# Numerical Solution of Oscillatory Reaction – Diffusion System of $\lambda-\omega$ Type

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## Abstract

Numerical solution of oscillatory reaction–diffusion system of $\lambda-\omega$ type was done by using two finite difference schemes . The first one is the explicit scheme and the second one is the implicit rank– Nicholson scheme with averaging of $f(u,v)$ and without averaging of $f(u,v)$. The comparison showed that Crank–Nicholson scheme is better than explicit scheme and Crank–Nicholson scheme with averaging of $f(u,v)$ is more accurate but it needs more time and double storage and double time steps than Crank–Nicholson scheme without averaging of $f(u,v)$.

## 1. Introduction and Mathematical Model

$\lambda-\omega$ Systems are a class of simple examples of two coupled reaction – diffusion equations whose kinetics have a stable limit cycle:

$$\left.\begin{aligned} u_t &= u_{xx} + \lambda(r)u - \omega(r)v \\ v_t &= v_{xx} + \omega(r)u + \lambda(r)v \end{aligned}\right\} \quad (1a)$$

Here $u$ and $v$ are functions of space $x$ and time $t$, with $x \in R$ and $t > 0$ , and $r = \sqrt{u^2+v^2}$ [1]. $\lambda-\omega$ Systems are an important prototype for the study of reaction – diffusion equations which have been used to model a number of biological and chemical systems [2]. Many systems in biology and chemistry are intrinsically oscillatory. In such cases, the stable state in the absence of spatial variation is not a stationary equilibrium , but rather consists of temporal oscillations in the interacting chemical or biological species . Examples include intracellular calcium signaling , the Belousov – Zhabotinskii reaction and some predator – prey interactions . These systems also exhibit spatial interactions , which are often modeled by diffusion . This combination of local oscillations and spatial diffusion produces a very wide range of spatiotemporal behaviors , including spiral waves , target patterns and spatiotemporal chaos . In one spatial dimension , the equivalent of both spiral waves and target patterns are periodic travelling waves , which are periodic functions of space and time, moving with constant shape and speed. Sherratt studied various features of the oscillatory reaction-diffusion systems of $\lambda-\omega$ type in series of papers. He presented numerical evidence for a complex sequence of bifurcations in the unstable region of parameter space [6]. He also used numerical techniques to give suggestion that the spatiotemporal irregularities are genuinely chaotic [3]. He also made a comparison between the numerical solutions of the oscillatory reaction-diffusion systems of $\lambda-\omega$ type [4]. He used numerical results suggesting that when this system is solved on semi-infinite domain subject to Dirichlet boundary conditions in which the variables are fixed at zero periodic travelling waves develop in the domain [5]. Borzi and Griesse presented the formulation, analysis, and numerical solution of distributed optimal control problems governed by lambda-omega systems [6]. Garvie and Blowey undertook the numerical analysis of a reaction-diffusion system of $\lambda-\omega$ type, their results are presented for a fully practical piecewise linear finite element method by mimicing results in the continuous case [7]. In this paper, the numerical solution of an oscillatory reaction-diffusion system of $\lambda-\omega$ type by the use of two finite difference schemes is done to the following system [4].

$$\left.\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + (1-u^2-v^2)(u-3v) \\ \frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial x^2} + (1-u^2-v^2)(3u+v) \end{aligned}\right\} \quad (1b)$$

With initial and boundary conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$ at $x=0$ and $u=v=0.01$ at $x=0$, $u=v=0$, for $x>0$ at $t=0$ and $u=v=0$ at $x=L$ where $L$ is sufficiently large number.

## 2. Discretization of the $\lambda-\omega$ System by Using Explicit Scheme

By using the finite difference approximations for the derivatives [8], we have

$$\frac{u_{i,j+1}-u_{i,j}}{k} = \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2} + \left(1-(u_{i,j})^2-(v_{i,j})^2\right)(u_{i,j}-3v_{i,j}) \Rightarrow$$

$$u_{i,j+1} = r(u_{i-1,j}+u_{i+1,j}) + (1-2r+k)u_{i,j} - k(u_{i,j})^3 + 3kv_{i,j}(u_{i,j})^2 - k(v_{i,j})^2 u_{i,j} - 3kv_{i,j} + 3k(v_{i,j})^2 \quad (2)$$

Where $r = k/h^2$

For the boundary condition $u_x = 0$, by using the central difference formula, we get

$$\frac{u_{1,j}-u_{-1,j}}{2h} = 0 \Rightarrow \quad (3)$$
$$u_{1,j} = u_{-1,j}$$

from (2) , we have , for $i=0$

$$u_{0,j+1} = r(u_{-1,j}+u_{1,j}) + (1-2r+k)u_{0,j} - k(u_{0,j})^3 + 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^3 \quad (4)$$

Substitute (3) in (4) , we obtain

$$u_{0,j+1} = 2ru_{1,j} + (1-2r+k)u_{0,j} - k(u_{0,j})^3 + 3kv_{0,j}(u_{0,j})^2 - k(v_{0,j})^2 u_{0,j} - 3kv_{0,j} + 3k(v_{0,j})^2 \quad (5)$$

Equation (5) represents the right boundary condition, with respect to equation (1b)

$$\frac{v_{i,j}+1-v_{i,j}}{k}=\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{h^2}$$
$$+\left(1-(u_{i,j})^2-(v_{i,j})^2\right)(3u_{i,j}+v_{i,j})$$
$$v_{i,j+1}=r(v_{i-1,j}+v_{i+1,j})+(1-2r+k)\,v_{i,j}-k(v_{i,j})^3 \qquad (6)$$
$$-kv_{i,j}(u_{i,j})^2-3k(v_{i,j})^2u_{i,j}+3ku_{i,j}-3k(u_{i,j})^3$$

The boundary condition $v_x=0$, by using the central difference formula, we have:

$$\frac{v_{1,j}-v_{-1,j}}{2h}=0\Rightarrow\ v_{1,j}=v_{-1,j} \qquad (7)$$

From (6) we have , for $i=0$

$$v_{0,j+1}=r(v_{-1,j}+v_{1,j})+(1-2r+k)v_{0,j}-k(v_{0,j})^3 \qquad (8)$$
$$-kv_{0,j}(u_{0,j})^2-3k(v_{0,j})^2u_{0,j}+3ku_{0,j}-3k(u_{0,j})^3$$

Substitute (7) in (8), we obtain

$$v_{0,j+1}=2rv_{1,j}+(1-2r+k)v_{0,j}-k(v_{0,j})^3 \qquad (9)$$
$$-kv_{0,j}(u_{0,j})^2-3k(v_{0,j})^2u_{0,j}+3ku_{0,j}-3k(u_{0,j})^3$$

which represents the right boundary condition, with respect to equation (1b).

**3. Discretization of the $\lambda-\omega$ System by Using of Crank – Nicholson Scheme without Averaging of $f(u,v)$**

The diffusion term $u_{xx}$ in this method is represented with central differences, with their values at the current and previous time steps averaged [8].

$$\frac{u_{i,j+1}-u_{i,j}}{k}=\frac{u_{i+1,j+1}-2u_{i,j}+u_{i-1,j+1}}{2h^2}+\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{2h^2}$$
$$+\left(1-(u_{i,j})^2-(v_{i,j})^2\right)(u_{i,j}-3v_{i,j})\Rightarrow$$
$$-r(u_{i-1,j+1}+u_{i+1,j+1})+(2+2r)u_{i,j+1}=r(u_{i-1,j}+u_{i+1,j})$$
$$+(2-2r+k)u_{i,j}-k(u_{i,j})^3+3kv_{i,j}(u_{i,j})^2-k(v_{i,j})^2u_{i,j}-3kv_{i,j}+3k(v_{i,j})^3$$
$$(10)$$

For the boundary condition $u_x=0$, from central difference formula , we have

$$\frac{u_{1,j}-u_{-1,j}}{2h}=0\ \Rightarrow\ u_{1,j}=u_{-1,j} \qquad (11)$$

from (10) , we have , for $i=0$

$$-r(u_{-1,j+1}+u_{1,j+1})+(2+2r)u_{0,j+1}=r(u_{-1,j}+u_{1,j})$$
$$+(2-2r+k)u_{0,j}-k(u_{0,j})^3+3kv_{0,j}(u_{0,j})^2-k(v_{0,j})^2u_{0,j}-3kv_{0,j}+3k(v_{0,j})^3$$

from (11) , we get

$$u_{i,j+1}=u_{-1,j+1} \qquad (12)$$

substitute (11) and (12) in (10), we obtain the equation for the left boundary condition which is

$$-2ru_{1,j+1}+(2+2r)u_{0,j+1}=2ru_{1,j}+(2-2r+k)u_{0,j}-k(u_{0,j})^3 \qquad (13)$$
$$+3kv_{0,j}(u_{0,j})^2-k(v_{0,j})^2u_{0,j}-3kv_{0,j}+3k(v_{0,j})^3$$

for (b) , we have

$$\frac{v_{i,j+1}-v_{i,j}}{k}=\frac{v_{i+1,j+1}-2v_{i,j+1}+v_{i-1,j+1}}{2h^2}+\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{2h^2}$$
$$+\left(1-(u_{i,j})^2-(v_{i,j})^2\right)(3u_{i,j}+v_{i,j})\Rightarrow$$
$$-r(v_{i-1,j+1}+v_{i+1,j+1})+(2+2r)v_{i,j+1}=r(v_{i-1,j}+v_{i+1,j})$$
$$+(2-2r+k)v_{i,0}-k(v_{i,j})^3-kv_{i,j}(u_{i,j})^2-3k(v_{i,j})^2u_{i,j}+3ku_{i,j}-3k(u_{i,j})^3$$
$$(14)$$

For the boundary condition $v_x=0$, from central difference formula, we have

$$\frac{v_{1,j}-v_{-1,j}}{2h}=0\ \Rightarrow\ v_{1,j}=v_{-1,j} \qquad (15)$$

from (14), we have

$$-r(v_{-1,j+1}+v_{1,j+1})+(2+2r)v_{0,j+1}=r(v_{-1,j}+v_{1,j})$$
$$+(2-2r+k)v_{0,j}-k(v_{0,j})^3-k(v_{0,j})(u_{0,j})^2$$
$$-3k(v_{0,j})^2u_{0,j}+3ku_{0,j}-3k(u_{0,j})^3$$

from (15), we have

$$v_{1,j+1}=v_{-1,j+1} \qquad (16)$$

Substitute (15) and (16) in (14), we have the equation for the left boundary condition, with respect to equation (1b)-

$$-2rv_{1,j+1}+(2+2r)v_{0,j+1}=2rv_{1,j}+(2-2r+k)v_{0,j}-k(v_{0,j})^3 \qquad (17)$$
$$-k(v_{0,j})(u_{0,j})^2-3k(v_{0,j})^2u_{0,j}+3ku_{0,j}-3k(u_{0,j})^3$$

The equations in above are especially pleasing to view in their tridiagonal matrix form $A_1X_1=B_1$, where $A_1$ is the coefficient matrix, X is the unknown vector and B is the known vector as shown below:

$$
\begin{bmatrix}
2+2r & -2r & & & & & \\
-r & 2+2r & -r & & & & \\
 & & & & & & \\
 & & -r & 2+2r & -r & & \\
 & & & -r & 2+2r & -r & \\
 & & & & -r & 2+2r & \\
 & & & & & 1 &
\end{bmatrix}
\begin{bmatrix}
u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{p,j+1} \\ \vdots \\ u_{n-2,j+1} \\ u_{n-1,j+1} \\ u_{n,j+1}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
(2-2r+k)u_{1,j}+2ru_{2,j}-k(u_{1,j})^3+3kv_{1,j}(u_{1,j})^2-k(v_{1,j})^2u_{1,j}-3kv_{1,j}+3k(v_{1,j})^3 \\
ru_{1,j}+(2-2r+k)u_{2,j}+ru_{3,j}-k(u_{2,j})^3+3kv_{2,j}(u_{2,j})^2-k(v_{2,j})^2u_{2,j}-3kv_{2,j}+3k(v_{2,j})^3 \\
 \\
ru_{p-1,j}+(2-2r+k)u_{p,j}+ru_{p+1,j}-k(u_{p,j})^3+3kv_{p,j}(u_{p,j})^2-k(v_{p,j})^2u_{p,j}-3kv_{p,j}+3k(v_{p,j})^3 \\
 \\
ru_{n-3,j}+(2-2r+k)u_{n-2,j}+ru_{n-1,j}-k(u_{n-2,j})^3+3kv_{n-2,j}(u_{n-2,j})^2-k(v_{n-2,j})^2u_{n-2,j}-3kv_{n-2,j}+3k(v_{n-2,j})^3 \\
ru_{n-2,j}+(2-2r+k)u_{n-1,j}-k(u_{n-1,j})^3+3kv_{n-1,j}(u_{n-1,j})^2-k(v_{n-1,j})^2u_{n-1,j}-3kv_{n-1,j}+3k(v_{n-1,j})^3 \\
0
\end{bmatrix}
$$

$$
\begin{bmatrix}
2+2r & -2r & & & & & \\
-r & 2+2r & -r & & & & \\
 & & & & & & \\
 & & -r & 2+2r & -r & & \\
 & & & -r & 2+2r & -r & \\
 & & & & -r & 2+2r & \\
 & & & & & 1 &
\end{bmatrix}
\begin{bmatrix}
v_{1,j+1} \\ v_{2,j+1} \\ \vdots \\ v_{p,j+1} \\ \vdots \\ v_{n-2,j+1} \\ v_{n-1,j+1} \\ v_{n,j+1}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
(2-2r+k)v_{1,j}+2rv_{2,j}-k(v_{1,j})^3-3ku_{1,j}(v_{1,j})^2-3k(u_{1,j})^2v_{1,j}+3ku_{1,j}-3k(u_{1,j})^3 \\
rv_{1,j}+(2-2r+k)v_{2,j}+rv_{3,j}-k(v_{2,j})^3-ku_{2,j}(v_{2,j})^2-3k(u_{2,j})^2v_{2,j}+3ku_{2,j}-3k(u_{2,j})^3 \\
 \\
rv_{p-1,j}+(2-2r+k)v_{p,j}+rv_{p+1,j}-k(v_{p,j})^3-ku_{p,j}(v_{p,j})^2-3k(u_{p,j})^2v_{p,j}+3ku_{p,j}-3k(u_{p,j})^3 \\
 \\
rv_{n-3,j}+(2-2r+k)v_{n-2,j}+rv_{n-1,j}-k(v_{n-2,j})^3-ku_{n-2,j}(v_{n-2,j})^2-3k(u_{n-2,j})^2v_{n-2,j}+3ku_{n-2,j}-3k(u_{n-2,j})^3 \\
rv_{n-2,j}+(2-2r+k)v_{n-1,j}-k(v_{n-1,j})^3-kv_{n-1,j}(u_{n-1,j})^2-3k(v_{n-1,j})^2u_{n-1,j}+3ku_{n-1,j}-3k(u_{n-1,j})^3 \\
0
\end{bmatrix}
$$

**4. Discretization of the $\lambda-\omega$ System by Using of Crank – Nicholson Scheme with Averaging of $f(u,v)$**

In this scheme the functions $u$ and $v$ in system (1b)will be replaced by their values at the current and previous steps averaged.

$$\frac{u_{i,j+1}-u_{i,j}}{k}=\frac{u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}}{2h^2}+\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{2h^2} \quad (18)$$
$$+\left(1-\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)^2-\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right)^2\right)$$
$$\left(\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)-3\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right)\right)\Rightarrow$$
$$\frac{u_{i,j+1/2}-u_{i,j}}{k/2}=\frac{u_{i+1,j+1/2}-2u_{i,j+1/2}+u_{i-1,j+1/2}}{2h^2} \quad (19a)$$
$$+\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{2h^2}+\left(1-\left(u_{i,j}\right)^2\right)-\left(u_{i,j}-3v_{i,j}\right)$$
$$\frac{u_{i,j+1}-u_{i,j+1/2}}{k/2}=\frac{u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}}{2h^2}$$
$$+\frac{u_{i+1,j+1/2}-2u_{i,j+1/2}+u_{i-1,j+1/2}}{2h^2}+\left(1-\left(u_{i,j+1/2}\right)^2-\left(v_{i,j+1/2}\right)^2\right) \quad (19b)$$
$$\left(u_{i,j+1/2}-3v_{i,j+1/2}\right)\Rightarrow$$
$$-r_1\left(u_{i-1,j+1}+u_{i+1,j+1}\right)+\left(2+2r_1\right)u_{i,j+1}=r_1\left(u_{i-1,j+1/2}+u_{i+1,j+1/2}\right) \quad (20a)$$
$$+\left(2-2r_1+k_1\right)u_{i,j+1/2}-k_1\left(u_{i,j+1/2}\right)^3+3k_1v_{i,j+1/2}\left(u_{i,j+1/2}\right)^2$$
$$-k_1\left(v_{i,j+1/2}\right)^2u_{i,j+1/2}-3k_1v_{i,j+1/2} \quad (20b)$$

Where $r_1=k/2h^2$

By the same manner we have followed in (4), with repeat to right boundary condition $u_x=0$ , we get

$$-2r_1u_{1,j+1/2}+\left(2+2r_1\right)u_{0,j+1/2}=2r_1u_{1,j}+\left(2-2r_1+k_1\right)u_{0,j} \quad (21a)$$
$$-k_1\left(u_{0,j}\right)^3+3k_1v_{0,j}\left(u_{0,j}\right)^2-k_1\left(v_{0,j}\right)^2u_{0,j}-3k_1v_{0,j}+3k_1\left(v_{0,j}\right)^3$$
$$-2r_1u_{1,j+1}+\left(2+2r_1\right)u_{0,j+1}=2r_1u_{1,j+1/2}+\left(2-2r_1+k_1\right)u_{0,j+1/2} \quad (21b)$$
$$-k_1\left(u_{0,j+1/2}\right)^3+3k_1v_{0,j+1/2}\left(u_{0,j+1/2}\right)^2-k_1\left(v_{0,j+1/2}\right)^2u_{0,j+1/2}$$
$$-3k_1v_{0,j+1/2}+3k_1\left(v_{0,j+1/2}\right)^3$$

with repeat to (1b) , we have

$$\frac{v_{i,j+1}-v_{i,j}}{k}=\frac{v_{i+1,j+1}-2v_{i,j+1}+v_{i-1,j+1}}{2h^2}+\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{2h^2} \quad (22)$$
$$+\left(1-\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)^2-\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right)^2\right)$$
$$\left(3\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)+\left(\frac{v_{i,j+1}+v_{i,j}}{2}\right)\right)\Rightarrow$$

$$\frac{v_{i,j+1/2}-v_{i,j}}{k/2}=\frac{v_{i+1,j+1/2}-2v_{i,j+1/2}+v_{i-1,j+1/2}}{2h^2}$$
$$+\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{2h^2}+\left(1-\left(u_{i,j}\right)^2-\left(v_{i,j}\right)^2\right)\left(3u_{i,j}+v_{i,j}\right) \quad (23a)$$
$$\frac{v_{i,j+1}-v_{i,j+1/2}}{k/2}=\frac{v_{i+1,j+1}-2v_{i,j+1}+v_{i-1,j+1}}{2h^2}$$
$$+\frac{v_{i+1,j+1/2}-2v_{i,j+1/2}+v_{i-1,j+1/2}}{2h^2}+\left(1-\left(u_{i,j+1/2}\right)^2-\left(v_{i,j+1/2}\right)^2\right) \quad (23b)$$
$$\left(3u_{i,j+1/2}+v_{i,j+1/2}\right)\Rightarrow$$
$$-r_1\left(v_{i-1,j+1/2}+v_{i+1,j+1/2}\right)+\left(2+2r_1\right)v_{i,j+1/2}=r_1\left(v_{i-1,j}+v_{i+1,j}\right) \quad (24a)$$
$$+\left(2-2r_1+k_1\right)v_{i,j}-k_1\left(u_{i,j}\right)^3-k_1v_{i,j}\left(u_{i,j}\right)^2$$
$$-3k_1\left(v_{i,j}\right)^2u_{i,j}+3k_1u_{i,j}-3k\left(u_{i,j}\right)^3$$
$$-r_1\left(u_{i-1,j+1}+u_{i+1,j+1}\right)+\left(2+2r_1\right)u_{i,j+1}=r_1\left(u_{i-1,j+1/2}+u_{i+1}\right)$$
$$+\left(2-2r_1+k_1\right)u_{i,j+1/2}-k_1\left(u_{i,j+1/2}\right)^3-k_1v_{i,j+1/2}\left(u_{i,j+1/2}\right)^2 \quad (24b)$$
$$-k_1\left(v_{i,j+1/2}\right)^2u_{i,j+1/2}+3k_1\left(u_{i,j+1/2}\right)^3$$

By the same manner we have already followed in (4) , with repeat to the right boundary condition , we get

$$-2r_1v_{1,j+1/2}+\left(2+2r_1\right)v_{0,j+1/2}=2r_1v_{1,j}+\left(2-2r_1+k_1\right)v_{0,j} \quad (25a)$$
$$-k_1\left(v_{0,j}\right)^3-k_1v_{0,j}\left(u_{0,j}\right)^2-3k_1\left(v_{0,j}\right)^2u_{0,j}+3k_1u_{0,j}-3k_1\left(u_{0,j}\right)^3$$

## Conclusions

In this study, we concluded that the Crank – Nicholson Schemes without averaging and with averaging of $f(u,v)$ are more accurate than the explicit scheme which is simpler and needs less time and storage . Crank – Nicholson scheme with averaging of $f(u,v)$ is more accurate than Crank – Nicholson scheme without averaging of $f(u,v)$ but it doubles the time steps and needs more storage as shown in table (1) below. The figures (1), (2) and (3) show a comparison among explicit, Crank-Nicholson without averaging and Crank-Nicholson with averaging schemes. The figures (4) and (5) show a comparison between explicit scheme and Crank-Nicholson without averaging scheme when $a=10,\ b=10,\ n=101,\ m=1001,\ h=0.2,\ k=0.01,\ r=0.4$ . From the figures we concluded that the solutions converge to the steady state solution $u=0,\ v=0$ when gets large.


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**Table (1)**

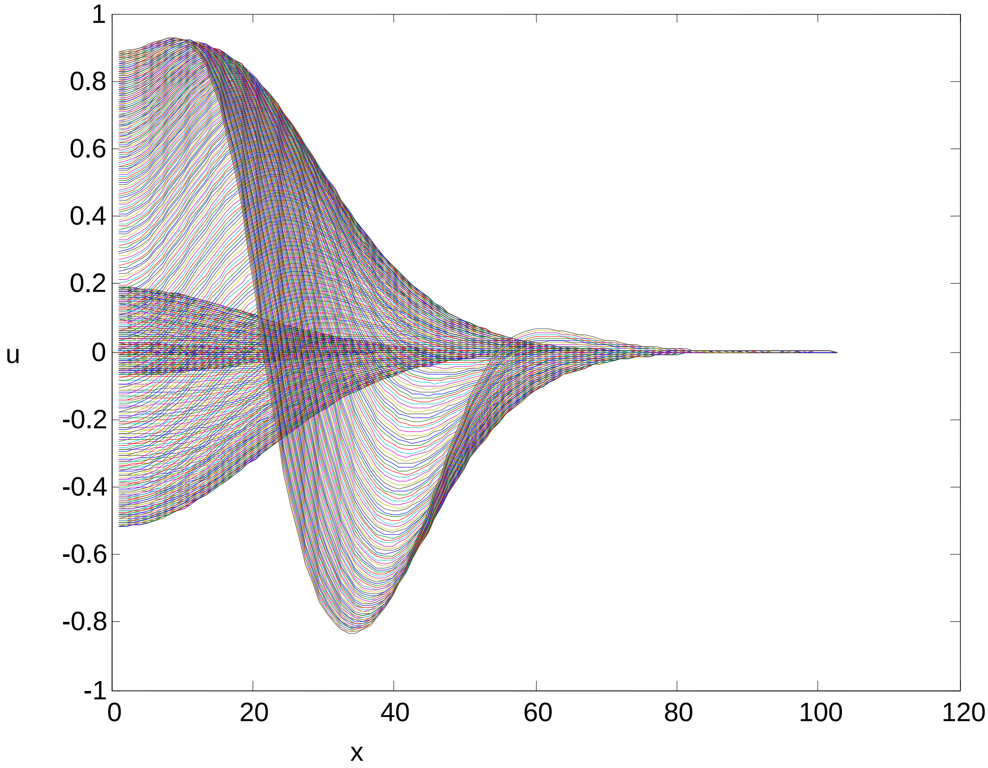
| **Explicit Scheme** | **Crank – Nicholson without averaging** | **Crank-Nicholson with averaging** |
|---|---|---|
| 0.01000 | 0.01000 | 0.01000 |
| 0.00192 | 0.00488 | 0.00689 |
| 0.00357 | 0.00334 | 0.00515 |
| 0.00191 | 0.00266 | 0.00412 |
| 0.00225 | 0.00227 | 0.00348 |
| 0.00161 | 0.00201 | 0.00304 |
| 0.00172 | 0.00182 | 0.00273 |
| 0.00149 | 0.00167 | 0.00241 |
| 0.00144 | 0.00155 | 0.00231 |
| 0.00131 | 0.00145 | 0.00217 |
| 0.00126 | 0.00121 | 0.00204 |
| 0.00117 | 0.00123 | 0.00164 |
| 0.00112 | 0.00118 | 0.00185 |
| 0.00106 | 0.00113 | 0.00177 |
| 0.00101 | 0.00106 | 0.00161 |
| 0.00965 | 0.00104 | 0.00164 |
| 0.00923 | 0.00101 | 0.00158 |
| 0.00883 | 0.00097 | 0.00153 |
| 0.00847 | 0.00064 | 0.00148 |
| 0.00812 | 0.00061 | 0.00144 |
| 0.00780 | 0.00086 | 0.00131 |
| 0.00749 | 0.00086 | 0.00136 |
| 0.00720 | 0.00084 | 0.00133 |
| 0.00693 | 0.00081 | 0.00121 |
| 0.00667 | 0.00079 | 0.00013 |

**Table (1) shows a comparison among explicit, Crank-Nicholson without averaging and Crank-Nicholson schemes with averaging of $u$ when** $a=1,\ b=1,\ n=11,\ m=251,\ r=0.4,\ k=0.004,\ h=0.1$.

**Figure (1)**

**explains the solution function $u$ of the system by using explicit scheme when**

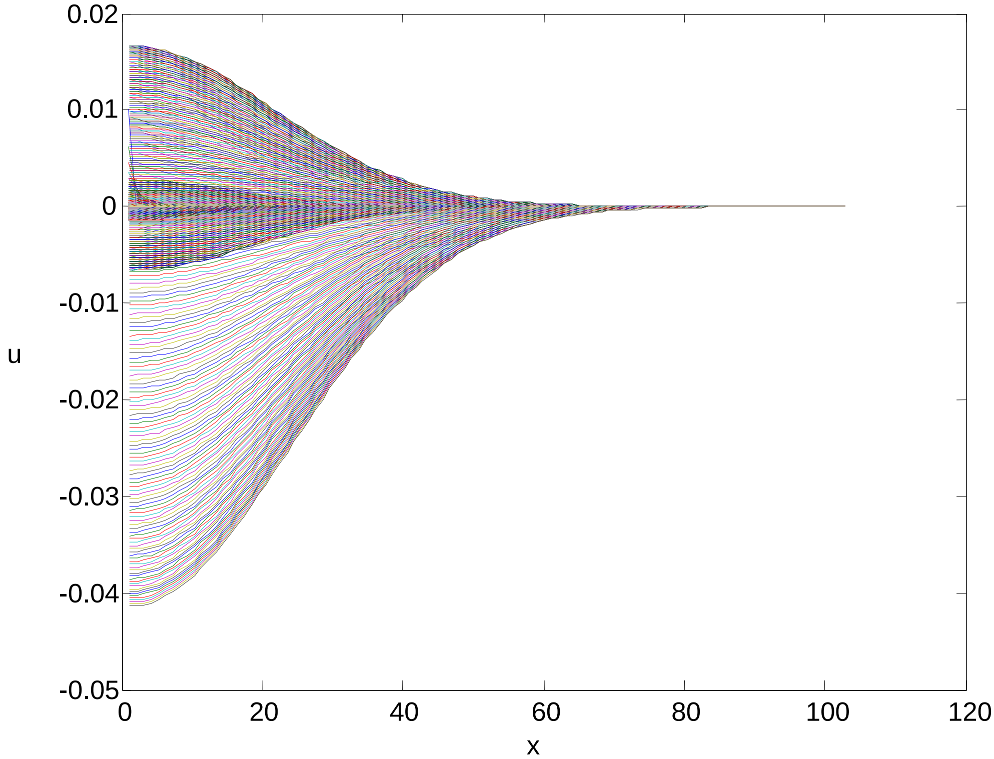
$a = 10,\ b = 10,\ n = 101,\ m = 1001,\ h = 0.2,\ k = 0.01,\ r = 0.4$




**Figure(2)**

**explains the solution function $u$ of the system by using Crank- Nicholson scheme without averaging when**

$a = 10,\ b = 10,\ n = 101,\ m = 1001,\ h = 0.2,\ k = 0.01,\ r = 0.4$.



**Figure(3)**

**explains the solution function $u$ of the system by using Crank-Nicholson with averaging when**

$a=10,\ b=10,\ n=101,\ m=1001,\ h=0.2,\ k=0.01,\ r=0.4$.

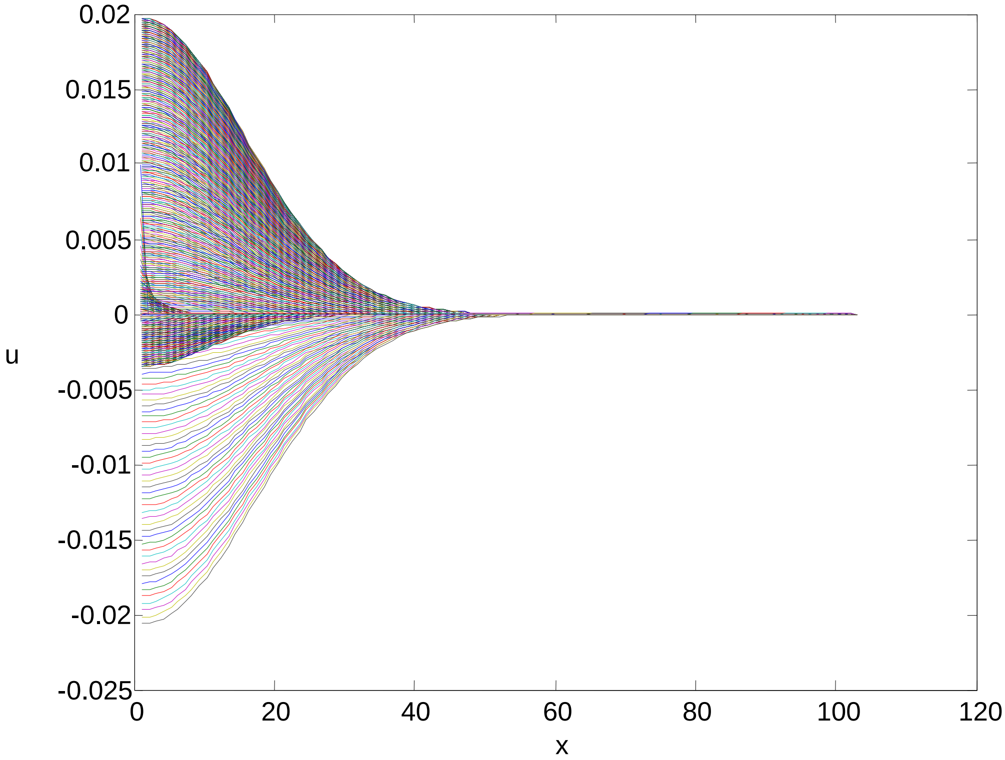



**Figure (4)**

**explains the solution function $v$ of the system by using explicit scheme when**

$a=10,\ b=10,\ n=101,\ m=1001,\ h=0.2,\ k=0.01,\ r=0.4$.

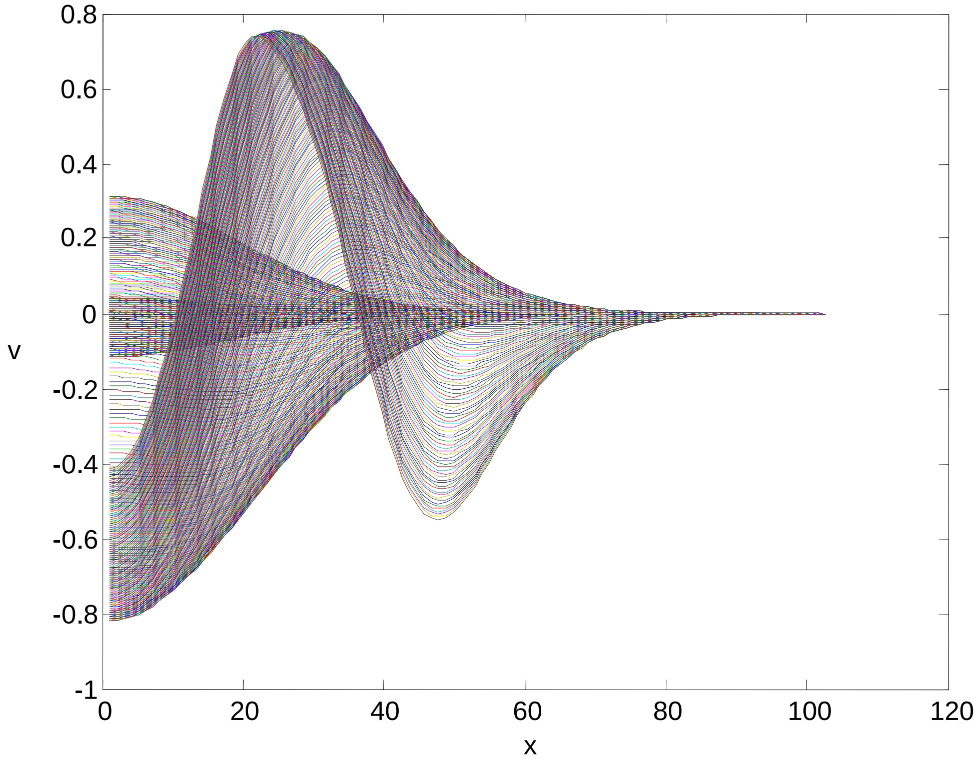


**Figure (5)**

**explains thee solution function $v$ of the system by using Crank-Nicholson scheme without averaging when**

$a=10,\ b=10,\ n=101,\ m=1001,\ h=0.2,\ k=0.01,\ r=0.4$.

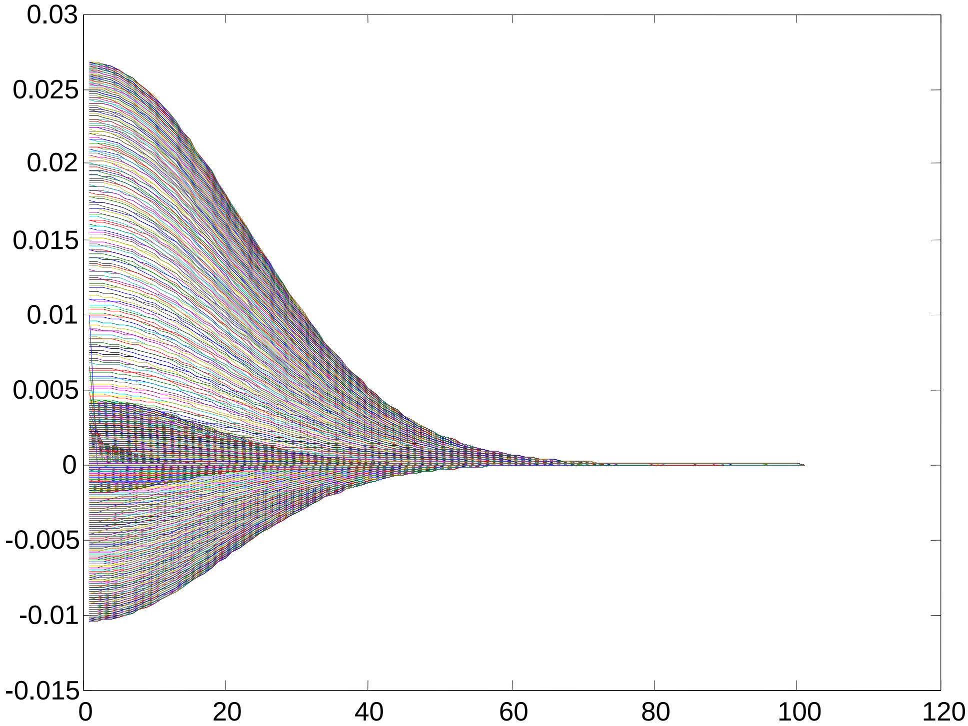
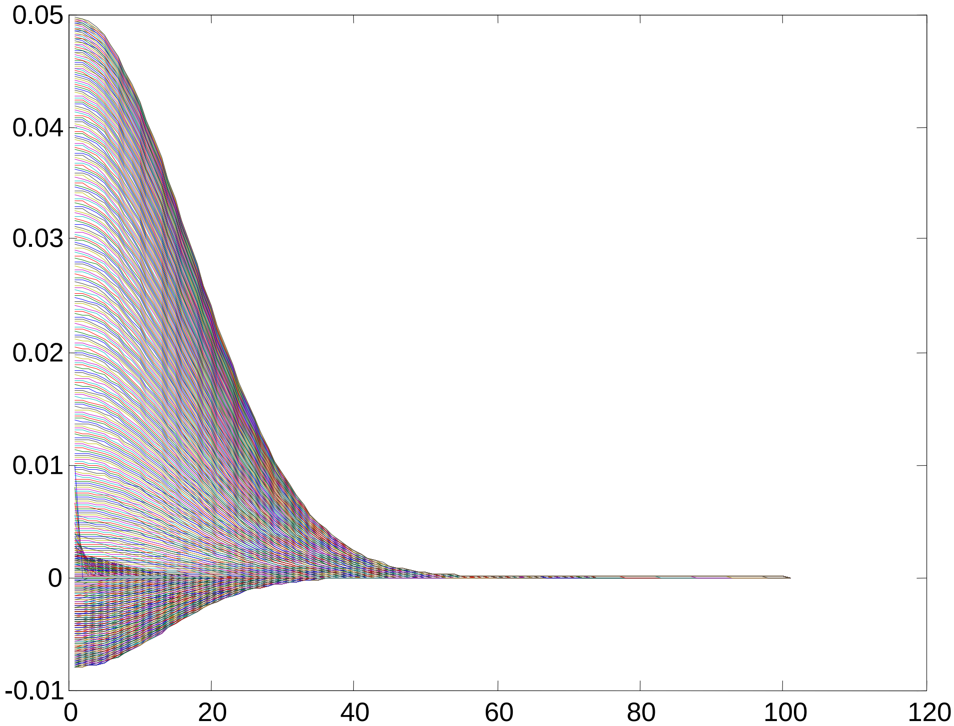



**Figure (6)**

**explains thee solution function $v$ of the system by using Crank-Nicholson scheme with averaging when**

$a=10,\ b=10,\ n=101,\ m=1001,\ h=0.2,\ k=0.01,\ r=0.4$.



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# الحل العددي لنظام الانتشار – التفاعل المتذبذب من النوع $\lambda-\omega$


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**الملخص**

تم حل نظام الانتشار – التفاعل المتذبذب من النوع $\lambda-\omega$ باستخدام طريقتين من طرائق الفروقات المنتهية . الأولى هي الطريقة الصريحة والثانية هي طريقة Crank – Nicholson الضمنية في حالتين : الأولى بأخذ المعدل لـ $f(u,v)$ ، والثانية بدون اخذ المعدل لـ $f(u,v)$ . إذ بينت المقارنة إن طريقة Crank – Nicholson هي أفضل من الطريقة الصريحة وان طريقة Crank – Nicholson بأخذ المعدل لـ $f(u,v)$ هي أكثر دقة ولكنها تحتاج إلى زمن أكثر والى خطوات زمنية وخزن مضاعف من طريقة Crank – Nicholson بدون اخذ المعدل لـ $f(u,v)$ .