {B^*}_{S2} \text{ and } {A^*}_{R3} \cup {B^*}_{S3}.$$

$$R^* \cap S^* \iff {A^*}_{R1} \cap {B^*}_{S1}\ , {A^*}_{R2} \cup {B^*}_{S2} \text{ and } {A^*}_{R3} \cup {B^*}_{S3}.$$

3. The neutrosophic union, $R^* \cup S^*$, of any two neutrosophic sets retract A^{τ} andB^{τ}, may be defined as follows:

$$R^* \cup S^* \iff {A^*}_{R1} \cup {B^*}_{S1}\ , {A^*}_{R2} \cup {B^*}_{S2} \text{ and } {A^*}_{R3} \cap {B^*}_{S3}.$$

$$R^* \cup S^* \iff {A^*}_{R1} \cup {B^*}_{S1}\ , {A^*}_{R2} \cap {B^*}_{S2} \text{ and } {A^*}_{R3} \cap {B^*}_{S3}.$$

8.1 Theorem

Let $R^*, S^*$and Q^* be three retract neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, then:

a. $R^* \subseteq S^* \to {R^*}^{-1} \subseteq {S^*}^{-1}$.

b. $(R^* \cup S^*)^{-1} \to {R^*}^{-1} \cup {S^*}^{-1}$, $(R^* \cap S^*)^{-1} \to {R^*}^{-1} \cap {S^*}^{-1}$.

c. $({R^*}^{-1})^{-1} = R^*$.

d. $R^* \cap (S^* \cup Q^*) = (R^* \cap S^*) \cup (R^* \cap Q^*), R^* \cup (S^* \cup Q^*) = (R^* \cup S^*) \cap (R^* \cup Q^*)$.

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e. If $S^* \subseteq R^*$, $Q^* \subseteq R^*$, then $S^* \cup Q^* \subseteq R^*$.

9 Composition of Star Neutrosophic Crisp Relations

Consider the three neutrosophic crisp sets: A of X, B of Y and C of Z; and the two star neutrosophic crisp relations: $R^* : A^* \rightarrow B^*$ and $S^* : B^* \rightarrow C^*$; where

$R^* = \langle R_1{}^*, R_2{}^*, R_3{}^* \rangle$, and $S^* = \langle S_1{}^*, S_2{}^*, S_3{}^* \rangle$. The composition of R^* and S^*, is denoted and defined as: $R^* \odot S^* = \langle R_1{}^* \circ S_1{}^*, R_2{}^* \circ S_2{}^*, R_3{}^* \circ S_3{}^* \rangle$, such that: $R_i{}^* \circ S_i{}^* : A_i{}^* \rightarrow C_i{}^*$, where

$$R_i{}^* \circ S_i{}^* = \{(a_i, b_i) : \exists \mathrm{b}_i \in B_i{}^*, (a_i, b_i) \in R_i{}^* \text{and } (b_i, c_i) \in S_i{}^*\}.$$

9.1 Corollary

For any two star neutrosophic crisp relations: $R^* : A^* \rightarrow B^*$ and $S^* : B^* \rightarrow C^*$;

$$Dom(R^* \odot S^*) \subseteq Dom(R^*)$$

$$Rng(R^* \odot S^*) \subseteq Rng(S^*)$$

9.2 Corollary

Consider the three star neutrosophic crisp relations: $R^* : A^* \rightarrow B^*$ and $S^* : B^* \rightarrow C^*$, and $K^* : C^* \rightarrow D^*$;

$$R^* \odot (S^* \odot K^*) = (R^* \odot S^*) \odot K^*$$

9.1 Example

From the Example 6.1 is easy to get $R_i{}^* \circ S_i{}^*$, $Dom(R^* \odot S^*)$ and $Rng(R^* \odot S^*)$

10 Retract Neutrosophic Crisp Relations

In this section,we consider $\mathrm{R}^{\curlyvee}$ and $\mathrm{S}^{\curlyvee}$ are two retract neutrosophic crisp relations between X and Y for every $(\mathrm{x}, \mathrm{y}) \in \mathrm{X} \times \mathrm{Y}$, retract neutrosophic crisp sets $\mathrm{A}^{\curlyvee}$ and $\mathrm{B}^{\curlyvee}$ in the form $\mathrm{A}^{\curlyvee} = \langle \mathrm{A_1}^{\curlyvee}, \mathrm{A_2}^{\curlyvee}, \mathrm{A_3}^{\curlyvee} \rangle$ in X, $\mathrm{B}^{\curlyvee} = \langle \mathrm{B_1}^{\curlyvee}, \mathrm{B_2}^{\curlyvee}, \mathrm{B_3}^{\curlyvee} \rangle$ on Y.

10.1 Definition

Consider any two neutrosophic crisp sets, A on X and B on Y; where $\mathrm{A} = \langle \mathrm{A_1}, \mathrm{A_2}, \mathrm{A_3} \rangle$ and $\mathrm{B} = \langle \mathrm{B_1}, \mathrm{B_2}, \mathrm{B_3} \rangle$, two retract neutrosophic crisp sets $\mathrm{A}^{\curlyvee}, \mathrm{B}^{\curlyvee}$ is the structure $\mathrm{A}^{\curlyvee} = \langle \mathrm{A_1}^{\curlyvee}, \mathrm{A_2}^{\curlyvee}, \mathrm{A_3}^{\curlyvee} \rangle, \mathrm{B}^{\curlyvee} = \langle \mathrm{B_1}^{\curlyvee}, \mathrm{B_2}^{\curlyvee}, \mathrm{B_3}^{\curlyvee} \rangle$ where $\mathrm{A_1}^{\curlyvee} = \mathrm{A_1} \cap \mathrm{co}(\mathrm{A_2} \cup \mathrm{A_3})$, $\mathrm{A_2}^{\curlyvee} = \mathrm{A_2} \cap \mathrm{co}(\mathrm{A_1} \cup \mathrm{A_3})$, $\mathrm{A_3}^{\curlyvee} = \mathrm{A_3} \cap \mathrm{co}(\mathrm{A_1} \cup \mathrm{A_2})$, $\mathrm{B_1}^{\curlyvee} = \mathrm{B_1} \cap \mathrm{co}(\mathrm{B_2} \cup \mathrm{B_3})$, $\mathrm{B_2}^{\curlyvee} = \mathrm{B_2} \cap \mathrm{co}(\mathrm{B_1} \cup \mathrm{B_3})$ and $\mathrm{B_3}^{\curlyvee} = \mathrm{B_3} \cap \mathrm{co}(\mathrm{B_1} \cup \mathrm{B_2})$. Then:

The Cartesian product of $A^{\curlyvee}$ and $B^{\curlyvee}$ is defined as the triple structure:

$$\mathrm{A}^{\curlyvee} \times \mathrm{B}^{\curlyvee} = \langle \mathrm{A}_1^{\curlyvee} \times \mathrm{B}_1^{\curlyvee}, \mathrm{A}_2^{\curlyvee} \times \mathrm{B}_2^{\curlyvee}, \mathrm{A}_3^{\curlyvee} \times \mathrm{B}_3^{\curlyvee} \rangle.$$

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where each component is a subset of the Cartesian product $X \times Y$;

$$A_i^{r} \times B_i^{r} = \{(a_i, b_i): a_i \in A_i^{r}, b_i \in B_i^{r}\}, \qquad \forall\, i = 1,2,3$$

10.2 Definition

A retract neutrosophic crisp relation R^{r} from a retract neutrosophic crisp set A^{r} to B^{r}, namely$R^{r} : A^{r} \to B^{r}$, is defined as a triple structure of the form $R^{r} = \langle R_1{}^{r}, R_2{}^{r}, R_3{}^{r} \rangle$, where $R_i^{r} \subseteq A_i^{r} \times B_i^{r}, \forall\, i = 1,2,3$, that is

$$R_i{}^{r} = \{(a_i, b_i): a_i \in A_i^{r}, b_i \in B_i^{r}\}$$

10.3 Definition

A retract neutrosophic crisp inverse relation $R^{r^{-1}}$is a retract neutrosophic crisp relation from a neutrosophic crisp set B^{r} toA^{r}, $R^{-1}: B^{r} \to A^{r}$, and to be defined as a triple structure of the form: $R^{r^{-1}} = \langle R_1^{r^{-1}}, R_2^{r^{-1}}, R_3^{r^{-1}} \rangle$, where $R_i^{r^{-1}} \subseteq B_i^{r} \times A_i^{r}, \forall\, i = 1,2,3$ that is: $R_i^{r^{-1}} = \{(a_i, b_i) : (a_i, b_i) \in R_i^{r^{-1}}\}$

10.1 Example

Let $X = \{a, b, c, d\}$, $A = \langle \{a, b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{b, d\} \rangle$, are neutrosophic crisp sets. Then $A^{r} = \langle \{a, b\}, \{c\}, \{d\} \rangle$, $B^{r} = \langle \{a\}, \{c\}, \{b, d\} \rangle$,then the products of two retractneutrosophic crisp sets are given by:

$$A^{r} \times B^{r} = \langle \{(a, a), (b, a)\}, \{(c, c)\}, \{(d, b), (d, d)\} \rangle,$$
$$B^{r} \times A^{r} = \langle \{(a, a), (a, b)\}, \{(c, c)\}, \{(b, d), (d, d)\} \rangle,$$
$$R_1^{r} = \langle \{(a, a)\}, \{(c, c)\}, \{(d, d)\} \rangle, R_1 \subseteq A \times B,$$
$$R_2^{r} = \langle \{(a, b)\}, \{(c, c)\}, \{(d, d), (b, d)\} \rangle, R_2 \subseteq B \times A,$$
$$R_1^{r-1} = \langle \{(a, a), (a, b)\}, \{(c, c)\}, \{(d, d), (b, d)\} \rangle,$$
$$R_2^{r-1} = \langle \{(b, a)\}, \{(c, c)\}, \{(d, d), (d, b)\} \rangle.$$

11 Domain and Range of Retract Neutrosophic Crisp Relations

For any neutrosophic crisp relation$R^{r} : A^{r} \to B^{r}$, we define the following:

– The domain of R^{r}, is defined as: $Dom(R^{r}) = \langle dom(R_1{}^{r}), dom(R_2{}^{r}), dom(R_3{}^{r}) \rangle$

– The range of R^{r}, is defined as: $Rng(R^{r}) = \langle Rng(R_1{}^{r}), Rng(R_2{}^{r}), Rng(R_3{}^{r}) \rangle$

– The Domain of R^{r}, is defined as: $Dom(R^{r}) = dom(R_1{}^{r}) \cup dom(R_2{}^{r}) \cup dom(R_3{}^{r})$

– The Range of R^{r}, is defined as: $Rng(R^{r}) = rng(R_1{}^{r}) \cup rng(R_2{}^{r}) \cup rng(R_3{}^{r})$

11.1 Corollary

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From the definitions given in 4.5, one may notice that for any retract neutrosophic crisp relation R^{r} : $A^{r} \rightarrow B^{r}$, we have:

– The domain of R^{r} is a crisp subset of X, namely, $\mathrm{Dom}(R^{r}) \subseteq$ X.

– The range of R^{r} is a crisp subset of Y, namely, $Rng(R^{r}) \subseteq Y$.

11.2 Corollary

For any retract neutrosophic crisp relation $R^{r} : A^{r} \rightarrow B^{r}$, we have that: $Dom\left(R^{r^{-1}}\right) = Rng(R^{r})$,

$$Rng\left(R^{r^{-1}}\right) = Dom(R^{r})$$

11.1 Example

From the Example 10.1 is easy to get $\mathrm{Dom}(R^{r})$, $\mathrm{Rng}(R^{r})$,

12 Retract Neutrosophic Crisp Relations' Operations

In this section, we call some definitions relations on retract neutrosophic crisp sets and its properties.

12.1 Definition

The complement of a retract neutrosophic crisp relation R^{r} ($\mathrm{co\ R}^{r}$, for short) may be defined as:

$$r co\ R^{r} = \langle R_3^{r}, R_2^{r}, R_1^{r} \rangle.$$

12**.2 Definition**

Consider R^{r} and S^{r} are two retract neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, retract neutrosophic crisp sets A^{r} and B^{r} in the form $\mathrm{A}^{r} = \langle \mathrm{A_1}^{r}, \mathrm{A_2}^{r}, \mathrm{A_3}^{r} \rangle$ in X, $\mathrm{B}^{r} = \langle \mathrm{B_1}^{r}, \mathrm{B_2}^{r}, \mathrm{B_3}^{r} \rangle$on Y.

4. The retract neutrosophic crisp relation R^{r} is a neutrosophic crisp subset of the retract neutrosophic crisp set S^{r} ($\mathrm{R}^{r} \breve{\subseteq} \mathrm{S}^{r}$), may be defined as one of the following two types:

$$\mathrm{R}^{r} \breve{\subseteq} \mathrm{S}^{r} \Leftrightarrow \mathrm{A}^{r}{}_{R1} \subseteq \mathrm{B}^{r}{}_{S1}, \mathrm{A}^{r}{}_{R2} \subseteq \mathrm{B}^{r}{}_{S2} \text{ and } \mathrm{A}^{r}{}_{R3} \subseteq \mathrm{B}^{r}{}_{S3}.$$

5. The neutrosophic intersection, $\mathrm{R}^{r} \breve{\cap} \mathrm{S}^{r}$, of any two neutrosophic sets retract A^{r} andB^{r}, may be defined as follows:

$$\mathrm{R}^{r} \breve{\cap} \mathrm{S}^{r} \Leftrightarrow \mathrm{A}^{r}{}_{R1} \cap \mathrm{B}^{r}{}_{S1}, \mathrm{A}^{r}{}_{R2} \cap \mathrm{B}^{r}{}_{S2} \text{ and } \mathrm{A}^{r}{}_{R3} \cap \mathrm{B}^{r}{}_{S3}.$$

6. The neutrosophic union, $\mathrm{R}^{r} \breve{\cup} \mathrm{S}^{r}$, of any two neutrosophic sets retract A^{r} andB^{r}, may be defined as follows:

$$\mathrm{R}^{r} \breve{\cup} \mathrm{S}^{r} \Leftrightarrow \mathrm{A}^{r}{}_{R1} \cup \mathrm{B}^{r}{}_{S1}, \mathrm{A}^{r}{}_{R2} \cup \mathrm{B}^{r}{}_{S2} \text{ and } \mathrm{A}^{r}{}_{R3} \cup \mathrm{B}^{r}{}_{S3}.$$

12. 1 Theorem

Let $\mathrm{R}^{r}, \mathrm{S}^{r}$ and Q^{r} be three retract neutrosophic crisp relations between X and Y for every$(x, y) \in X \times Y$, then:

a) $\mathrm{R}^{r} \breve{\subseteq} \mathrm{S}^{r} \rightarrow \mathrm{R}^{r^{-1}} \breve{\subseteq} \mathrm{S}^{r^{-1}}$.

b) $(\mathrm{R}^{r} \breve{\cup} \mathrm{S}^{r})^{-1} \rightarrow \mathrm{R}^{r^{-1}} \breve{\cup} \mathrm{S}^{r^{-1}}$, $(\mathrm{R}^{r} \breve{\cap} \mathrm{S}^{r})^{-1} \rightarrow \mathrm{R}^{r^{-1}} \breve{\cap} \mathrm{S}^{r^{-1}}$.

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c) $(R^{r^{-1}})^{-1} = R^{r}$.

d) $R^{r} \cap (S^{r} \,\breve{\cup}\, Q^{r}) = (R^{r} \,\breve{\cap}\, S^{r}) \,\breve{\cup}\, (R^{r} \,\breve{\cap}\, Q^{r}), R \,\breve{\cup}\, (S^{r} \,\breve{\cup}\, Q^{r}) = (R^{r} \,\breve{\cup}\, S^{r}) \cap (R^{r} \,\breve{\cup}\, Q^{r})$.

e) If $S^{r} \breve{\subseteq} R^{r}$, $Q^{r} \breve{\subseteq} R^{r}$, then $S^{r} \,\breve{\cup}\, Q^{r} \subseteq R^{r}$.

13 Composition of Neutrosophic Crisp Relations

Consider the three neutrosophic crisp sets: A of X, B of Y and C of Z; and the two retract neutrosophic crisp relations: $R^{r} : A^{r} \to B^{r}$ and $S^{r} : B^{r} \to C^{r}$; where $R^{r} = \langle R_1{}^{r}, R_2{}^{r}, R_3{}^{r} \rangle$, and $S^{r} = \langle S_1{}^{r}, S_2{}^{r}, S_3{}^{r} \rangle$. The composition of R^{r} and S^{r}, is denoted and defined as: $R^{r} \odot S^{r} = \langle R_1{}^{r} \circ S_1{}^{r}, R_2{}^{r} \circ S_2{}^{r}, R_3{}^{r} \circ S_3{}^{r} \rangle$, such that: $R_i{}^{r} \circ S_i{}^{r} : A_i{}^{r} \to C_i{}^{r}$, Where $R_i{}^{r} \circ S_i{}^{r} = \{(a_i, b_i): \exists b_i \in B_i{}^{r}, (a_i, b_i) \in R_i{}^{r} \text{and} (b_i, c_i) \in S_i{}^{r}\}$.

13.1 Corollary

For any two retract neutrosophic crisp relations: $R^{r} : A^{r} \to B^{r}$ and $S^{r} : B^{r} \to C^{r}$;

$$Dom(R^{r} \odot S^{r}) \subseteq Dom(R^{r})$$

$$Rng(R^{r} \odot S^{r}) \subseteq Rng(S^{r})$$

13.2 Corollary

Consider the three retract neutrosophic crisp relations: $R^{r} : A^{r} \to B^{r}$ and $S^{r} : B^{r} \to C^{r}$, and $K^{r} : C^{r} \to D^{r}$;

$$R^{r} \odot (S^{r} \odot K^{r}) = (R^{r} \odot S^{r}) \odot K^{r}$$

13.1 Example

From the Example 10.1 is easy to get $\mathrm{Dom}(R^{r})$, $\mathrm{Rng}(R^{r})$, and $Dom(R^{r} \odot S^{r})$

Conclusion

In this work, the concepts of star neutrosophic crisp relations and retract neutrosophic crisp relations were introduced. Added to, we have generalized the notion of crisp relation. Also, the main properties related to the neutrosophic crisp relations have been studied. Future work will be directed to study the notion of the neutrosophic crisp mapping for other types of relations based on neutrosophic crisp sets.

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