Harmonic meromorphic starlike functions of complex order involving Mittag-Leffler operator

Mays S. Abdul Ameer
Department of Mathematics
College of Education for Pure Sciences
Tikrit University
Salahaddin
Iraq
mays33@st.tu.edu.iq

Abdul Rahman S. Juma
Department of Mathematics
College of Education for Pure Science
University of Anbar
Ramadi
Iraq
eps.abdulrahman.juma@uoanbar.edu.iq

Raheam A. Al-Saphory*
Department of Mathematics
College of Education for Pure Sciences
Tikrit University
Salahaddin
Iraq
saphory@tu.edu.iq

Abstract. The main goal of the present paper is to introduce the class $\check{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$ of harmonic meromorphic starlike functions in order to define the Mittag-Leffler operator. Thus, we give some relations devoted to different classes of harmonic meromorphic functions to be a starlike function. Furthermore, we provide the fundamental and sufficient conditions for these considered functions to characterize some important subclasses of $\check{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Keywords: meromorphic univalent function, Mittag-Leffler operator, starlike function.

1. Introduction

A continuous function $f = u + iv$ is harmonic in the complex domain $F \subset \mathbb{C}$, where the real harmonic are u and v in F. Thus, we can express $f(z) = h(z) + \overline{g(z)}$, such that $h(z)$ and $g(z)$ are analytic in F. In addition, we say that $h(z)$ the analytic part and $g(z)$ the co-analytic part of f, see Clunie and Sheil-Small [5]. Then, as showing in [1]-[3] the class of harmonic functions

$*$. Corresponding author

$f(z) = h(z) + \overline{g(z)}$ are described by $\mathcal{H}$ in which have sense-preserving for the disk of type $U = \{z : |z| < 1\}$, where:

$$f(z) = h(z) + \overline{g(z)} = \frac{1}{z} + \sum_{n=2}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^n, \ |b_1| < 1. \tag{1}$$

Assume that the family of functions $f(z) = h(z) + \overline{g(z)}$ denote by $\mathcal{MT}_{\mathcal{H}}$, that are harmonic in $\mathcal{U}$ with the normalization:

$$h(z) = \frac{1}{z} - \sum_{n=2}^{\infty} a_n z^n, g(z) = \sum_{n=1}^{\infty} b_n z^n, \ \ a_n \geq 0, b_n \geq 0, \ b_1 < 1. \tag{2}$$

Note that $\mathcal{H}$ lessens to $\mathcal{S}$, the normalized analytic functions class such that, the co-analytic part $g \equiv 0$.

Let $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha, \gamma, d)$ denote the class of any functions in $\mathcal{MT}_{\mathcal{H}}$, for which:

$$\Re e \left\{ 1 + \frac{1}{d} \left(\frac{z\left(E_{\alpha,\gamma}h(z)\right)' - z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'}{E_{\alpha,\gamma}h(z) + \overline{E_{\alpha,\gamma}g(z)}} - 1 \right) \right\} > 0 \tag{3}$$

with $d \in \mathbb{C}\backslash\{0\}$ (see, [9]).

Now, consider the operator $E_{\alpha,\gamma}$ of Mittag-Leffler type [15] described by $E_{\alpha,\gamma}f(z) = \frac{1}{z} + \sum_{n=2}^{\infty} D_n(\alpha, \gamma) a_n z^n$, where:

$$D_n(\alpha, \gamma) = \frac{\Gamma(\gamma)}{\Gamma(\alpha(n - 1 + \gamma)}, \ z, \alpha, \gamma \in \mathbb{C}; \ \Re e(\alpha) > 0; \ \gamma \neq 0, -1, -2, \ldots$$

and $\Gamma(z)$ is gamma function. Currently, we can define the Mittag-Leffler operator as:

$$E_{\alpha,\gamma}f(z) = E_{\alpha,\gamma}h(z) + \overline{E_{\alpha,\gamma}g(z)} \tag{4}$$

where:

$$E_{\alpha,\gamma}h(z) = \frac{1}{z} - \sum_{n=2}^{\infty} D_n(\alpha, \gamma) a_n z^n$$

and

$$E_{\alpha,\gamma}g(z) = \sum_{n=1}^{\infty} D_n(\alpha, \gamma) b_n \bar{z}^n.$$

We note that $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}$ is a harmonic function class in the unit disk studied such as in [8], [10].

Let $\breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}(\alpha, \gamma, d)$ indicates the subclass at $\mathcal{MT}_{\mathcal{H}}$ of function $f(z) = h(z) + \overline{g(z)} \in \mathcal{MT}_{\mathcal{H}}$, such that satisfied the condition:

$$\sum_{n=1}^{\infty} (2(n - 1 + |d|) D_n(\alpha, \gamma) a_n + (n + 1 + |n + 1 - 2d|) D_n(\alpha, \gamma) b_n) \leq 4|d|. \tag{5}$$

Also let $\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$ the subclass of $\mathcal{MT}_{\mathcal{H}}$ that verified the following:

$$\sum_{n=1}^{\infty}\left[(n-1)\frac{\Re e(d)}{|d|}+|d|\right]D_n(\alpha,\gamma)a_n + \left[(n+1)\frac{\Re e(d)}{|d|}-|d|\right]D_n(\alpha,\gamma)b_n \leq 2|d|. \tag{6}$$

Numerous researchers was investigated the class $\mathcal{H}$ of harmonic univalent functions in different situations [1], [5], [6], [10], [13], [14]. Thus, there were many related research papers are explored the class $\mathcal{H}$ associated with its subclass which is presented by Jahangiri and Noor et al. as in [6], [12] for meromorphic functions. So, Silverman and Jahangiri in 1999 are studied these results to the case of meromorphic functions through the negative coefficients [14]. More recently, Juma et al. 2021 contribute an important extension on bi-univalent functions is associated with the operator of Srivastava Attiya type, through the class of starlike or convex functions of order α [11]. Recently, it was proved that the sufficient coefficient conditions:

$$\sum_{n=2}^{\infty} n\left(|a_n|+|b_n|\right)\leq 1, b_1=0$$

is sufficient for $f(z)=h(z)+\overline{g(z)}$ to be harmonic univalent and starlike function [4]. Furthermore, the constant condition is also needed if $b_1=0$ and a_n, $b_n<0$, in equation (1), and hence Jahangiri in [6] is shown that if $f(z)=h(z)+\overline{g(z)}$, then we have:

$$\sum_{n=2}^{\infty}\left(\frac{n-\lambda}{1-\lambda}|a_n|+\frac{n+\lambda}{1-\lambda}|b_n|\right)\leq 1, \tag{7}$$

where $0\leq\lambda<1$ and then f is harmonic univalent and starlike function in $\mathcal{U}$. Consequently, the condition (7) is necessary if $h(z)$ and $g(z)$ are of the form (1), for example $\lambda=0$ is and for $\lambda=b_1=0$, (see [6], [7], [14]).

2. The main results

In this section, the main important results are stated to give some relations devoted to different classes in the next outcomes.

Theorem 2.1. $\breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)\subset\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Proof. Let $f\in\breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$, and according to the equation (2) allows to show that condition (5) holds, and then:

$$\Re e\left\{\frac{(d-1)\left[E_{\alpha,\gamma}h(z)+\overline{E_{\alpha,\gamma}g(z)}+z\left(E_{\alpha,\gamma}h(z)\right)'-z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'\right]}{d\left(E_{\alpha,\gamma}h(z)+\overline{E_{\alpha,\gamma}}g(z)\right)}\right\}>0,$$

where $d \in \mathbb{C}\backslash\{0\}$. Now by using the fact that:

$$\alpha \geq 0 \text{ and } \Re e(w) > \alpha \Leftrightarrow |1+w| > |1-w|, \ \forall w \in \mathbb{C}$$

then, we can get:

$$\begin{aligned}
&\left|(2d-1)\left[E_{\alpha,\gamma}h(z)+\overline{E_{\alpha,\gamma}g(z)}\right]+z\left(E_{\alpha,\gamma}h(z)\right)'-z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'\right| \\
&-\left|E_{\alpha,\gamma}h(z)+\overline{E_{\alpha,\gamma}g(z)}-z\left(E_{\alpha,\gamma}h(z)\right)'-z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'\right| \\
&=(2d-1)\left[\frac{1}{z}-\sum_{n=2}^{\infty}D_n(\alpha,\gamma)a_nz^n+\sum_{n=1}^{\infty}D_n(\alpha,\gamma)b_n\bar{z}^n\right]-\frac{1}{z} \\
&+\left|\sum_{n=2}^{\infty}D_n(\alpha,\gamma)na_nz^n+\sum_{n=1}^{\infty}D_n(\alpha,\gamma)nb_n\bar{z}^n\right| \\
&-\left|\frac{1}{z}-\sum_{n=2}^{\infty}D_n(\alpha,\gamma)na_nz^n+\sum_{n=1}^{\infty}D_n(\alpha,\gamma)nb_n\bar{z}^n\right. \\
&\left.-\frac{1}{z}+\sum_{n=2}^{\infty}D_n(\alpha,\gamma)na_n\bar{z}^n+\sum_{n=1}^{\infty}D_n(\alpha,\gamma)nb_n\bar{z}^n\right| \\
&\left|2dz^{-1}-\sum_{n=2}^{\infty}(2d-1+n)D_n(\alpha,\gamma)a_nz^n\right. \\
&\left.-\sum_{n=1}^{\infty}(n+1-2d)D_n(\alpha,\gamma)b_n\bar{z}^n\right|-\left|\sum_{n=2}^{\infty}(n-1)D_n(\alpha,\gamma)a_nz^n\right. \\
&\left.-\sum_{n=1}^{\infty}(n+1)D_n(\alpha,\gamma)b_n\bar{z}^n\right| \\
&\geq 2|d||z|^{-1}-\sum_{n=2}^{\infty}2(n-1+|d|)D_n(\alpha,\gamma)a_n|z|^n \\
&-\sum_{n=1}^{\infty}(n+1+|n+1-2d|)D_n(\alpha,\gamma)b_n|\bar{z}|^n 2d \\
&-\left(\sum_{n=2}^{\infty}2(n-1+|d|)D_n(\alpha,\gamma)a_n+\right. \\
&\left.\sum_{n=1}^{\infty}(n+1+\mid n+1-2d\mid)D_n(\alpha,\gamma)b_n\right)\geq 0.
\end{aligned}$$

Consequently, the function:

$$f(z)=\frac{1}{z}-\sum_{n=2}^{\infty}\frac{|d|}{(n-1+|d|)}l_nz^n+\frac{2|d|}{n+1+\mid n+1-2d}t_n\bar{z}^n, \tag{8}$$

where $l_n, t_n \geq 0$. Then, in this case:

$$\sum_{n=2}^{\infty} l_n + \sum_{n=1}^{\infty} t_n = 1$$

and hence the functions $f(z)$ in (8) belong to $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$, since:

$$\sum_{n=2}^{\infty} 2(n-1+|d|)n(\alpha,\gamma)a_n + \sum_{n=1}^{\infty}(n+1+|n+1-2d|)D_n(\alpha,\gamma)b_n$$
$$= 2|d|\left(\sum_{n=2}^{\infty} l_n + \sum_{n=1}^{\infty} t_n\right) = 4|d|. \qquad \square$$

Next, we show that $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d) \subset \breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Theorem 2.2. $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d) \subset \breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Proof. Assume that $f \in \breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$, from (3), we get:

$$\Re e\left\{\frac{1}{d}\left(\frac{z\left(E_{\alpha,\gamma}h(z)\right)'^{-z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'}}{E_{\alpha,\gamma}h(z)+\overline{(E_{\alpha,\gamma}g(z))}}\right)\right\} \geq -1,$$

$$\Re e\left\{\frac{1}{d}\left(\frac{\frac{-1}{z}-\sum_{n=2}^{\infty} D_n(\alpha,\gamma)na_nz^n+\sum_{n=1}^{\infty} D_n(\alpha,\gamma)nb_n\bar{z}^n}{\frac{1}{z}-\sum_{n=2}^{\infty} D_n(\alpha,\gamma)a_nz^n+\sum_{n=1}^{\infty} D_n(\alpha,\gamma)b_n\bar{z}^n}\right)\right\} > -1$$

therefore, we get:

$$\Re e\left\{\frac{1}{d}\left(\frac{-\sum_{n=2}^{\infty}(n-1)D_n(\alpha,\gamma)a_nz^n-\sum_{n=1}^{\infty}(n+1)D_n(\alpha,\gamma)b_n\bar{z}^n}{\frac{1}{z}-\sum_{n=2}^{\infty} D_n(\alpha,\gamma)a_nz^n+\sum_{n=1}^{\infty} D_n(\alpha,\gamma)b_n\bar{z}^n}-1\right)\right\} > -1.$$

Taking $z \to 1^-$ on the real axis, we get:

$$\frac{\sum_{n=2}^{\infty}(n-1)D_n(\alpha,\gamma)a_n-\sum_{n=1}^{\infty}(n+1)D_n(\alpha,\gamma)b_n}{1-\sum_{n=2}^{\infty} D_n(\alpha,\gamma)a_n+\sum_{n=1}^{\infty} D_n(\alpha,\gamma)b_n}\Re e\left(\frac{1}{d}\right) \leq 1,$$

then:

$$\sum_{n=2}^{\infty}(n-1)D_n(\alpha,\gamma)a_n-\sum_{n=1}^{\infty}(n+1)D_n(\alpha,\gamma)b_n$$
$$\leq \frac{|d|^2}{\Re e(d)}\left(1-\sum_{n=2}^{\infty} D_n(\alpha,\gamma)a_n+\sum_{n=1}^{\infty} D_n(\alpha,\gamma)b_n\right)$$

and so:

$$\sum_{n=1}^{\infty}\left[(n-1)\frac{\Re e(d)}{|d|}+|d|\right]D_n(\alpha,\gamma)a_n+\left[(n+1)\frac{\Re e(d)}{|d|}-|d|\right]D_n(\alpha,\gamma)b_n \leq 2|d|.$$

Then, by (7), we get $f \in \breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$. $\square$

Now, we demonstrate that the subclass $\breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)$ is equal to subclass $\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Theorem 2.3. $\mathcal{R}\breve{\mathcal{T}}_{\mathcal{H}}(\ \alpha,\gamma,d)=\breve{\mathcal{M}}\mathcal{T}_{H}(\alpha,\gamma,d)=\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$ *where*

$$(0<d\leq 1,z,\alpha,\gamma\in\mathbb{C},\Re e(\alpha)>0;\gamma\neq 0,-1,-2,\ldots).$$

Proof. If $d\in(0,1]$, then the inequalities (5) and (6) are equivalent. Thus, by using Theorems 2.1 and 2.2, we getting $\breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)=\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)$. □

The following outcome proves $\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)\nsubseteq\ \breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$.

Theorem 2.4. $\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)\nsubseteq\ \breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,d)$ *if* $\Re e(d)\leq 0$ *and* $\Re e(d)\neq-\frac{1}{2}$ *otherwise* $d\in\left(\frac{3}{2},\infty\right)$.

Proof. Assume that $f(z)=z-\frac{1}{\alpha+1}z^2,\alpha>-1,f\in\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)$. Since:

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[(n-1)\frac{\Re e(d)}{|d|}+|d|\right]D_n(\alpha,\gamma)a_n+\left[(n+1)\frac{\Re e(d)}{|d|}-|d|\right]D_n(\alpha,\gamma)b_n\\&=|d|.1+\frac{\Re e(d)}{|d|}+|d|=2|d|+\frac{\Re e(d)}{|d|}\leq 2|d|,\end{aligned}$$

when $d\in\mathbb{C}\backslash\{0\}$ and $\Re e(d)<0$. Moreover, if we suppose:

$$r=\Re e(d)<0,k\geq 0,\ni 1+2r(1-k)>0.$$

Now, in this situation, if we choose $z=\frac{d(1-\hbar)}{1+d(1-k)}$, then $z\in U$.

Since $E_{\alpha,\gamma}f(z)=z-z^2$, we get:

$$1+\frac{1}{d}\left(\frac{z\left(E_{\alpha,\gamma}h(z)\right)'-z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'}{E_{\alpha,\gamma}h(z)+\overline{(E_{\alpha,\gamma}g(z))}}-1\right)=k<0$$

and hence $f\notin\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)$. Now, let:

$$f(z)=z-\frac{1}{\alpha+1}\bar{z}^2.$$

Then $f\in\breve{\mathcal{S}}\mathcal{T}_{\mathcal{H}}(\ \alpha,\gamma,d)$, where $d\in\left(\frac{3}{2},+\infty\right)$, because:

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[(n-1)\frac{\Re e(d)}{|d|}+|d|\right]D_n(\alpha,\gamma)a_n+\left[(n+1)\frac{\Re e(d)}{|d|}-|d|\right]D_n(\alpha,\gamma)b_n\\&=|d|.1+\left(3\frac{\Re e(d)}{|d|}-|d|\right)\cdot 1\\&=3\frac{\Re e(d)}{|d|}\leq 2|d|.\end{aligned}$$

Suppose that $k \geq 0, \ni d + d(k-1) < 0$. We can take $z = -\frac{d(\mathfrak{k}-1)}{3+d(\mathfrak{k}-1)}$, then $z \in \mathcal{U}$ and through the definition of f, we have:

$$1 + \frac{1}{d}\left(\frac{z\left(E_{\alpha,\gamma}h(z)\right)' - z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'}{E_{\alpha,\gamma}h(z) + \left(E_{\alpha,\gamma}g(z)\right)} - 1\right) = k < 0$$

and hence $f \notin \breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}\,(\alpha, \gamma, d)$. □

In the next theorem, we verify that the $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}\,(\ \alpha, \gamma, d) \nsubseteq \breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}\,(\ \alpha, \gamma, d)$.

Theorem 2.5. $\breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}\,(\ \alpha, \gamma, d) \nsubseteq \breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}\,(\ \alpha, \gamma, d)$, *whenever* $d < -1, \gamma \geq 0$.

Proof. Let the function:

$$f_\delta = z - \delta z^2. \tag{9}$$

Then $f \in \breve{\mathcal{M}}\mathcal{T}_{\mathcal{H}}\,(\alpha, \gamma, d)$ and $\delta > \frac{d}{1+d}$, we have:

$$\Re e\left\{1 + \frac{1}{d}\left(\frac{z\left(E_{\alpha,\gamma}h(z)\right)' - z\left(\overline{E_{\alpha,\gamma}g(z)}\right)'}{E_{\alpha,\gamma}^{A(z)+(E_{\alpha,\gamma}g(z))}} - 1\right)\right\} = \left\{1 + \frac{\delta z}{d(\delta z - 1)}\right\} > 0, z \in \mathcal{U}.$$

Also:

$$\sum_{n=1}^{\infty}\left(2(n-1+|d|)D_n(\alpha,\gamma)a_n + (n+1+|n+1-2d|)D_n(\alpha,\gamma)b_n\right)$$
$$= 2|d| + 2(1 + |d|\delta > 4)$$

because $\delta > \frac{d}{1+d} > 1$, then $f \notin \breve{\mathcal{R}}\mathcal{T}_{\mathcal{H}}\,(\alpha, \gamma, d)$. □

3. Conclusion

We have been shown that the harmonic meromorphic functions which are starlike functions of complex order. More precisely, various interesting results concerning some relations devoted to different classes of harmonic meromorphic functions in connection to the Mittag-Leffler operator are demonstrated and analyzed.

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